Black-Box Optimization Benchmarking the IPOP-CMA-ES on the Noisy Testbed

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Comparison to the BIPOP-CMA-ES

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ABSTRACT

We benchmark the IPOP-CMA-ES on the noisy testbed of the BBOB 2010 workshop. The performances of the IPOP-CMA-ES are compared to those of the BIPOP-CMA-ES. Both algorithms are shown to perform comparably on the BBOB noisy testbed.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolution strategies

1. ALGORITHM PRESENTATION

The algorithm Covariance Matrix Adaptation-Evolution Strategy (CMA-ES) [9] is a stochastic search method based on a population. We choose to apply the $(\mu/\mu_w, \lambda)$-CMA-ES [3, 7, 8] in this paper. The Increasing POPulation-size (IPOP) restart policy was proposed for the CMA-ES in [1]. The resulting IPOP-CMA-ES algorithm uses a population doubling in size at each restarts.

We compare the performances of the IPOP-CMA-ES to those of the BIPOP-CMA-ES [4] which was proposed to the BBOB 2009 workshop. The BIPOP-CMA-ES distributes the allocated budget —number of function evaluations— between a doubling population size and a small population size policy. The BIPOP-CMA-ES showed good performances on the function testbeds of the BBOB 2009 workshop.

2. EXPERIMENTAL PROCEDURE

The IPOP-CMA-ES was tested using the same experimental set-up as that of the BIPOP-CMA-ES [4] tested for the BBOB 2009 workshop. In particular parameter c_1 and c_μ, terms of the learning rate c_{cov} are set to one fifth of the values that used for the noiseless testbed.

The only difference with the BIPOP-CMA-ES is that all the budget in terms of number of function evaluations is allocated to the doubling population size policy.

The crafting effort for IPOP-CMA-ES [5] computes to $\mathrm{CrE} = 0$.

3. RESULTS

Results of the CPU timing experiment are given in the paper benchmarking IPOP-CMA-ES on the noiseless testbed.

Results from experiments according to [5] on the benchmark functions given in [2, 6] are presented in Figures 1, 2, 3 and 4 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and tables, depends on a given target function value, $f_{\mathrm{t}} = f_{\mathrm{opt}} + \Delta f_{\mathrm{t}}$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_{t}, summed over all trials and divided by the number of trials that actually reached f_{t} [5, 10]. **Statistical significance** is tested with the rank-sum test for a given target Δf_{t} (10^{-8} in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_{t} (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

The performances of both IPOP-CMA-ES and BIPOP-CMA-ES are pretty close with IPOP-CMA-ES being slightly faster overall but not significantly. The exception is function f_{117} (Ellipsoid function with uniform noise model) in 20-D where the IPOP-CMA-ES is significantly faster but only by a factor smaller than two.

4. REFERENCES

[1] A. Auger and N. Hansen. A restart CMA evolution strategy with increasing population size. In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2005)*, pages 1769–1776. IEEE Press, 2005.

