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Abstract: A new approach to estimate the road maximum adherence is presented. It is based on a combination of new algebraic tools for estimation and diagnosis via numerical differentiation of noisy signals. Instantaneous friction is first computed within this framework. Then, extended braking stiffness concept is exploited to detect which braking efforts allow to distinguish a road type from another. A weighted Dugoff model is used during these "distinguishable" intervals to estimate the maximum friction coefficient. Promising results have been obtained in noisy simulations and real experimentations.

Keywords: Automotive vehicles, active safety, friction estimation, nonlinear estimation, nonlinear diagnosis, numerical differentiation.

1. INTRODUCTION

Automobile manufacturers have dedicated enormous efforts on developing intelligent systems for the dynamic performance of road vehicles in the last years. Thus, many systems have been deeply studied in order to increase safety and improve handling characteristics. As such systems become more advanced, they increasingly depend on accurate information about the state of the vehicle and its surroundings. Much of this information can be obtained by direct measurement, but the appropriate sensors may be unreliable, inaccurate, or prohibitively expensive. This is why these enhancements must be a priori related to an optimal usability of the existing hardware.[1] Besides, most of these systems are based on an efficient transmission of the forces from vehicle wheels to the road surface.

Friction is the major mechanism for generating these forces on the vehicle. Hence, knowing the longitudinal and vertical tire-road efforts (F_x, F_z), and therefore the maximum friction coefficient

$$\mu_{x_{max}} = \left.\frac{F_x}{F_z}\right|_{max} \tag{1}$$

turns out to be crucial, because the maximum braking performance is related with the maximum tire-road friction coefficient.

[1] Let us recall that only measurements from encoders, longitudinal and lateral accelerometers, and yaw rate gyroscope are usually available trough the CAN bus.

The goal of this work is then to find a computationally affordable estimator of the maximum tire-road friction coefficient with actual on-board hardware and sensors.

1.1 State of the art

Static and dynamic tire force models have been developed for accurate simulation of advanced control systems (see, e.g., (Canudas-de-Witt *et al.*, 2002; Kiencke *et al.*, 1995; Pasterkamp *et al.*, 1994; Svendenius *et al.*, 2009; Yi *et al.*, 2003)). Extensive testing is however required to determine the parameters of these analytic models; it is then extremely difficult to determine all those parameters in real-time for every potential tire, tire pressure, and wear state.

Nevertheless, many authors have tried to use robust analytic techniques to determine tire-road friction coefficient from tire force models. Thus, simplified models have been coupled with vehicle dynamics to produce different observation and filtering techniques: Matusko *et al.* (2008) employed a neural-network based identification; Ono *et al.* (2003), Tanelli *et al.* (2009) and Yi *et al.* (1999) developed several least-square methods; Dakhlallah *et al.* (2008), Grip *et al.* (2007), Lee *et al.* (2004), Ray (1997), or Shim *et al.* (2004) utilize various nonlinear asymptotic observers. Most of the above references try to obtain reliable tire effort estimates and, thereafter, the maximum tire friction value by various polynomial fitting techniques. Those approaches, unfortunately, are based either on too restrictive hypotheses (e.g. only longitudinal dynamics situations) or on nonstandard measurements (wheel torque). Moreover, most of them concentrate their efforts on precise tire forces estimation, but they do not go into maximum friction estimation in depth.

A different research line is focused on the effects that are generated by friction. Gustafsson (1997), for instance, used the idea that more slip at a given tire force would indicate a more slippery road. Observing the correlation between slip and friction coefficient can provide μ_{max} information. However, under low slip situations, it becomes really hard to distinguish between different road types from noisy measurements.

A possible solution to this problem is to exploit what Ono *et al.* (2003) and Umeno (2002) call the extended braking stiffness (XBS). It can be defined as the slope of friction coefficient against slip velocity at the operational point. Its value is related to the friction coefficient because the maximum braking force can be obtained when XBS is equal to zero (see [2] Fig. 1).

Note that snow, and specially ice, exhibit a very short transition between linear and nonlinear zones. Therefore, the XBS based algorithm presented here will correctly behaves for wet and dry roads, but it will probably not be that efficient for ice conditions.

