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Résumé. Nous proposons un nouveau modèle afin de modéliser les séries temporelles avec de la persistance variable dans le temps. Ce modèle permet au paramètre de mémoire longue d'avoir un ou plusieurs changements structurels continus. Nous développons un test de Multiplicateur de Lagrange (LM) afin de tester le changement structurel, et une stratégie permettant de déterminer le nombre de changements dans le paramètre de mémoire longue. Des études de simulation montrent que le test a de bonnes propriétés au niveau de la taille et la puissance; elles montrent aussi que la stratégie proposée identifiant le nombre de régimes est très satisfaisante.

Une application empirique sur les valeurs absolues des rentabilités du CAC40 indique que de telles données sont caractérisées par une instabilité dans leur persistance expliquée par un changement structurel dans le paramètre de mémoire longue.

Mots clés: Longue mémoire, Changement structurel, Volatilité persistante

Summary. In this paper, we propose a new model to describe time series processes with time varying persistence. The model allows the long memory parameter to have one or multiple continuous structural changes. LM-type tests for structural changes in the long memory parameter and a strategy for determining the number of changes have been developed. The simulation study shows that these tests have good sizes and powers properties and that the proposed strategy has a satisfactory performance in detecting the number of structural changes. An empirical application to the absolute returns of CAC 40 index indicates that these data are characterized by instability in their persistence explained by one structural change in the long memory parameter.

Key words Volatility persistence; Long memory; Structural changes.

In several recent research studies, attention has been focused on structural changes or long memory to describe the persistency observed in many time series. Some of these studies argued that long memory and structural changes can be easily confused, see Granger and Teräsvirta (1999), Diebold and Inoue (2001), and Granger and Hyung (2004), among others. Other studies have suggested that both long memory and structural change characterize the structure of financial returns volatility, see for instance Lobato and Savin (1998), Beine and Laurent (2001), Morana and Beltratti (2004) and Martens et al. (2004), and Baillie and Morana (2009). In another direction, the persistence observed in many economic time series can be analyzed jointly by long memory and nonlinearity, see for example van Dijk, Frances, and Paap (2002) and Ajmi, Ben Nasr and Boutahar (2008). However, it is possible for some data sets that the long memory parameter displays structural instability. Few attempts have been made to consider structural change in a long-term dependence structure. Granger and Ding (1996) provided empirical evidence of changing long memory parameter in the S&P 500 stock absolute returns. They found that the parameter estimates of the $I(d)$ process vary considerably from one subseries to the next. The first test for a change in the long memory parameter was proposed by Beran and Terrin (1996). The approach is based on the comparison of Whittle estimator to detect a change in the long memory parameter, but their approach is not applicable in empirical work. The correct limit distribution of their test statistic was obtained, based on quadratic forms, by Horváth and Shao (1999). Horváth (2001) suggested another test based on quadratic forms of Whittle estimator of the long memory parameter. Ray and Tsay (2002) proposed a Bayesian method for detecting structural change in the fractional differencing parameter d. They supposed that the parameter d changes over time as a random walk. Wang and Wang (2006) proposed a semiparametric test and estimation procedures for a change in the long-memory parameter. Their test is constructed based on the averaged periodogram estimator proposed by Robinson (1994). Chong (2007) used the approach of the minimum distance estimator to estimate the change point and the differencing parameters. More recently, Reisen et al. (2009) proposed a fully semiparametric test for the stationarity of the long memory parameter. Dufrénot et al. (2005a,b, 2008) considered SETAR specifications of the long memory process by allowing for regime switching in the differencing parameter d. Boutahar et al. (2008) considered a fractionally integrated model with smoothly transition long memory parameter. They allowed the parameter d to change smoothly using a logistic transition function. The structural changes in the long memory parameter may be caused by changes in the physical mechanism that generates the data, for example, time series with long memory in telecommunication engineering where changes may occur according to the mechanism generating the trafic (Beran et al., 1995), or by changes in the way observations are taken. Even small changes of the long memory parameter have significant effect on the long-term behavior of the process, see Beran and Terrin (1996) and Ray and Tsay (2002).

In this paper, we extend the model proposed by Boutahar et al. (2008) to describe continuous structural change in the long memory parameter. We also deal with possible extensions of the basic model by allowing it for more than one continuous structural change.

We propose LM tests for long memory parameter constancy against the time varying fractionally integrated model and for m changes against $m+1$ changes in the parameter of long memory. These tests are particularly useful in that they allow a specific to general modeling procedure to determine the appropriate number of changes in the long memory parameter.

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