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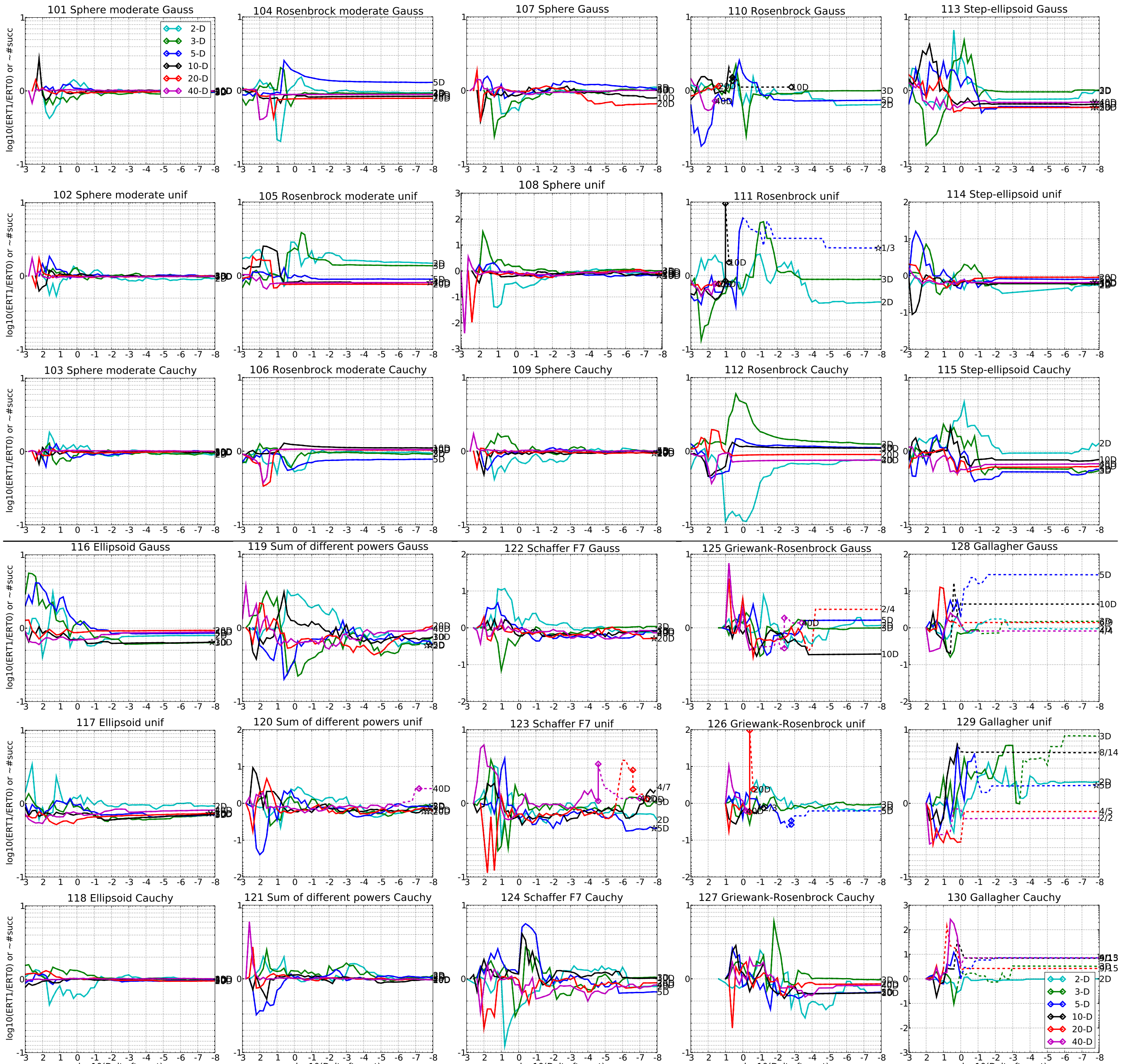

Figure 1: Ratio of the expected running times (ERT) of IPOP-CMA divided by BIPOP-CMA versus $\log_{10}(\Delta f)$ for f_{101}–f_{130} in 2, 3, 5, 10, 20, 40-D. Ratios $< 10^0$ indicate an advantage of IPOP-CMA, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for IPOP-CMA. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \ldots 9\}$ for IPOP-CMA (1st number) and non-zero for BIPOP-CMA (2nd number). Results are statistically significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