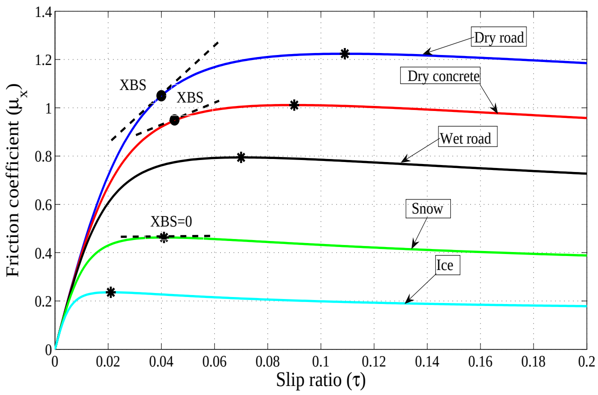

Fig. 1. Adhesion coefficient characteristic curve for several tire-road interfaces (top); XBS definition (bottom)

Since in the $\tau - \mu_x$ linear zone $\mu_{x_{max}}$ is hardly identifiable, our work relies on the accurate estimation of XBS when non-linear behavior takes shape (soon enough to avoid wheel saturation). Diagnosis tools will be used, combined with new algebraic filtering techniques and a weighted Dugoff model, to consecutively estimate longitudinal forces and the maximum friction coefficient.

In addition to accuracy and reliability, production cost is an important matter in vehicle serial production. In that sense, our estimation techniques, which are borrowed from Fliess *et al.* (2008), are especially efficient in terms of computational cost, at least when compared with most of the above mentioned observer-based approaches. Furthermore, only standard and low cost sensors will be required for implementing the proposed algorithm.

[2] Friction coefficient is plotted in terms of slip ratios for several tire-road interfaces following the pseudo-static Pacejka tire model (Pacejka *et al.*, 1991).

1.2 Outline of the article

Sect. 2 presents an overview of a new algebraic estimation framework. The first example of that approach is introduced in Sect. 3, where a pitch diagnosis-based estimator allows to obtain a good estimate of the instantaneous friction coefficient. Sect. 4 deals with the problem of distinguishing different road surfaces enough ahead of time to avoid undesirable control actions. In Sect. 5 a simulated scenario is used to test the quality of the estimator on multi-adherence roads. Preliminary experimental results are presented in Sect. 6, where the algorithms are tested with real experimental recorded data. Some concluding remarks are given in Sect. 7.

2. ALGEBRAIC ESTIMATION FRAMEWORK

In diagnosis terminology, a residual is defined as the amount by which an observation differs from its expected value. It is often used in fault tolerant control to detect a failure (for instance, in sensors or actuators) and act consequently. This idea will be used here in an estimation context to detect an abnormal behavior with respect to an ideal prediction model. In other words, the estimated variable can be considered as the sum of an ideal term and a "disturbing" one. [3]

The condition to decide wether the ideal term is valid or not to estimate the unknown variable is usually hard to obtain. Indeed, highly corrupted signals provided by the vehicle sensors and fixed integration step determined by signals sampling rate impose a signal pre-treatment. In addition, robust and real-time efficient numerical differentiators are also needed to render this approach feasible.

This is made possible by recent advances (Fliess *et al.* (2008), Mboup *et al.* (2009)). It is important to point out that the fast filters and estimators, which are obtained in that manner, are not of asymptotic nature, and do not require any statistical knowledge of the corrupting noises. [4]

3. TIRE-ROAD FRICTION ESTIMATION

Using Newton's second law of motion, front and rear longitudinal efforts can be expressed as follows

$$F_{x_f} = M_{eq_f}\gamma_x,\ M_{eq_f} = \frac{F_{z_f}}{g};\ \ F_{x_r} = M_{eq_r}\gamma_x,\ M_{eq_r} = \frac{F_{z_r}}{g}$$

where $M_{eq_{(f,r)}}$ are the front and rear equivalent masses, respectively. Hence, the front and rear friction coefficients (1) turn out to be equal and on only dependent on the longitudinal acceleration γ_x

$$\mu_{x_f} = \mu_{x_r} = \frac{F_{x_f}}{F_{z_f}} = \frac{F_{x_r}}{F_{z_r}} = \frac{\gamma_x}{g} \tag{2}$$

Since this approach does take into account neither the vertical nor the pitch dynamics, an additional term will be introduced in order to achieve better estimations of μ_x. Experimental measurements have shown that the addition of a corrective term $\Delta\mu_{x_\phi}$, proportional to the integral of

[3] Those disturbing terms are nothing else than "poorly known" effects. See (Fliess *et al.*, 2008) for more references and details.

