Coding Strategies for UWB Interference-Limited Peer-To-Peer Networks

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Younes Souilmi, Raymond Knopp and Giuseppe Caire *
Institut Eurécom
2229 route des Crêtes, B.P. 193
06904 Sophia Antipolis Cedex
France

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Abstract

In this work we study the achievable rates of memoryless signaling strategies adapted to UltraWideBand (UWB) multipath fading channels. We focus on strategies which do not have explicit knowledge of the instantaneous channel realization, but may have knowledge of the channel statistics. We evaluate the average mutual information of general binary flash-signaling and achievable rates for m-PPM as a function of the channel statistics. Finally, we briefly examine the robustness of flash-signaling for interference-limited systems.

Notations

Throughout the paper, small letters $'a'$ will be used for scalars, capital letters $'A'$ for vectors, and bold capital letters $'\mathbf{A}'$ for matrices.

1. Introduction

In this work, we consider achievable rates for transmission strategies suited to *Ultra-wideband (UWB)* systems and focus non-coherent receivers (i.e. those which do not perform channel estimation, but may have prior knowledge of the second-order channel statistics). Here we take a UWB system to be loosely defined as any wireless transmission scheme that occupies a bandwidth between 1 and 10 GHz and more than 25% of it's carrier frequency in the case of a passband system.

The most common UWB transmission scheme is based on transmitting information through the use of short-term impulses, whose positions are modulated by a binary information source [1]. This can be seen as a special case of *flash signaling* coined by Verdu in [6]. Similar to direct-sequence spread-spectrum, the positions can further be modulated by a non-binary sequence (known as a *time-hopping sequence*) for mitigating inter-user interference in a multiuser setting [2]. This type of UWB signaling is a promising candidate for military imaging systems as well as other non-commercial sensor network applications because of its robustness to interference from signals (potentially from other non-UWB systems) occupying the same bandwidth. Based on recent documentation from the FCC it is also being considered for commercial adhoc networking applications based on peer-to-peer communications, with the goal being to provide low-cost high-bandwidth connections to the internet from small handheld terminals in both indoor and outdoor settings. Proposals for indoor wireless personal area networks (WPAN) in the 3-5 GHz band (802.15.3) are also considering this type of transmission scheme.

In this work, we focus on the case of non-coherent detection since it is well known [7][8] that coherent detection is not required to achieve the *wideband* AWGN channel capacity, $C_\infty = \frac{P_R}{N_0 \ln 2}$ bits/s, where P_R is the received signal power in watts, and N_0 is the noise power spectral density. In [8] Telatar and Tse showed this to be the case for arbitrary channel statistics in the limit of infinite bandwidth and infinite carrier frequency. Their transmission model was based on frequency-shift keying (FSK) and it was shown that channel capacity is achieved using very impulsive signals.

In [6] Verdu addresses the spectral efficiency of signaling strategies in the wideband regime under different assumptions regarding channel knowledge at the transmitter and receiver. The characterization is in terms of the minimum energy-per-bit to noise spectral density ratio $(E_b/N_0)_{\min}$ and the wideband slope $\mathcal{S}_0$. The latter quantity is measured in bits/s/Hz/3dB and represents growth of spectral efficiency at the origin as a function of E_b/N_0. Verdu's work is fundamental to our problem since is shows that approaching C_∞ with non-coherent detection is impossible for practical data rates (>100 kbit/s) even for the vanishing spectral efficiency of UWB systems. This is due to the fact that $\mathcal{S}_0$ is zero at the origin for non-coherent detection. To get an idea of the loss incurred, consider a system with a 2GHz bandwidth and data rate of 20 Mbit/s (this would correspond to a memoryless transmission strategy for channels with a 50ns delay-spread) yielding a spectral-efficiency of .01 bits/s/Hz. For Rayleigh statistics the loss in energy efficiency is on the order of 3dB, which translates into a factor 2 loss in data rate compared to a system with perfect channel state information at the receiver. The loss becomes less significant for lower data rates and/or higher bandwidths.

The main goal of this work is to examine under what conditions different non-coherent signaling strategies can approach the wideband channel capacity with perfect channel knowl-

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edge at the receiver subject to a large but finite bandwidth constraint and different propagation conditions. Section II deals with the underlying system model for transmission and reception as well as the channel model. In section III we evaluate expressions for the achievable information rates of different signaling schemes based on reasonably simple analog filter receivers. In section IV we present some numerical evaluations of the expressions from section III. Finally in section V we examine some issues related to multiple-access interference.