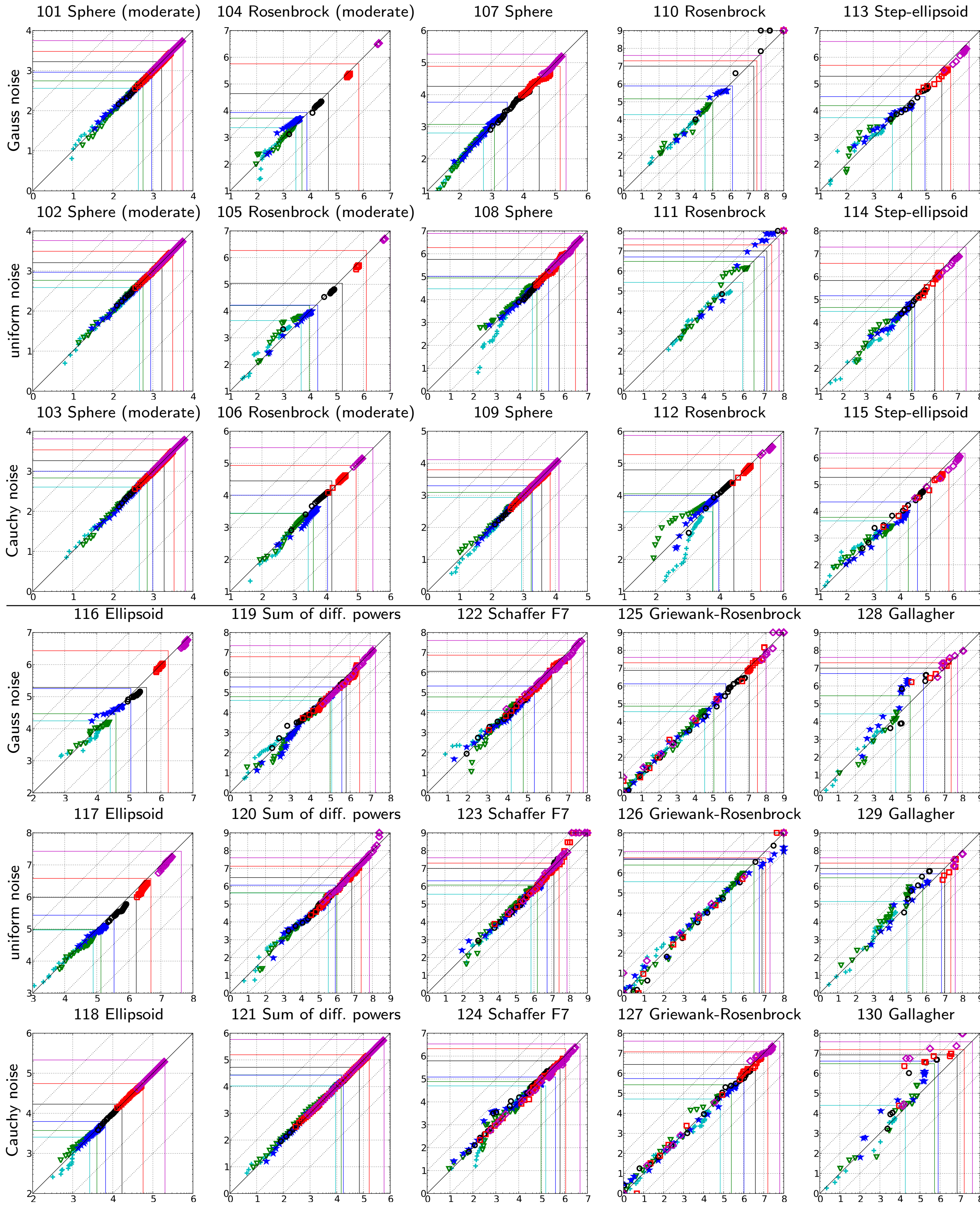

Figure 2: Expected running time (ERT in log10 of number of function evaluations) of IPOP-CMA versus BIPOP-CMA for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_{101}–f_{130}. Markers on the upper or right egde indicate that the target value was never reached by IPOP-CMA or BIPOP-CMA respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□, 40:◇.

