Cop and robber games when the robber can hide and ride

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In the classical cop and robber game, two players, the cop $\mathcal{C}$ and the robber $\mathcal{R}$, move alternatively along edges of a finite graph $G = (V, E)$. The cop captures the robber if both players are on the same vertex at the same moment of time. A graph G is called *cop win* if the cop always captures the robber after a finite number of steps. Nowakowski, Winkler (1983) and Quilliot (1983) characterized the cop-win graphs as dismantlable graphs. In this talk, we will characterize in a similar way the class $\mathcal{CWFR}(s, s')$ of cop-win graphs in the game in which the cop and the robber move at different speeds s' and s, $s' \leq s$. We also establish some connections between cop-win graphs for this game with $s' < s$ and Gromov's hyperbolicity. In the particular case $s' = 1$ and $s = 2$, we prove that the class of cop-win graphs is exactly the well-known class of dually chordal graphs. We show that all classes $\mathcal{CWFR}(s, 1)$, $s \geq 3$, coincide and we provide a structural characterization of these graphs. We also investigate several dismantling schemes necessary or sufficient for the cop-win graphs (which we call *k-winnable* and denote by $\mathcal{CWW}(k)$) in the game in which the robber is visible only every k moves for a fixed integer $k > 1$. We characterize the graphs which are k-winnable for any value of k.

We present now our main results (for the full version, see [1]). A graph $G = (V, E)$ is (s, s')-*dismantlable* if its vertices can be ordered $v_1, \ldots, v_n$ so that for each $v_i, 1 \leq i < n$, there exists v_j with $j > i$, such that $N_s(v_i, G \setminus \{v_j\}) \cap \{v_i, v_{i+1}, \ldots, v_n\} \subseteq N_{s'}(v_j)$. A graph G is *dually chordal* if its clique hypergraph is a hypertree. Dually chordal graphs are exactly the graphs $G = (V, E)$ admitting a *maximum neighborhood ordering*.

Theorem 1. *For any $s, s' \in \mathbb{N} \cup \{\infty\}$, $s' \leq s$, a graph $G = (V, E)$ belongs to $\mathcal{CWFR}(s, s')$ iff G is (s, s')-dismantlable.*

Theorem 2. *For a graph $G = (V, E)$, the following conditions are equivalent: (i) $G \in \mathcal{CWFR}(2)$; (ii) G is $(2, 1)$-dismantlable; (iii) G is dually chordal.*

A graph G is a *big brother graph* if its blocks can be ordered $B_1, \ldots, B_r$ such that B_1 is dominated and, for any $i > 1$, the block B_i is a leaf in the block-decomposition of $\cup_{j \leq i} B_j$ and is dominated by the articulation point connecting B_i to $\cup_{j<i} B_j$. A graph G is a *big two-brother graph* if G can be represented as a union of its subgraphs $G_1, \ldots, G_r$ labeled in such a way that G_1 has a dominating vertex, and for any $i > 1$, either the subgraph G_i intersects $\cup_{j<i} G_j$ in two adjacent vertices x_i, y_i belonging to a common subgraph $G_j, j < i$, so that y_i dominates G_i, or G_i has a dominating vertex y_i and intersects $\cup_{j<i} G_j$ in a single vertex x_i (that may coincide with y_i).

Theorem 3. *For a graph $G = (V, E)$ the following conditions are equivalent: (i) $G \in \mathcal{CWFR}(3)$; (i′) G is $(3, 1)$-dismantlable; (ii) $G \in \mathcal{CWFR}(\infty)$; (ii′) G is $(\infty, 1)$-dismantlable; (iii) G is a big brother graph. In particular, the classes of graphs $\mathcal{CWFR}(s), s \geq 3$, coincide.*

Theorem 4. *A graph $G = (V, E)$ is k-winnable for all $k \geq 1$ if and only if G is a big two-brother graph.*

A graph $G = (V, E)$ is *k-bidismantlable* if its vertices can be ordered $v_1, \ldots, v_n$ in such a way that for each vertex $v_i, 1 \leq i < m$, there exist two adjacent or coinciding vertices x, y with $y = v_j, x = v_\ell$ and $j, \ell > i$ such that $N_k(v_i, G \setminus \{x, y\}) \cap X_i \subseteq N_1(y)$, where $X_i := \{v_i, v_{i+1}, \ldots, v_m\}$.

Theorem 5. *Any graph $G = (V, E)$ of $\mathcal{CWW}(2)$ is 2-bidismantlable, however there exist 2-bidismantlable graphs G with $G \notin \mathcal{CWW}(2)$. For any odd integer $k \geq 3$, if a graph G is k-bidismantlable, then $G \in \mathcal{CWW}(k)$.*

[1] J. Chalopin, V. Chepoi, N. Nisse, Y. Vaxès, Cop and robber games when the robber can hide and ride, arXiv:1001.4457.