2. System Models

We restrict our study to strictly time-limited memoryless real-valued signals, both at the transmitter and receiver. The time-limited and memoryless assumptions are made possible due to the virtually unlimited bandwidth of UWB signals. The transmitted pulse, of duration T_p, is passed through a linear channel, $h(t,u)$, representing the response of the channel at time t to an impulse at time u. We assume that the impulse response of the channel is of duration $T_d \gg T_p$. For examples, the channel is further assumed to be a zero-mean process.
The received signal bandwidth W is roughly $1/T_p$, in the sense that the majority of the signal energy is contained in this finite bandwidth. The received signal is given by

$$r(t) = \int_0^{T_p} x(u)h(t,u)du + z(t) \tag{1}$$

where $z(t)$ is white Gaussian noise with power spectral density $N_0/2$. The channel is further assumed to satisfy

$$\int_0^{T_d+T_p} \int_0^{T_p} h^2(t,u)dtdu < \infty \tag{2}$$

which rules out impulsive channels and practically models the bandlimiting nature of analog transmit and receive chains. The transmitted signal is written as

$$x(t) = \sum_{k=0}^{N} s(u_k)\sqrt{E_s}p\left(t - kT_s\right) \tag{3}$$

where k is the symbol index, T_s the symbol duration, $E_s = PT_s$ the transmitted symbol energy, $u_k \in \{1,\ldots,m\}$ is the transmitted symbol at time k, $p(t)$ and $s(u_k)$ are the assigned pulse and amplitude for symbol u_k. For all k, $p(t)$ is a unit-energy pulse of duration T_p. The considered model encompasses modulation schemes such as flash signaling, m-ary PPM, amplitude, and differential modulations. A guard interval of length T_d is left at the end of each symbol (from our memoryless assumption) so that $T_s \geq T_p + T_d$. From the point of view of spectral efficiency, we have that $\frac{E_b}{N_0} = \frac{P}{N_0 C(P/N_0)}$, where $C(P/N_0)$ is the average mutual information of the underlying signaling scheme as a function of the SNR.
The large bandwidths considered here (>1GHz) provide a high temporal resolution and enable the receiver to resolve a large number of paths of the impinging wavefront. Providing that the channel has a high diversity order (i.e. in rich multipath environments), the total channel gain is slowly varying compared to its constituent components. It has been shown [3, 4, 5] through measurements that in indoor environments, the UWB channel can contain several hundreds of paths of significant strength. We may assume, therefore, that for all practical purposes, the total received energy should remain constant at its average path strength, irrespective of the particular channel realization. Variations in the received signal power will typically be caused by shadowing rather than fast fading.
The finite-energy random channel may be decomposed as

$$h(t,u) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} h_{i,j}\theta_j(u)\phi_i(t) \tag{4}$$

where $h_{i,j}$ are the projections of the channel on the the input and output eigenspaces, $\{\theta_j(u)\}$ is the set of eigenfunctions (for $L^2(0,T_p)$) of the transmit pulse and $\{\phi_i(t)\}$ is the set of eigenfunctions (for $L^2(0,T_p+T_d)$) of the received signal. Since the input in (3) is one-dimensional, the most appropriate choice for $p(t)$ is the one which maximizes the expected energy of the channel output

$$p(t) = \underset{f(t)}{\operatorname{argmax}}\, \mathrm{E}\left(\int_0^{T_p} h(t,u)f(u)du\right)^2 = \theta_1(t) \tag{5}$$

where $\theta_1(t)$ is the eigenfunction corresponding to the maximum eigenvalue, μ_1, of the input cross-correlation function

$$R_{\mathrm{i}}(u,u') = \mathrm{E}\int_0^{T_s} h(t,u)h(t,u')dt = \int_0^{T_s} R_h(t,t;u,u')dt \tag{6}$$

and $R_h(t,t';u,u') = \mathrm{E}h(t,u)h(t',u')$. The use of this input filter is conditioned on the emmision requirments of UWB systems, and thus it may not be possible to satisfy the maximal energy solution in practice.
The above decomposition allows us to write (1), for each symbol k, as the equivalent channel

$$r_{k,i} = \sqrt{\mu_1}h_i s(u_k) + z_i, i = 1, ..., \infty \tag{7}$$

where z_i is $\mathcal{N}(0, N_0/2)$. For notational convenience we have dropped the index corresponding to the input projection from h_{ij} since we are constrained one-dimensional inputs. Furthermore, if we choose the output eigenfunctions to be the solution to

$$\lambda_i\phi_i(t) = \int_0^{T_d+T_p}\int_0^{T_p}\int_0^{T_p} R_h(t,u;t',u')\theta_1(u)\theta_1(u')\phi_i(t')dudu'dt' \tag{8}$$

we have that the h_i are uncorrelated and have variance λ_i.
Because of the bandlimiting nature of the channels in this study, the channel will be characterized by a finite number, D, of significant eigenvalues which for rich environments will be roughly equal to $1 + 2WT_d$, in the sense that a certain proportion of the total channel energy will be contained in these D components. Under our rich scattering assumption D is limited by bandwidth and not insufficient scattering and we may in some cases make the following approximation

$$\mu_1 \sum_{i=1}^{D} h_i^2 \approx 1 \tag{9}$$

for all channel realizations. This assumption essentially says that the received signal energy is not impaired by signal fading due to the rich scattering environment.