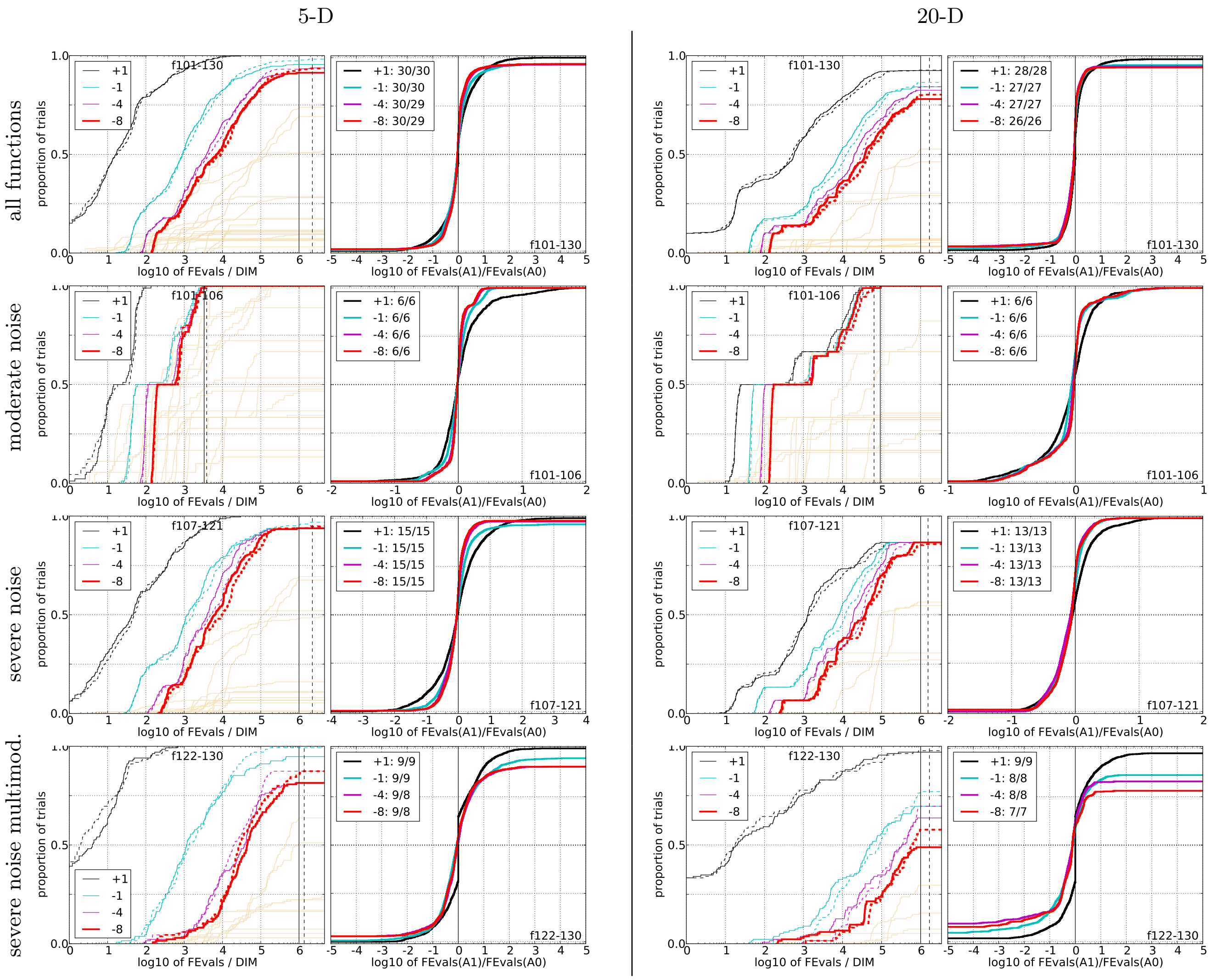

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of necessary function evaluations divided by dimension D (FEvals/D) to reached a target value $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for IPOP-CMA (solid) and BIPOP-CMA (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of IPOP-CMA divided by BIPOP-CMA, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (IPOP-CMA first).