[4] See, e.g., (García Collado *et al.*, 2009) for a nice introductory presentation.

pitch angle ϕ, remarkably corrects the estimation error obtained with (2):

$$\mu_x = \frac{\gamma_x}{g}\left(1 + \Delta\mu_{x_\phi}\right) \quad (3)$$

Following the approach of Tseng *et al.* (2007), the kinematic relationship between the outputs of an inertial measurement unit and the derivatives of the Euler angles can be written, in its longitudinal component

$$\dot{V}_x = \gamma_x + \dot{\psi}V_y - \dot{\phi}V_z + g\sin\phi \quad (4)$$

with V_y, V_z and ψ the lateral and vertical velocities, and the yaw rate, respectively. If the vertical and lateral velocities are neglected (Tseng *et al.*, 2007), and the longitudinal velocity is considered equal to $r\omega$, the following pitch angle estimator $\hat{\phi}$ can be obtained from (4)

$$\hat{\phi} = \arcsin\left(\frac{\gamma_x - r\dot{\omega}}{g}\right)$$

The corrective term $\Delta\mu_{x_\phi}$ in Eq. (3) is therefore numerically computed with the next algorithm

$$\begin{cases} \Delta\mu_{x_\phi}(t) = \displaystyle\int_{T_{i_\phi}}^{T_{f_\phi}} K_\phi\hat{\phi}(t)dt, \text{ if } |\hat{\phi}(t)| > \epsilon_1,\ 0 < \epsilon_1 \ll 1 \\ \Delta\mu_{x_\phi}(t) = 0, \text{ elsewhere} \end{cases}$$

where T_{i_ϕ} and T_{f_ϕ} are respectively the initial and final time where the pitch variation is significative (i.e. $|\hat{\phi}(t)|$ is greater than a threshold ϵ_1), and K_ϕ is an off-line identified parameter, which represents the normalized pitch stiffness.

Fig. 2 shows the different behavior between (2) and (3) when demanding braking efforts are applied to the vehicle. These results have been obtained from experimental data recorded on a real vehicle with noisy measurements (see Sect. 6 for more details).

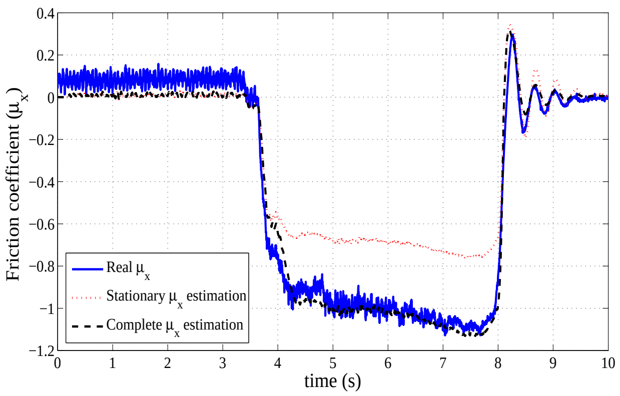

Fig. 2. Longitudinal friction estimation comparison between equations (2) and (3) from real data measurements.

4. FROM INSTANTANEOUS FRICTION TO MAXIMUM FRICTION ESTIMATION

While the vehicle is in the linear ($\mu_x - \tau$) zone, there is a strong risk of indistinguishability of the type of surface, and therefore, to fail in predicting the maximum friction coefficient.

Hence, our approach tends to take advantage of the presented numerical algorithms to be able to detect danger zones in a reliable way. Once the 'failure' is detected, a simple tire behavior model (section 4.1) will help to obtain a good estimation of $\mu_{x_{max}}$ slightly ahead of time.

4.1 Dugoff model

Several tire models have been developed to understand the nonlinear and complex physics of tire force generation (see (Svendenius *et al.*, 2009) and the references therein). The pseudo-static model from Pacejka *et al.* (1991) gives a good approximation to experimental results and is widely used in automotive research and industries. However, this model has a complex analytical structure and its parameters are difficult to identify. For these reasons, it is mainly used for simulation rather than for control or estimation purposes.