For notational convenience, we will assume that the eigenvalues are ordered by decreasing amplitude. An example of an eigenvalue distribution is shown in Fig. 1. This corresponds to an exponentially decaying multipath intensity profile with delay-spread 50ns filtered by a window function of width 1ns, resulting in a system bandwidth of approximately 1 GHz.

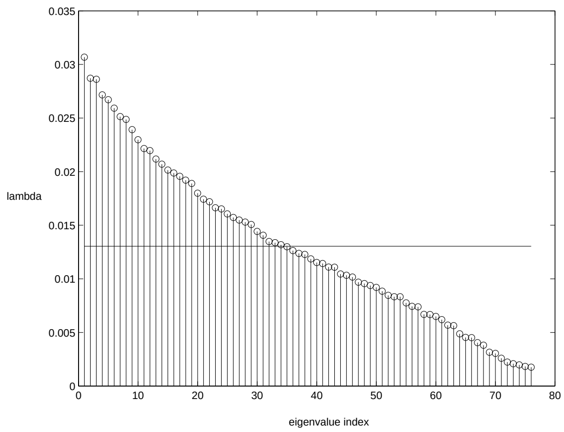


Figure 1: Example Eigenvalue Distribution: $T_d = 50$ ns, $W \approx 1$ GHz.

3. Non-Coherent Detection

In this section we consider non-coherent receivers, that may or may not have access to the second-order channel statistics. The motivation for such this study is to derive receivers that are reasonable from an implementation point of view. We particularly focus on solutions whose front-end can be implemented with analog technology, as shown in Fig. 2. We assume that the transmitter does not have any side information about the channel and that is constrained to the use of flash-like signaling. We first numerically compute the average mutual information for such a system, then we derive a lower bound on the achievable rates for three receivers, based on energy-detection with m-ary PPM modulation, using different front-end filters. This modulation can be seen as a specially-designed channel code for flash-signaling.

3.1 Average Mutual Information

From the results of Verdu in [6] we have in our rich scattering case that $(E_b/N_0)_{\min}$ is $\frac{\ln 2}{\mu_1 \mathrm{E} \sum_{i=1}^{D} h_i^2} \approx \ln 2$. In the case of vanishing spectral efficiency, binary flash-signaling is first-order optimal, and achieves this minimal E_b/N_0. Using the notation from the previous section, we express the binary flash signaling scheme as

$$u_k = \begin{cases} 1 & \text{with probability } \eta \\ 0 & \text{with probability } (1-\eta) \end{cases} \tag{10}$$

$s(0) = 0$, $s(1) = \sqrt{\frac{E_s}{\eta}}$, and $T_s = T_d + T_p$. We assume that the h_i are Gaussian ergodic sequences, which implies that the system's temporal resolution is not fine enough to resolve all the degrees of freedom of the considered channel and that the projection of $h(t,u)$, on each of the kernel's directions, is the combination of a relatively large number of independent multipath components. Measurements of UWB channels[3] have shown that channel components can be considered to fade according to Rayleigh statistics, indicating that this assumption is quite reasonable. Conditioned on u_k, R_k is a zero-mean Gaussian vector with covariance matrix $\mathrm{E}\left[R_k R_k^T\right] = \mathrm{diag}(s(u_k)E_s\lambda_i + \frac{N_0}{2})$. It is easily shown that

$$\begin{aligned} I(U;R) &= -\frac{1}{T_s} \mathop{\mathrm{E}}_{Y}\Big[\eta \log\left(\eta + (1-\eta)\sqrt{\det(\mathbf{I} + \mathrm{A}^{-1})} \exp\left(-\tfrac{1}{2} Y^T \mathbf{A}^{-1} Y\right)\right) \\ &+ (1-\eta)\log\left((1-\eta) + \frac{\eta}{\sqrt{\det(\mathbf{I}+\mathrm{A}^{-1})}} \exp\left(\tfrac{1}{2} Y^T (\mathbf{A}+\mathbf{I}))^{-1} Y\right)\right)\Big] \text{ bits/s} \end{aligned} \tag{11}$$

with Y a zero-mean gaussian random vector with covariance matrix $\mathbf{I}$ and $\mathbf{A} = \mathrm{diag}(\frac{N_0 \eta}{2E_s\lambda_i}))$. This is easily computed numerically.