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_{101}	11	37	44	62	69	75	15/15
0: BIP	3.2	3.1	4.6	6.1	8.0	10	15/15
1: IPO	3.3	3.4	4.7	6.0	7.8	9.3	15/15
f_{102}	11	35	50	72	86	99	15/15
0: BIP	2.7	3.0	4.0	5.1	6.3	7.2	15/15
1: IPO	3.4	3.1	4.1	5.1	6.5	7.3	15/15
f_{103}	11	28	30	31	35	115	15/15
0: BIP	3.5	4.7	7.4	13	17	6.9	15/15
1: IPO	3.6	4.0	6.6	12	17	7.1	15/15
f_{104}	173	773	1287	1768	2040	2284	15/15
0: BIP	1.4	1.9	2.0	2.0	1.9	1.8	15/15
1: IPO	1.4	3.4	2.9	2.7	2.5	2.4	15/15
f_{105}	167	1436	5174	10388	10824	11202	15/15
0: BIP	1.7	3.7	1.7	1	1	1	15/15
1: IPO	1.6	3.8	1.6	0.90	0.90	0.90	15/15
f_{106}	86	529	1050	2666	2887	3087	15/15
0: BIP	3.6	4.3	3.2	1.6	1.7	1.7	15/15
1: IPO	3.3	2.5	2.2	1.2	1.3	1.3	15/15
f_{107}	40	228	453	940	1376	1850	15/15
0: BIP	1.7	1	1	1	1	1	15/15
1: IPO	2.1	0.98	1.1	1.3	1.2	1.1	15/15
f_{108}	87	5144	14469	30935	58628	80667	15/15
0: BIP	6.1	1.0	1	1	1	1	15/15
1: IPO	9.1	0.80	0.67	0.77	$0.62^{\downarrow}$	0.69	15/15
f_{109}	11	57	216	572	873	946	15/15
0: BIP	3.5	2.2	1.1	1.1	1.1	1.5	15/15
1: IPO	2.9	2.2	1.2	1.0	1.1	1.5	15/15
f_{110}	949	33625	1.20e5	5.93e5	6.03e5	6.11e5	15/15
0: BIP	1	4.8	3.7	1	1	1	15/15
1: IPO	0.73	8.3	3.4	0.72	0.73	0.74	15/15
f_{111}	6856	6.12e5	8.83e6	2.30e7	3.10e7	3.13e7	3/15
0: BIP	1	2.5	1	1	1	1	3/15
1: IPO	0.78	15	3.9	3.2	2.4	2.4	1/15
f_{112}	107	1684	3421	4502	5132	5596	15/15
0: BIP	4.0	1	1.2	1.3	1.3	1.3	15/15
1: IPO	2.1	1.4	1.4	1.5	1.5	1.5	15/15
f_{113}	133	1883	8081	24128	24128	24402	15/15
0: BIP	1.5	1.3	1.7	1.1	1.1	1.1	15/15
1: IPO	3.7	1.4	1.4	0.67	0.67	0.67	15/15
f_{114}	767	14720	56311	83272	83272	84949	15/15
0: BIP	2.2	1	1	1	1	1	15/15
1: IPO	3.2	0.45	0.48	0.79	0.79	0.80	15/15
f_{115}	64	485	1829	2550	2550	2970	15/15
0: BIP	1.5	2.6	6.5	5.9	5.9	5.7	15/15
1: IPO	1.7	2.4	2.7	3.1	3.1	2.7	15/15
f_{116}	5730	14472	22311	26868	30329	31661	15/15
0: BIP	1.2	2.0	1.9	2.1	2.0	2.0	15/15
1: IPO	3.1	2.3	1.9	1.8	1.7	1.7	15/15
f_{117}	26686	76052	1.10e5	1.37e5	1.73e5	1.92e5	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	1.1	0.95	0.77	0.73	0.67	0.69	15/15
f_{118}	429	1217	1555	1998	2430	2913	15/15
0: BIP	3.2	2.0	1.9	2.1	2.0	1.8	15/15
1: IPO	3.2	2.0	1.9	2.0	1.9	1.7	15/15
f_{119}	12	657	1136	10372	35296	49747	15/15
0: BIP	1.9	1	1	1	1.5	2.3	15/15
1: IPO	1.1	0.35	0.70	0.83	1.0	1.4	15/15
f_{120}	16	2900	18698	72438	3.33e5	5.48e5	15/15
0: BIP	17	1.1	1	1	1	1	15/15
1: IPO	6.0	1.6	0.68	0.69	0.55	0.83	15/15
f_{121}	8.6	111	273	1583	3870	6195	15/15
0: BIP	2.7	1.1	1	1.1	2.0	2.2	15/15