Dugoff tire model (Dugoff *et al.*, 1969) assumes a uniform vertical pressure distribution on the tire contact patch. This is a simplification compared to the more realistic parabolic pressure distribution assumed in Pacejka. However, it has the advantage of its conciseness and its results are considered to be on the safe side in an emergency situation. Furthermore, longitudinal forces are directly related to the maximum friction coefficient in more transparent equations than in Pacejka model.

Dugoff model accuracy will then be evaluated as a good candidate to estimate $\mu_{x_{max}}$. Longitudinal efforts are modeled as follows

$$F_x = f(\lambda)K_x\tau$$

where τ is the slip ratio [5]

K_x is the longitudinal stiffness coefficient and $f(\lambda)$ is a piecewise function

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda & , \lambda < 1 \\ 1 & , \lambda \geqslant 1 \end{cases}, \quad \lambda = \frac{\mu_{x_{max}}F_z}{2|K_x\tau|}$$

It is not difficult to see that $\mu_{x_{max}}$ can be expressed in terms of 4 a priori known variables [6] $\mu_{x_{max}} = g(F_x, F_z, \sigma, K_x)$. Let us take the nonlinear zone case, i.e. $f(\lambda) = (2-\lambda)\lambda$

$$F_x = \left(\frac{2 - \mu_{x_{max}}F_z}{|K_x\tau|}\right)\frac{\mu_{x_{max}}F_z}{|K_x\tau|}K_x\tau$$

This expression can be rewritten as a second order algebraic equation of the maximum friction coefficient:

$$\mu_{x_{max}}^2 F_z^2 - 2\mu_{x_{max}}|K_x\tau|F_z + |K_x\tau|F_x = 0$$

whose two solutions

$$\mu_{x_{max}} = \frac{\left(|K_x\tau| \pm \sqrt{K_x\tau(K_x\tau - F_x)}\right)}{F_z} \quad (6)$$

are always real because $K_x\tau(t) - F_x(t) \geqslant 0,\ \forall t$.

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[5] Slip ratio is defined as

$$\begin{cases} \tau = \dfrac{V_x - \omega r}{V_x} \text{ if } V_x > \omega r,\ V_x > 0 \text{ (braking)} \\ \tau = \dfrac{\omega r - V_x}{\omega r} \text{ if } V_x < \omega r,\ \omega r > 0 \text{ (accelerating)} \end{cases} \quad (5)$$

where r and ω are the wheel's radius and angular speed, respectively.

[6] While F_x, F_z and σ are byproducts of our global estimation scheme, a nominal K_x could be obtained either by an off-line or an online identification, following techniques introduced in Sect. 2.

As a result, our $\mu_{x_{max}}$ prediction model will be obtained with the minimum value of (6):

$$\hat{\mu}^D_{x_{max}}(t_k) = \frac{1}{\hat{F}_z(t)}\left(|K_x\hat{\tau}(t_k)| - \sqrt{K_x\hat{\tau}(t_k)(K_x\hat{\tau}(t_k) - \hat{F}_x(t_k))}\right), \ \lambda(t_k) < 1$$
$$\hat{\mu}^D_{x_{max}}(t_k) = \hat{\mu}^D_{x_{max}}(t_{k-1}), \ \lambda(t_k) \geqslant 1 \quad (7)$$

Remark 4.1. Slip $\hat{\tau}$ and vertical force $\hat{F}_x$ estimation has to be provided to compute (7).

4.2 Detection algorithm

As stated in the introduction, the extended braking stiffness (XBS) will be used to detect the entrance in danger zone (or, in other words, to signal the distinguishability between road surfaces).

XBS was defined by Ono *et al.* (2003) as the derivative of the friction coefficient with respect to slip ratio. Therefore, our switching function will be given by the following XBS estimator

$$XBS(t) = \frac{d\mu_x}{d\tau} = \frac{d\mu_x}{dt}\frac{dt}{d\tau} = \frac{\hat{\dot{\mu}}_x}{\hat{\dot{\tau}}}$$

where $\hat{\dot{\mu}}_x$ and $\hat{\dot{\tau}}$ are obtained using (Mboup *et al.*, 2009).

The main difficulty in this computation is to obtain numerical derivative estimators such that a good trade-off between filtering and reactivity is achieved. Algebraic techniques introduced in (Fliess *et al.*, 2008) are used to obtain for example a denoised $\dot{\mu}$:

$$\hat{\dot{\mu}} = \int_0^T (T - 2t)\,\mu(t)dt$$

where $[0, T]$ is a quite short and sliding time window.