3.2 m-ary PPM with Energy Detection

In this section we consider three versions of energy detectors, two involving time-varying filters, and a third based on a simple time-invariant filter. We use base-band m-ary PPM signals to transmit the information bits. Each PPM symbol corresponds to choosing one out of m symbol times in which to emit the transmit pulse $p(t)$, which is a special case of the flash signaling system described above with $\eta = 1/m$. In practice, we would consider a coded modulation scheme with a binary code mapped to m-PPM symbols. Here, $T_s = T_p + T_d$, so that the channel can be considered to be memoryless. The general coding scheme is shown in Fig. **??**, where here we ignore the scrambling (time-hopping) sequence required for multiuser operation.

The data is encoded using a randomly generated codebook $\mathcal{C} = \{C_1, C_2, \ldots, C_M\}$ of cardinality M and codeword length N. Each codeword C_l is a sequence $C_l = (c_{1,l}, c_{2,l}, \ldots, c_{N,l})$ of m-PPM symbols. Let C_w be the transmitted codeword, using the notations of model (3) we have $u_k = c_{k,w}$, and $s(u_k) = 1$. For all $n \in [1,N]$ and $k \in [1,m]$ let

$$\begin{aligned} R_{n,k} &= [i \in [0,D]\,, \ < r(t), \phi_{i,n}(t-(k-c_{n,w})T_s) >] \\ &= S_{n,k} + Z_{n,k} \end{aligned} \tag{12}$$

where $S_{n,k}$ and $Z_{n,k}$ are the signal and noise components of $R_{n,k}$, and w denotes the index of the transmitted codeword.

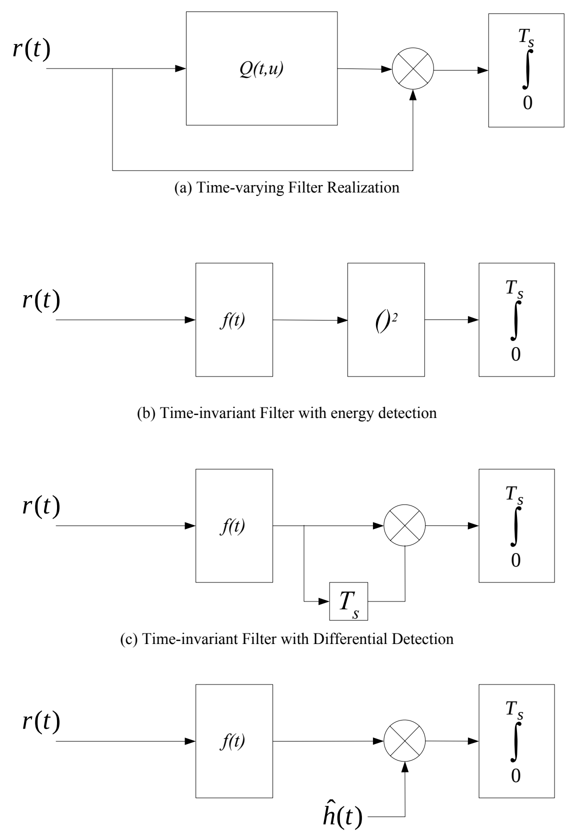
$$Z_{n,k} = [i \in [0,D]\,, \ < z(t), \phi_{i,n}(t-(k-c_{n,w})T_s) >] \tag{13}$$

$Z_{n,k}$ is a Gaussian random vector with mean zero and covariance matrix $\mathbf{K_z} = \frac{N_0}{2}\mathbf{I}$. Using the same reasoning as in the previous section, we will assume that $S_{n,k}$ has a Gaussian distribution.

Optimal Time-Variant Filter- Consider the weighted energy detector used for all the possible pulse emission positions at each symbol,

$$q_{n,k} = R_{n,k} Q^{-1} R_{n,k}^T \tag{14}$$

with $Q = \mathbf{diag}(\frac{N_0}{2} + E_s\mu_1\lambda_i)$ This front-end is equivalent to passing the received signal through the time-variant linear



(a) Time-varying Filter Realization

(b) Time-invariant Filter with energy detection

(c) Time-invariant Filter with Differential Detection

(d) Time-invariant Filter with Imperfect Coherent Detection

Figure 2: Receiver Structures

filter [11]

$$
\begin{aligned}
Q(t,u) &= \sum_{i=1}^{D} \frac{1}{\frac{N_0}{2} + E_s \mu_1 \lambda_i} \, \phi_{n,i}(t)\phi_{n,i}(u) \\
q_{n,k} &= \int_{t_{n,k}}^{t_{n,k}+T_d+T_p} r(t) \int_{t_{n,k}}^{t_{n,k}+T_d+T_p} Q(t,u) r(u) \, du \, dt
\end{aligned}
$$

which is optimum for symbol-by-symbol detection. The decoder forms the decision variables

$$
q_k = \frac{1}{N} \sum_{n=1}^{N} q_{k,n} \tag{15}
$$

and uses the following threshold rule to decide on a message: if q_k exceeds a certain threshold ρ for exactly one value of k, say $\hat{k}$, then it will declare that $\hat{k}$ was transmitted. Otherwise, it will declare a decoding error. This is the same sub-optimal decoding scheme considered in [8].
An upper-bound on the decoding error probability is then given by the following theorem