1: IPO	1.9	1.1	1.0	1.1	2.1	2.3	15/15
f_{122}	10	1727	9190	30087	53743	1.11e5	15/15
0: BIP	2.2	1	1	1	1	1	15/15
1: IPO	4.8	0.94	0.44	0.56	0.68	$0.67^{\downarrow}$	15/15
f_{123}	11	16066	81505	3.36e5	6.71e5	2.22e6	15/15
0: BIP	8.1	1	1	1	1	1	15/15
1: IPO	23	0.62	0.52	0.74	0.65	0.45	15/15
f_{124}	10	202	1040	20478	45337	95200	15/15
0: BIP	1.5	1.1	1	1.1	1.2	1	15/15
1: IPO	2.8	1.3	4.0	1.2	0.93	0.65	15/15
f_{125}	1	1	1	2.39e5	2.43e5	2.46e5	15/15
0: BIP	1.1	17	3443	1	1	1	15/15
1: IPO	1	27	2599	0.78	1.3	1.3	15/15
f_{126}	1	1	1	∞	∞	∞	0
0: BIP	1	160	13292	∞	∞	∞	0/15
1: IPO	1	63	10254	*1.21e7*	*1.88e7*	*1.89e7*	2/15
f_{127}	1	1	1	3.42e5	3.89e5	3.95e5	15/15
0: BIP	1	19	2136	1	1	1	15/15
1: IPO	1	15	1542	0.58	0.64	0.65	15/15
f_{128}	111	4248	7808	12447	17217	21162	15/15
0: BIP	2.2	6.9	10	6.6	4.8	3.9	15/15
1: IPO	1.0	14	166	183	132	108	10/15
f_{129}	64	10710	59443	2.85e5	5.11e5	5.80e5	15/15
0: BIP	12	7.1	9.2	3.9	2.2	1.9	13/15
1: IPO	8.5	13	18	6.7	3.8	3.3	11/15
f_{130}	55	812	3034	32823	33889	34528	10/15
0: BIP	1.9	57	55	5.1	5.0	5.0	15/15
1: IPO	1.2	59	321	37	36	35	12/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
f_{101}	59	361	513	700	739	783	15/15
0: BIP	6.1	1.8	1.8	2.1	2.7	3.3	15/15
1: IPO	6.0	1.7	1.7	2.0	2.6	3.2	15/15
f_{102}	231	399	579	921	1157	1407	15/15
0: BIP	1.6	1.6	1.6	1.6	1.8	1.8	15/15
1: IPO	1.6	1.6	1.6	1.6	1.7	1.8	15/15
f_{103}	65	417	629	1313	1893	2464	14/15
0: BIP	5.5	1.6	1.5	1.2	1.2	1.2	15/15
1: IPO	5.5	1.5	1.4	1.2	1.2	1.2	15/15
f_{104}	23690	85656	1.71e5	1.82e5	1.89e5	1.96e5	15/15
0: BIP	10	3.2	1.7	1.6	1.6	1.6	15/15
1: IPO	7.5	2.5	1.3	1.3	1.3	1.2	15/15
f_{105}	1.92e5	6.11e5	6.32e5	6.49e5	6.60e5	6.70e5	15/15
0: BIP	2.7	1	1	1	1	1	15/15
1: IPO	1.9	0.76	0.76	0.77	0.77	0.76	15/15
f_{106}	11480	21668	23746	25470	26492	27360	15/15
0: BIP	1.0	1.3	1.4	1.5	1.5	1.5	15/15
1: IPO	1.0	1.4	1.5	1.5	1.5	1.5	15/15
f_{107}	8571	13582	16226	27357	52486	65052	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	1.1	0.95	1.1	0.96	0.68	0.65	15/15
f_{108}	58063	97228	2.03e5	4.46e5	6.30e5	8.98e5	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	0.72	0.87	0.66	0.77	0.94	1.0	15/15
f_{109}	333	632	1138	2287	3583	4952	15/15
0: BIP	1.2	1.2	1.1	1.1	1.1	1.0	15/15
1: IPO	1.1	1.2	1.1	1.1	1.0	1.00	15/15
f_{110}	∞	∞	∞	∞	∞	∞	0
0: BIP	∞	∞	∞	∞	∞	∞	0/15
1: IPO	∞	∞	∞	∞	∞	∞	0/15
f_{111}	∞	∞	∞	∞	∞	∞	0
0: BIP	∞	∞	∞	∞	∞	∞	0/15
1: IPO	∞	∞	∞	∞	∞	∞	0/15
f_{112}	25552	64124	69621	73557	76137	78238	15/15
0: BIP	1	1.1	1.1	1.2	1.2	1.2	15/15