Algebraic derivative estimators are compared for a particular scenario to their exact analytic values in Fig. 3 (a and c). Our estimators perform in a satisfactory way, even with singular behaviors such as sudden changes of $\mu_{x_{max}}$ (i.e. at t=3s). The analytic values present a hard discontinuity at that point, but it is pretty well filtered by our estimators.

Also in Fig. 3 (b and d) a comparison between μ_x and filtered XBS evolutions is shown. Similar trends can be appreciated in both graphs, i.e. when μ_x reaches a local peak, XBS is close to local minima. Furthermore, the closest μ_x is to $\mu_{x_{max}}$, the lower value of XBS is obtained. As a result of this, an XBS validity range ($[XBS_{max}, XBS_{min}]$ can be selected as significative for $\mu_{x_{max}}$ detection. Thus, when XBS values are greater than XBS_{max} or lower than XBS_{min}, we will consider μ_x remains equal to the last obtained value within the validity range. If μ_x falls into the validity range, a corrective factor will be applied to the $\mu_{x_{max}}(t)$ predicted value by equation (7). Finally, a $[0, 1.2]$ saturation function is used to correct the previous value in case the estimation exceeds realistic friction limits. To sum up, the final algorithm can be concisely written as follows:

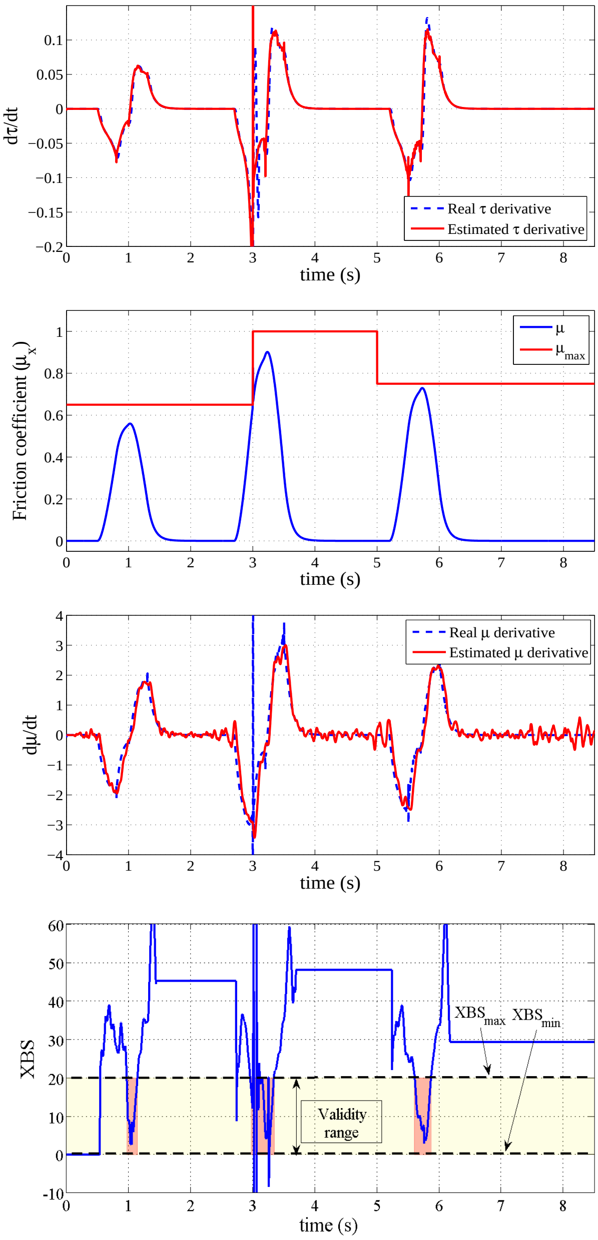

Fig. 3. eXtended Braking Stiffness estimation. a) Comparison between τ algebraic derivative estimation and its analytical value. b) Real μ_x and $\mu_{x_{max}}$ evolution. c) Comparison between μ_x algebraic derivative estimation and its analytical value. d) XBS estimation, validity range and $\mu_{x_{max}}$ distinguishable zones.