Theorem 1 *The probability of codeword error is upper bounded by*

$$
\Pr[\text{error}] \leq M \min_{t>0} \exp -N \left[t\rho - \ln \left((1-\eta)(1-2t)^{-\frac{D}{2}} + \right.\right. \tag{16}
$$

$$
\left.\left. \eta \prod_{i=1}^{D} \left(1 - \frac{2t}{1 + \frac{2E_s \mu_1 \lambda_i}{N_0}} \right)^{-\frac{1}{2}} \right) \right] \tag{17}
$$

with $\rho = (1-\epsilon)$, *and* $\eta = 1/m$.

Proof: The decision variable for the transmitted codeword C_w is given by

$$
\frac{1}{N} \sum_{n=1}^{N} (S_{n,w} + Z_{n,w}) \, Q^{-1} \, (S_{n,w} + N_{n,w})^T \tag{18}
$$

by the ergodicity of the noise process, this time-average will exceed the threshold with probability arbitrarily close to 1 for any $\epsilon > 0$ as N gets large. For all $k \neq w$ we bound the probability $\Pr[q_k \geq \rho]$ using a Chernoff bound

$$
\begin{aligned}
\Pr[q_k \geq \rho] &= \Pr[N q_k \geq N\rho] \\
&\leq \min_{t>0} e^{-tN\rho} \prod_{n=1}^{N} \mathrm{E}[\exp(t q_{n,k})]
\end{aligned} \tag{19}
$$

We have that for all $c_{n,k} = c_{n,w}$

$$
\mathrm{E}[\exp(t q_{n,k})] = (1-2t)^{-\frac{D}{2}} \tag{20}
$$

and for all $c_{n,k} \neq c_{n,w}$

$$
\mathrm{E}[\exp(t q_{n,k})] = \prod_{i=1}^{D} \left(1 - \frac{2t}{1 + \frac{2E_s \mu_1 \lambda_i}{N_0}} \right)^{-\frac{1}{2}} \tag{21}
$$

Let l be the number of collisions between codewords C_w and C_k, then we have that

$$
\Pr[q_k \geq \rho] \leq \min_{t>0} \left(e^{-Nt\rho} \left(\prod_{i=1}^{D} \left(1 - \frac{2t}{1 + \frac{2E_s \mu_1 \lambda_i}{N_0}} \right)^{-\frac{1}{2}} \right)^{l} \cdot \right. \tag{22}
$$

$$
\left. (1-2t)^{-\frac{(N-l)D}{2}} \right) \tag{23}
$$

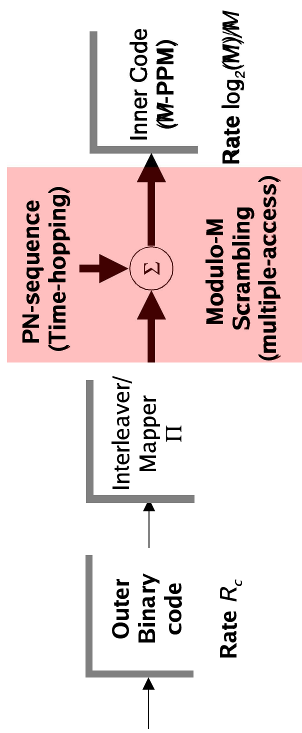


Figure 3: m-PPM Transmitter Structure

Averaging over all the realizations of the randomly generated codebook and using a union bound we obtain the desired result.

The decoding error probability decays to zero exponentially in N as long as the transmission rate R satisfies

$$R = \frac{1}{MNT_s}\log(M)$$
$$\leq \max_{t>0}\frac{1}{T_s}\left(t\rho - \ln\left((1-\eta)(1-2t)^{-\frac{D}{2}} + \right.\right. \quad (24)$$
$$\left.\left.\eta\prod_{i=1}^{D}\left(1-\frac{2t}{1+\frac{2E_s\mu_1\lambda_i}{N_0}}\right)^{-\frac{1}{2}}\right)\right) \quad (25)$$