1: IPO	0.95	0.94	1.0	1.1	1.1	1.1	15/15
f_{113}	50123	3.64e5	5.60e5	5.88e5	5.88e5	5.91e5	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	1.0	0.53	$0.58^{\downarrow}$	$0.59^{\downarrow}$	$0.59^{\downarrow}$	$0.59^{\downarrow}$	15/15
f_{114}	2.08e5	1.12e6	1.45e6	1.57e6	1.57e6	1.58e6	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	$0.59^{\downarrow}$	0.68	0.84	0.91	0.91	0.92	15/15
f_{115}	2405	30268	91749	1.27e5	1.27e5	1.29e5	15/15
0: BIP	1	6.5	3.9	3.0	3.0	3.0	15/15
1: IPO	1.1	4.8	2.2	1.8	1.8	1.9	15/15
f_{116}	4.98e5	6.94e5	8.93e5	1.03e6	1.08e6	1.12e6	15/15
0: BIP	1.4	1.2	1.1	1	1	1	15/15
1: IPO	1.2	1.1	1.00	0.92	0.93	0.93	15/15
f_{117}	1.79e6	2.46e6	2.60e6	2.91e6	3.24e6	3.62e6	15/15
0: BIP	1	1	1	1	1	1	15/15
1: IPO	0.55	**0.61**$^{\star 2\downarrow 2}$	**0.66**$^{\star\downarrow 2}$	**0.69**$^{\star\downarrow}$	**0.71**$^{\star\downarrow}$	**0.72**$^{\star\downarrow 2}$	15/15
f_{118}	6908	11786	17514	26342	30062	32659	15/15
0: BIP	1.9	1.8	1.6	1.5	1.6	1.6	15/15
1: IPO	2.0	1.8	1.7	1.5	1.5	1.5	15/15
f_{119}	2771	29365	35930	4.11e5	1.40e6	1.90e6	15/15
0: BIP	1.6	1	1	1	1.3	1.1	15/15
1: IPO	1.9	0.58	0.69	0.58	**0.59**$^{\star 3\downarrow 2}$	0.97	15/15
f_{120}	36040	1.79e5	2.81e5	1.59e6	6.74e6	1.35e7	13/15
0: BIP	1	1	1	1	1	1	13/15
1: IPO	0.69	0.60	0.74	$0.67^{\downarrow}$	0.69	0.69	15/15
f_{121}	249	769	1426	9304	34434	57404	15/15
0: BIP	1.2	1.0	1.2	1.1	1.3	1.9	15/15
1: IPO	1.3	1.1	1.1	1.1	1.4	1.9	15/15
f_{122}	692	52008	1.40e5	7.93e5	2.00e6	5.82e6	15/15
0: BIP	1.8	1	1	1	1	1	15/15
1: IPO	2.0	0.92	0.74	0.63	0.95	0.64	15/15
f_{123}	1063	5.30e5	1.49e6	5.29e6	2.71e7	1.58e8	0
0: BIP	5.7	1	1	1	1	1	0/15
1: IPO	7.2	0.72	0.61	0.80	0.62	∞*2.0e7*	0/15
f_{124}	192	1959	40840	1.27e5	3.89e5	7.99e5	15/15
0: BIP	1.1	1.0	1	1	1	1	15/15
1: IPO	1.1	0.99	0.75	0.98	0.84	0.78	15/15
f_{125}	1	1	1	2.50e7	8.03e7	8.06e7	4/15
0: BIP	1	383	9.82e6	1	1	1	4/15
1: IPO	1	957	7.10e6	0.79	1.8	1.8	2/15
f_{126}	1	1	1	∞	∞	∞	0
0: BIP	1	5781	∞	∞	∞	∞	0/15
1: IPO	1	5759	∞	∞	∞	∞	0/15
f_{127}	1	1	1	4.43e6	7.27e6	7.43e6	15/15
0: BIP	1	176	8.99e5	1	1	1	15/15
1: IPO	1	267	9.58e5	0.81	0.84	0.85	15/15
f_{128}	1.40e5	1.34e7	1.72e7	1.72e7	1.72e7	1.72e7	9/15
0: BIP	1	1	1	1	1	1	9/15
1: IPO	12	1.0	1.4	1.4	1.4	1.4	6/15
f_{129}	7.81e6	4.13e7	4.15e7	4.18e7	4.21e7	4.24e7	5/15
0: BIP	1	1	1	1	1	1	5/15
1: IPO	0.29	0.30	0.77	0.77	0.77	0.77	4/15
f_{130}	4904	93149	2.52e5	2.54e5	2.55e5	2.57e5	7/15
0: BIP	1.9	33	14	14	14	14	15/15
1: IPO	4.9	76	37	37	37	37	9/15