$$\text{if } XBS(t_k) \leqslant XBS_{max}$$
$$\mu_{x_{max}}(t_k) = \max\left(0, \min\left(1.2, \mu^*_{x_{max}}(t_k)\right)\right)$$
$$\mu^*_{x_{max}}(t_k) = \hat{\mu}^D_{x_{max}}(t_k)\left(1 + \chi\frac{XBS(t_k)}{XBS_{max}}\right)$$
$$\text{if } XBS_{max} \leqslant XBS(t_k)$$
$$\mu_{x_{max}}(t_k) = \mu_{x_{max}}(t_{k-1}) \quad (8)$$

Remark 4.2. Parameter χ acts as a confidence factor of the friction value provided by Dugoff model within the validity range. It can be seen from Eq. (8) that the nearest XBS is to XBS$_{max}$, the closest the factor $\dfrac{XBS(t_k)}{XBS_{max}}$ is from 1. On the contrary, when XBS tends to 0, the same stands for $\dfrac{XBS(t_k)}{XBS_{max}}$. The value of parameter χ is always close to 1 and its optimum value depends on the measurement noise nature.

5. SIMULATION RESULTS OF FRICTION ESTIMATION

A multi-adherence scenario has been used to test the algorithm in a simulation environment.[7] Each of the three braking phases is carried out under different friction conditions. Thus, $\mu_{x_{max}}(t) = 0.65, 0 \leq t \leq 3$, $\mu_{x_{max}}(t) = 1, 3 \leq t \leq 5$, $\mu_{x_{max}}(t) = 0.75, 5 \leq t \leq 8.5$. The applied braking efforts have been chosen to be useful for our failure detection algorithm, i.e., they are strong enough to leave the linear zone, but soft enough to avoid tire saturation.

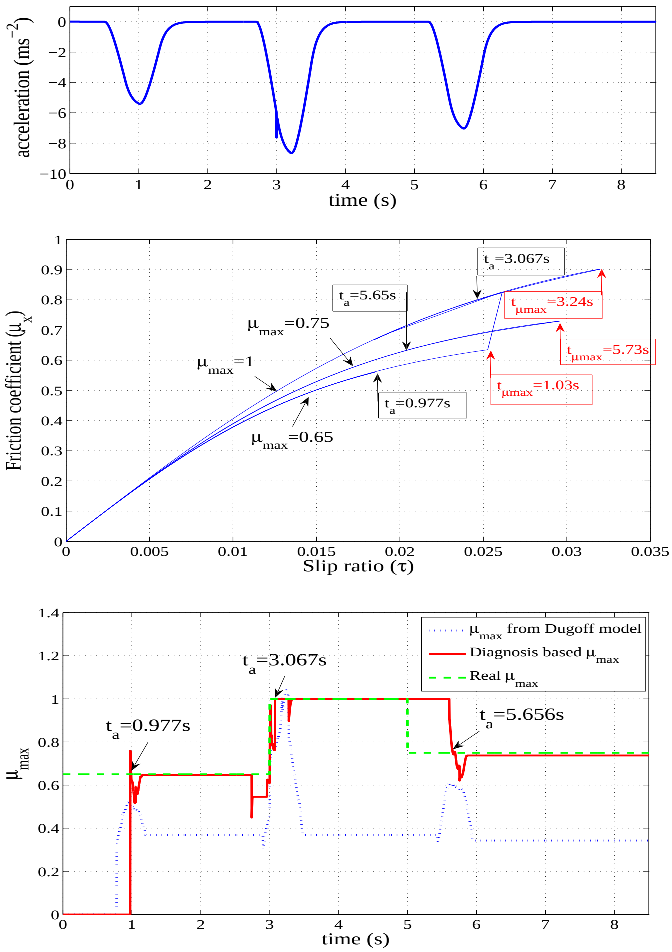

Fig. 4. a)Longitudinal acceleration b)$\mu_x - \tau$ graph with maximum friction and detection instants c)Maximum adherence estimation during the first multi-adherence scenario

Fig. 4 displays our simulation results. A vehicle begins to move at 15 ms^{-1} and three braking actions are consecutively applied so that the resulting longitudinal accelerations are those of Fig. 4a. The bottom graph of the same figure plots the friction coefficient and its estimation, at the instants $t_{\mu_{max}}$ where minimum values are attained. This information can be complemented with Fig. 4b, where the $(\mu_x - \tau)$ evolution can be very well distinguished for all three tire-road interfaces. Moreover, alarm times (t_a, in black), coming from the estimation algorithm, always arise sufficiently in advance with respect to maximum friction instants $t_{\mu_{max}}$ (in red).