Due to the finite cardinality of the symbol alphabet our information rate is bounded by

$$R \leq \frac{1}{T_s}\log_2(m) \quad \text{bits/s} \quad (26)$$

White Low-Pass Filter- If the significant eigenvalues λ_i have approximately the same magnitude, we can approximate the time-variant filter in (15) by the white low-pass filter

$$Q(t,u) = \sum_{i=1}^{D}\phi_{n,i}(t)\phi_{n,i}(u) \quad (27)$$

which weighs all significant eigenmodes of the channel equally. We use the same detector and decoding rule as in the previous case, except that we remove the requirement that the channel coefficients are ergodic sequences. An upper-bound on the decoding error probability conditioned on a particular channel realization is then given by

$$\Pr[\text{error}|h(t,u)] \leq M\min_{t>0} e^{-N\left[t\rho+\frac{D}{2}\ln(1-N_0t)-\ln\left((1-\eta)+\eta e^{\frac{t}{1-N_0t}\alpha\mu_1E_s}\right)\right]} \quad (28)$$

with $\rho = (1-\epsilon)E_s\alpha + D\frac{N_0}{2}$, $\alpha = \sum_{i=1}^{D}\lambda_i s_{n,i}^2$, $\eta = 1/m$. Under the rich scattering assumption, the random variable α will be close to 1, so that the error probability is virtually independent of the particular channel realization. We should notice that this result does not assume any particular distribution for the vectors $S_{n,k}$. The proof of this result is similar to that of Theorem 1.

Sub-optimal time-invariant filter- We now consider the case where the receiver does not have access to channel statistics and/or is constrained to use a time-invariant front-end filter because of implementation considerations. The received signal is first filtered by the time-limited unit-energy filter $q(t)$ of duration $T_p/2$, which allows us to reduce the number of degrees of freedom of the received signal while capturing the majority of its information bearing part. If we constrain the channel to be time-invariant, in order to maximize the signal energy at the output of the filter we choose $q(t)$ such that $q(t)*q(t) = \theta_1(t)$, and use $q(t)$ in the transmitter as well. The resulting output,

$$\begin{aligned} y(t) = (r*q)(t) &= ((s+z)*q)(t) \\ &= s_q(t) + z_q(t) \end{aligned} \quad (29)$$

For each potential emission position $t_{n,k} = (n-1)mT_s + kT_s$ we compute the received energy on the interval from $t_{n,k}$ to $t_{n,k} + T_d + T_p$

$$q_{n,k} = \int_{t_{n,k}}^{t_{n,k}+T_d+T_p} y^2(t)\,dt \quad (30)$$

Let $\{\phi_{n,i}(t), i = 1,\cdots,\infty\}$ be the set of basis functions of the Karhunen-Loeve expansion of the noise part of the signal $z_p(t)$ on the interval from t_n to $t_n + T_d + T_p$, then $Z_{n,k}(i) =< z_q(t), \phi_{n,i}(t) >$ with $Z_{n,k}$ is a Gaussian random vector with mean zero and covariance matrix $\mathbf{diag}(\nu_i)$. Let $\sqrt{E_s\lambda_i}s_{n,i} =< s(t), \phi_{n,i}(t) >$ and $\mathrm{E}\sum_{i=1}^{\infty}\lambda_i s_{n,i}^2 = 1$, an upper bound of the decoding error probability conditioned on the channel response is

$$\Pr[\text{error}|h(t)] \leq M\min_{t>0} e^{-N\left[t\rho+\frac{D}{2}\ln(1-N_0t)-\ln\left((1-\eta)+\eta e^{\frac{t}{1-N_0t}\alpha\mu_1E_s}\right)\right]} \quad (31)$$

with $\rho = (1-\epsilon)\alpha E_s + D\frac{N_0}{2}$, $\alpha = \sum_{i=1}^{\infty}\lambda_i s_{n,i}^2$, and $\eta = 1/m$. The derivation of this upper-bound is similar to that of Theorem 1. Under the rich scattering assumption, α will be very close to 1.

3.3 Quasi-Coherent Detection

Noisy Channel Estimation with m-PPM- Consider the case where a noisy estimate of the channel $\tilde{h}(t,u) = h(t,u) + n(t,u)$ is available at the receiver, where $n(t,u)$ is a white Gaussian zero-mean random process with variance σ_h^2. Projecting $\tilde{S}$ over the same set of basis functions as in (7) we obtain

$$\begin{aligned} \tilde{S_n} &= [i\in[0,D],\ < \tilde{s}(t), \phi_{i,n}(t)) >] \\ &= S_{n,w} + N_{n,w} \end{aligned} \quad (32)$$

The received signal is then correlated against the channel estimate for all the possible pulse emission positions in each symbol period as

$$\begin{aligned} q_{n,k} &= R_{n,k}\tilde{S_n}^T \\ &= (S_{n,k} + Z_{n,k})(S_{n,w} + N_{n,w})^T. \end{aligned} \quad (33)$$

An upper bound of the decoding error probability conditioned on the channel realization is given by

$$\Pr[\text{error}|h(t,u)] \leq M\min_{t>0} e^{-N\left[t\rho+\frac{D}{2}\ln(1-\frac{N_0\sigma_h^2t^2}{2})-\frac{\alpha\mu_1E_sN_0t^2}{4}-\ln((1-\eta)+\eta V)\right]} \quad (34)$$

with

$$V = \exp\frac{\alpha\mu_1E_s\left(2t+\sigma_h^2\frac{N_0}{2}t^4\right)\left(\sigma_h^2+\frac{N_0}{2}\right)}{2\left(1-\frac{N_0\sigma_h^2t^2}{2}\right)} \quad (35)$$

and $\rho = (1-\epsilon)\alpha E_s$, $\alpha = \sum_{i=1}^{D}\lambda_i s_{n,i}^2$, and $p = 1/m$. The proof of this result is similar to Theorem 1. Again, under the rich scattering assumption, α will be close to 1 so that the error probability is independent of the particular channel realization.