Table 1: ERT in number of function evaluations divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_{101}–f_{130}. #succ is the number of trials that reached the final target $f_{\rm opt} + 10^{-8}$. 0: BIP is BIPOP-CMA and 1: IPO is IPOP-CMA. #succ is the number of trials that reached the final target $f_{\rm opt} + 10^{-8}$. 0: BIP is BIPOP-CMA and 1: IPO is IPOP-CMA. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 60.

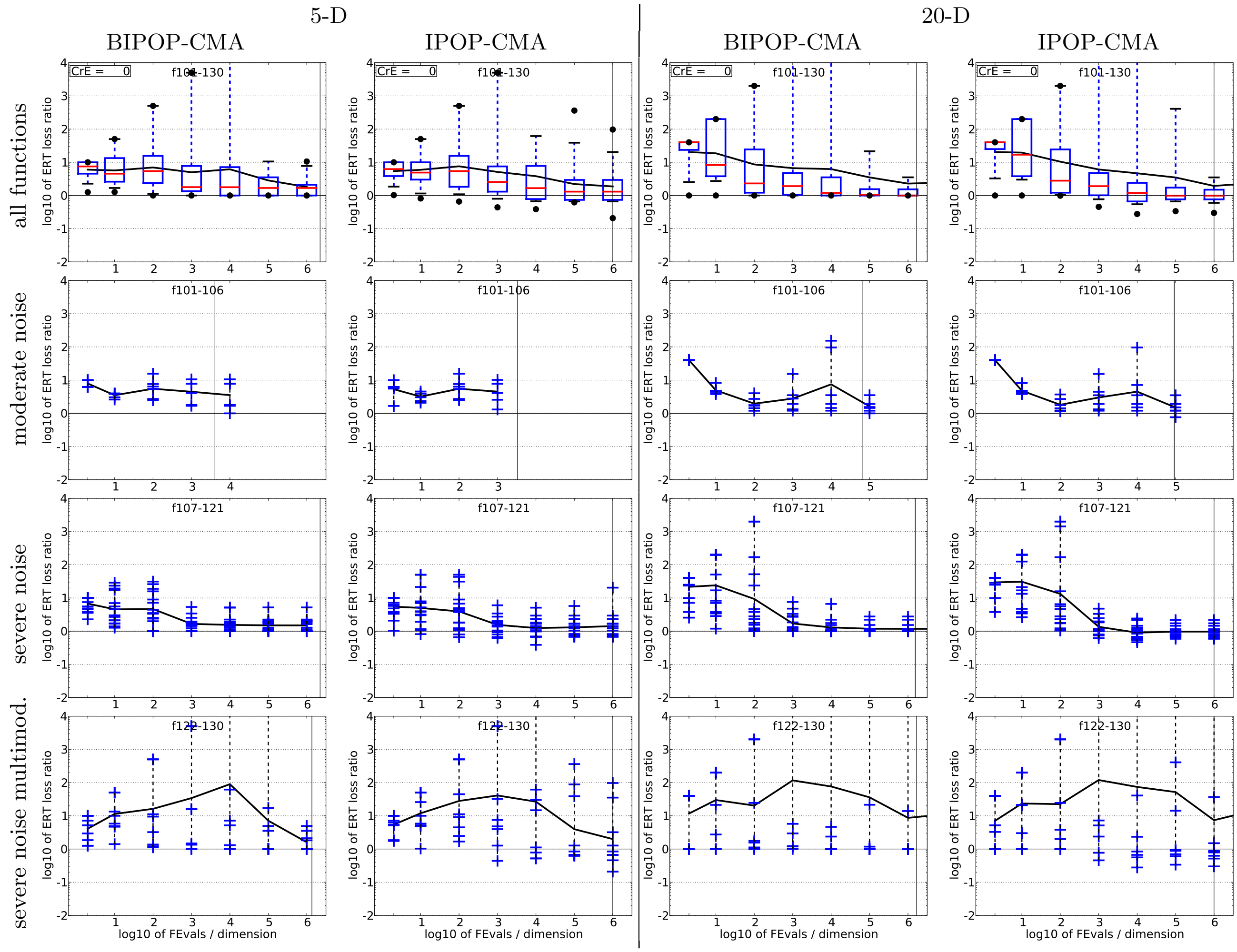

Figure 4: ERT loss ratio versus given budget FEvals. The target value f_t for ERT is the smallest (best) recorded function value such that $\mathrm{ERT}(f_t) \leq \mathrm{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\mathrm{ERT}(f_t)$ from BBOB-2009 for functions f_{101}–f_{130} in 5-D and 20-D. Each ERT is multiplied by $\exp(\mathrm{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

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Table 2: ERT **loss ratio (see Figure 4) compared to the respective best result from BBOB-2009 for budgets given in the first column. The last row $\mathbf{RL_{US}/D}$ gives the number of function evaluations in unsuccessful runs divided by dimension. Shown are the smallest, 10%-ile, 25%-ile, 50%-ile, 75%-ile and 90%-ile value (smaller values are better). ERT Loss ratio is equal to zero if the algorithm considered outperformed all algorithms from BBOB-2009.**

BIPOP-CMA

	***f*101–*f*130 in 5-D**, maxFE/D=2.23e6					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.3	2.1	4.5	7.3	10	10
10	1.3	1.5	2.6	4.3	13	39
100	1.0	1.1	2.4	4.9	16	2.7e2
1e3	1.0	1.0	1.3	1.8	7.8	2.5e3
1e4	1.0	1.0	1.0	1.7	7.2	2.5e4
1e5	1.0	1.0	1.0	1.7	3.5	9.2
1e6	1.0	1.0	1.0	1.7	2.1	6.5
RL_{US}/D	8e5	8e5	9e5	1e6	1e6	1e6

IPOP-CMA

	***f*101–*f*130 in 5-D**, maxFE/D=1.00e6					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	1.8	3.8	6.2	10	10
10	0.81	1.1	3.0	4.7	10	50
100	0.66	1.1	1.8	5.2	16	2.8e2
1e3	0.44	0.73	1.3	2.3	7.5	2.5e3
1e4	0.39	0.60	0.78	1.5	7.8	46
1e5	0.62	0.66	0.74	1.3	2.9	25
1e6	0.21	0.66	0.74	1.3	2.9	15
RL_{US}/D	4e5	4e5	5e5	8e5	1e6	1e6

BIPOP-CMA

	***f*101–*f*130 in 20-D**, maxFE/D=1.68e6					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	1.8	24	40	40	40
10	1.0	2.0	3.8	8.3	2.0e2	2.0e2
100	1.0	1.0	1.2	2.0	24	2.0e3
1e3	1.0	1.0	1.1	1.9	4.8	2.0e4
1e4	1.0	1.0	1.0	1.2	3.5	1.0e5
1e5	1.0	1.0	1.0	1.0	1.5	12
1e6	1.0	1.0	1.0	1.0	1.5	3.1
1e7	1.0	1.0	1.0	1.0	1.5	3.1
RL_{US}/D	4e5	5e5	1e6	1e6	1e6	1e6

IPOP-CMA

	***f*101–*f*130 in 20-D**, maxFE/D=1.00e6					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	2.1	25	40	40	40
10	1.0	2.8	3.8	15	2.0e2	2.0e2
100	0.99	1.0	1.2	2.8	24	2.0e3
1e3	0.46	0.76	1.0	1.9	4.8	2.0e4
1e4	0.28	0.54	0.66	1.2	2.4	1.0e5
1e5	0.34	0.63	0.76	0.96	1.7	2.1e2
1e6	0.30	0.60	0.76	0.96	1.5	2.8
1e7	0.52	0.63	0.77	1.1	1.7	3.7
RL_{US}/D	1e5	2e5	6e5	9e5	1e6	1e6