Bottom graph of Fig. 4 compares the real $\mu_{x_{max}}$ obtained values, on the one hand, with Dugoff prediction model, and on the other hand, with the weighted Dugoff prediction model. Besides the fact that the alarm times seem to be sufficiently ahead of time, the estimation error is significantly small.

6. EXPERIMENTAL RESULTS OF FRICTION ESTIMATION

Several braking maneuvers have been realized on roads with different maximum friction coefficient. In every test, a large set of dynamic variables has been recorded at high frequency rates (around 250 Hz) on an instrumented vehicle.

The promising results obtained in simulation have been confirmed with real data in severe braking maneuvers. Two examples of this satisfactory results are plotted in figure 5. In the first case, the vehicle is moving at a constant

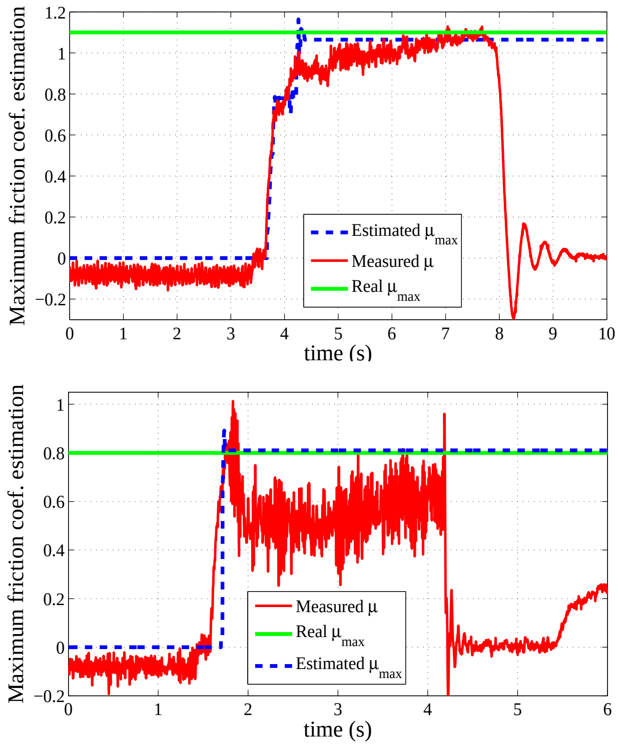

Fig. 5. Maximum friction estimation for two experimental maneuvers.

speed (100 kmh^{-1}) on a dry road ($\mu_{x_{max}} = 1.1$) , when a

[7] A realistic simulator of a vehicle with 14 degrees of freedom, and with complete suspension and tire models (Pacejka *et al.*, 1991) has been used in all simulations. Additionally, a realistic white gaussian noise has been added to every measurement.

sudden and hard braking effort is applied at $t = 3.5$ s. The vehicle remains braking close to the maximum tire-road friction coefficient until it stops at $t = 8.5$. The real friction coefficient μ_x increases rapidly around $t = 3.5$, but it does not attain its maximum value almost until the end of the braking maneuver. This monotone behavior, probably due to the the fact that wheels do not get locked, is not an obstacle to our estimator, which obtains a constant value of $\hat{\mu}_{x_{max}} = 1.07$ at $t = 4.2$, sufficiently in advance of the μ_x peak.

The second case is again a vehicle moving at a constant speed (60 kmh^{-1}) in a straight line on a wet road ($\mu_{x_{max}} = 0.8$). As it can be appreciated in Fig. 5, the results are even more satisfactory than in the previous case. On the one hand, the estimated value is now even closer to the real value, and on the other hand, the transient state is much shorter than before.

7. CONCLUSION

A new approach to estimate vehicle tire forces and road maximum adherence is presented. It is based on the combination of elementary diagnosis tools and new algebraic techniques for filtering and estimating derivatives, which were introduced in (Fliess *et al.*, 2008). First of all, instantaneous friction is estimated. Then, extended braking stiffness concept is exploited to detect which braking efforts allows to distinguish a road type from another. Very promising results have been obtained in noisy simulations and in real experiments.

Let us emphasize finally that the practical usefulness of our algebraic techniques is confirmed by other successful applications to the automotive industry (Villagra *et al.*, 2008, 2009, 2010).

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