The decoding error probability depicted in equation (34) decays to zero exponentially in N as long as the transmission rate R satisfies

$$\begin{aligned} R &= \frac{1}{mNT_s}\log(M) \\ &\leq \max_{t>0}\frac{1}{T_s}\left(t\rho + \frac{D}{2}\ln(1-\frac{N_0\sigma_h^2 t^2}{2}) - \frac{E_s N_0 t^2}{4} - \ln\left((1-p)+pV\right)\right. \end{aligned} \tag{36}$$

Differential Modulation- A practical way to obtain an estimate of the channel at the receiver is to use differential antipodal modulation. Using the notations of (3) we have that $T_s = T_d + T_p$, and $u_k = s(u_k)s(u_{k-1}) \in \{-1, 1\}$. Providing the assumption that the channel does not vary between two consecutive symbol times, we can use the received signal at symbol-time $k-1$ as a channel estimate for the following symbol-time k so that

$$\begin{aligned} q_{n,k} &= x_{n,k}R_{n,k}R_{n-1,k}^T \\ &= x_{n,k}\left(S_n + Z_{n,k}\right)\left(S_n + Z_{n-1,k}\right)^T \end{aligned} \tag{37}$$

An upper bound on the decoding error probability conditioned on a particular channel realization is given by

$$\Pr[\text{error}|h(t,u)] \leq \tag{38}$$

$$M \min_{t>0} e^{-N\left[t\rho + \frac{D}{2}\ln(1-\frac{N_0^2 t^2}{4}) - \frac{\alpha\mu_1 E_s t\left(1+\frac{N_0}{2}\right)}{1-\frac{t^2 N_0}{4}} - \ln\left(\frac{1}{2}(1+V)\right)\right]} \tag{39}$$

with

$$V = \exp\frac{t\alpha\mu_1 E_s\left(\frac{t^2N_0^2}{4} - 2\right)}{1-\frac{t^2N_0}{4}} \tag{40}$$

and $\rho = (1-\epsilon)\alpha E_s$ and $\alpha = \sum_{i=1}^{D}\lambda_i s_{n,i}^2$. The proof of this result is similar to Theorem 1.

4. Numerical Results

We show the numerical evaluation of the bounds in the previous sections in Figs. 4,5, where the symbol alphabet size (i.e. m) has been optimized for each SNR. The effective bandwidth of the system is 1 GHz and the channel is an exponentially decaying multipath channel with a delay spread of 50ns (the example of section II). We see that the typical information rate losses with respect to optimal flash signaling are less than a factor 2 with m-PPM and reseasonably simple linear filter analog receivers. We note, however, that significant performance degradation can be expected with differential detection of antipodal modulation compared to flash signaling.

5. Multi-access Interference

The networks which will likely employ UWB signaling, for example *Wireless Personal Area Networks(WPAN)* and *sensor networks*, are characterized not only by a rich scattering propagation environment but also by requirements for adhoc and peer-to-peer communications. This latter requirement has a significant impact on systems design, since the signaling schemes must be robust to strong impulsive interference (from nearby interferers) as shown in Fig. 6. Here we show a

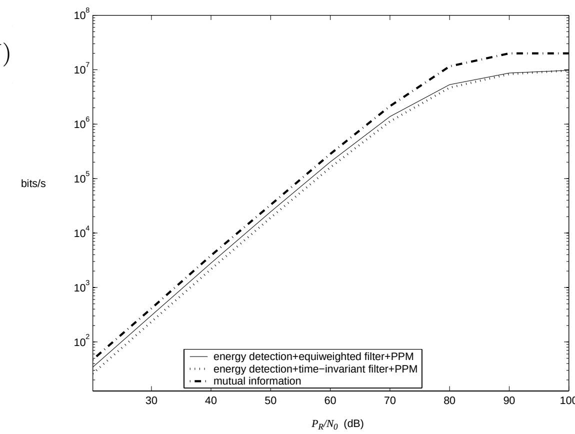


Figure 4: Achievable rates of energy detection based receivers: $T_d = 50$ ns, $W = 1$ GHz

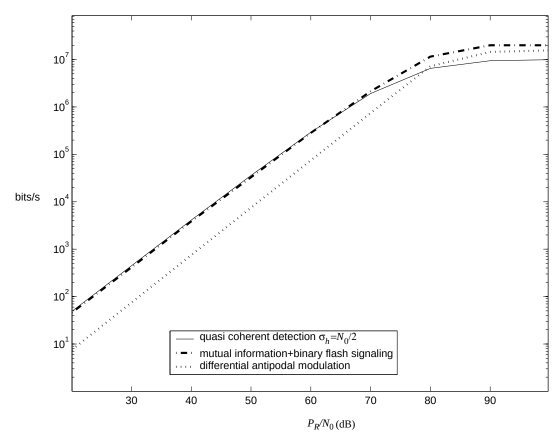


Figure 5: Achievable rates of quasi-coherent detection schemes: $T_d = 50$ ns, $W = 1$ GHz

small network consisting of 2 transmitter-receiver pairs. The receiving nodes are both far from their respective transmitters and suffer from strong interference. In contrast to CDMA networks with a basestation/mobile topology, UWB adhoc networks will likely not benefit significantly from centralized or distributed power control resulting in extreme near-far interference. Even the enourmous processing gains (with coherent detection) of these systems will do little in such an environment. The purpose of this section is to analyze the robustness of flash-signaling with respect to near-far interference.

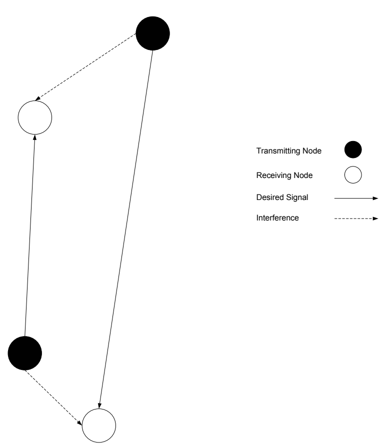


Figure 6: Simple Peer-to-Peer Network Example

Generalizing the model of the previous sections by adding a single interferer we have

$$r_{k,i}^{(1)} = \sqrt{\frac{P_1 T_s \mu_1}{\rho}} h_{k,i}^{(1)} s^{(1)}(u_k^{(1)}) + \underbrace{\sqrt{\frac{P_2 T_s \mu_1}{\rho}} h_{k,i}^{(2)} s^{(2)}(u_k^{(2)})}_{\text{interference}} + z_{k,i}^{(1)}, \quad i = 1, \ldots, \infty \tag{41}$$

We have assumed for simplicity that the interferer is synchronous to the desired signal. Ignoring the constraint on the data rate from the point-of-view of the receiver of $s^{(2)}(u_k)$ we can show that a decoder using knowledge of the codebooks of both the desired signal and the inteferer is governed by the following rate region

$$\begin{aligned} R_1 &\leq I(U^{(1)}; R^{(1)}, U^{(2)}) \\ R_1 + R_2 &\leq I(U^{(1)}, U^{(2)}; R^{(1)}) \end{aligned} \tag{42}$$

This region reflects the influence of the data rate of user 2 on the the achievable rate of user 1, when it is considered as "decodable" interference. It is most meaningful in the case of very strong interference, where we first decode the interferer (provided $R_2 \leq I(U^{(2)}; R^{(1)})$), and then decode the desired signal. The rate of the desired user's signal is

$$\begin{aligned} R_1 &\leq I(U^{(1)}; R^{(1)}, U^{(2)}) \\ &= I(U^{(1)}; R^{(1)} | U^{(2)}) \\ &= (1-\eta) I(U^{(1)}; R^{(1)} | U^{(2)} = 0) + \eta I(U^{(1)}; R^{(1)} | U^{(2)} = 1) \end{aligned} \tag{43}$$

Each of the mutual information functionals in (43) can be expressed as in (11), and in the case of very strong interference, the second term will be negligible. We see, therefore, that the influence of the interference is a reduction in throughput by a factor $1-\eta$. To achieve this throughput, however, knowledge of the interfering positions is required. In practice this could be achieved using a threshold rule on the filter output, which is chosen so that the probability of detecting the presence of strong interference (and thus declaring an erasure) is very close to 1 when an interferer is transmitting.

5. Conclusions

In this work we studied the achievable rates of memoryless signaling strategies over ultrawideband (UWB) multipath fading channels. In particular we focused on strategies which do not have explicit knowledge of the instantaneous channel realization, but may have knowledge of the channel statistics. We evaluated the mutual information of general binary flash-signaling and achievable rates for m-PPM as a function of the channel statistics, which can be seen as a practical coded-modulation strategy for implementing flash-signaling. We can conclude that m-PPM combined with simple analog front-end receivers can provide virtually the same data rates as general binary flash-signaling. Finally, we briefly examine the robustness of flash-signaling for interference-limited systems, where it is shown that flash-signaling seems to be an efficient means for combatting severe near-far interference.

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