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Abstract

The Probabilistic Traveling Salesman Problem is in general *NP-Hard,* the objective of this paper is to look for particular cases which can be solved in an easy way. First we give the relation between the two problems deterministic and probabilistic. Second, we study the stability of the deterministic problem. We prove that in the small traveling salesman problem, several tours were optimal, but there is only one stable among them : The absence of certain cities does not disturb the tour induced by the method of modification of the optimal tour a priori.

Keyword : Processus, Statistique mathématique

1 Introduction

Over the past fifty years, the study of Combinatorial Optimization Problems (COP) has become an active area of research in discrete mathematics. Precisely, there is a growing body of research devoted to both introducing new concepts and solving a large number of extremely complicated practical problems. The most important results obtained so far include : The development of the basic rules for ranking and evaluating the inherent difficulties of combinatorial optimization problems (for example, the classification of the problems' categories *P, NP, NP-Complete, NP -Hard)*; clarifying fundamental principles such as the algorithmic complexity and the heuristic performance analysis; introducing new techniques of solving algorithms that have become very popular in other fields, (for example, dynamic programming, "branch and bound" methods); and finally, these concepts and retour technics have been used successfully in several areas of applications. Of course, the scientific literature has been extended.

The most common way in wich probabilities are associeted with COP is to consider that the data of the problem are deterministic and randomness carries over the relation between these data (we have a particular problem). More recently, in the late 1980, another approach to the randomness was developed : Probabilities are associated with data. Such formulations give rise to what we call Probabilistic Combinatorial Optimisation Problems POCP. Like example of these problems, we quote the Traveling Salesman Problem (TSP) which consists in seeking the tour minimal length through a fixed number n of cities [10]. A very natural probabilistic extension of TSP is introduced for the first time by Jaillet in 1985, when he supposes that the number of the cities presents is a random variable [9].

Since then, this approach has been extended to the other problems such as the vehicle routing problem [7], the spanning tree problem [6]. The success of this probabilistic approach continues, it shows itself more and more in many domains, such as the probabilistic maximum independent set problem [22]et [19], the probabilistic longest path problem [21], the probabilistic minimum vertex covering problem [20], the probabilistic minimum coloring problem [18], the probabilistic steiner tree problem [14], the probabilistic graph-coloring in bipartite problem [11], and an example of particular elevance tothis papers is the review volume on the Probabilistic combinatorial optimization on graphs [17]. This approach has been extended to the combinatorial problems not defined on graph [2], [3] and [4]. The motivation behind this work is the desire to formulate and analyze models which are more appropriate for real-world problems where randomness is present.

Another motivation is an attempt to analyze the stability when the perturbation of the optimally solved deterministic problem instance induced by the absence of some data still gives an optimal solution if restricted to the sub-instance. So the study of the stability relies on the optimal solution; then we should deal with polynomial cases of the Traveling Salesman Problem (TSP) by treating some polynomial cases, which can be solved in an easy way. Then we study a well-solved special case of traveling salesman problem : the Small TSP [10]. We will aim mainly two goals: the relation between the two problems deterministic and probabilistic, and the stability of the deterministic one when the data of the problem is modified.

2 Probabilistic Traveling Salesman Problem (PTSP)

Very often in the applications, after having solved a particular instance of a given combinatorial optimization problem, we must solve in a repeated way a great number of instances of the same problem. These other instances are generally only simple variations of the initial problem; however they are sufficiently different to require individual treatment.

The most natural approach to answer this kind of situation consists in solving in an optimal way the various instances. We call this strategy: the reoptimization strategy. This approach has nevertheless many disadvantages. Moreover, and with regard to TSP, we have an exponential number of instances to treat, from where an effort of significant storage. Or, the wish to obtain a very fast tour of manner, without having computer equipement.

It is thus necessary to adopt a different strategy. Rather than of reoptimization of each sub-instance, one can try to determine a priori tour for the initial problem which can change simple way successively (according to a method of modification) in order to generate a tour for each sub-instance.

For an illustration of this methodology, we will take for example the TSP. This problem supposes that : An instance$V$ of n cities must be visited, the distances between the cities are given by a matrix, TSP consists in finding a tour minimal length through the cities of V. Thereafter, only certain S_1, S_2,..,subsets of V will be really visited, these subsets being successively and independently selected according to a probability definite on 2^V.

The $PTSP$ rests on the following assumptions: We have a law of probability defined on 2^V, $P(S)$ probability of presence of subsets S. We have a method of modification $\mathcal{U}$: given a priori tour T defined on V we obtain , from T , a feasible tour for sub-instance S of V using a method of modification $\mathcal{U}$. PTSP consists in minimizing this expression:

$$\mathbb{E}(L\,(T,\mathcal{U})) = \sum_{S\subset V} P(S)L_{(T,\mathcal{U})}(S)$$

Where $L_{(T,\mathcal{U})}(S)$ indicates, for any $S \subseteq V$, the length of the tour T modified by $\mathcal{U}$ and restricted on the subset S.

In practice, the method of modification $\mathcal{U}$ corresponds to a behavior observed of the real problem.

the choice of modification depends strongly on the real-world application modelled [8].

We consider in this communication that each city has the same probability of presence equal to p. In the next paragraph, we will apply this methodology to special cases of $PTSP$.

Let us call an $n \times n$ matrix C small if there exist $n-$dimensional vectors a and b such that $c_{ij} = \min\{a_i, b_j\}$. A small matrix where all of the a_i and b_j are distinct is said to have distinct values.

For the formalism, we adopt the following notations:

- Let d_i be the Ith smallest of the $2n$ distinct values a_i et b_j.
- We note d :

$$d = \sum_{i=1}^{n} d_i$$

The whole of the d_i, $i \in \{1, ..., 2n\}$ can be partitioned into two sets:

- $D = \{d_1, ..., d_n\}$
- $\overline{D} = \{d_{n+1}, ..., d_{2n}\}$

The cities can be partitioned into four sets:

- $D_2 = \{i \text{ such that } a_i \text{ and } b_i \in D\}$

- $D_0 = \left\{ i \text{ such that } a_i \text{ and } b_i \in \overline{D} \right\}$
- $D_a = \left\{ i \text{ such that } a_i \in D \text{ and } b_i \in \overline{D} \right\}$
- $D_b = \left\{ i \text{ such that } b_i \in D \text{ and } a_i \in \overline{D} \right\}$

3 The probabilistic small TSP

First, we recall that PTSP consists in determining a tour a priori which minimizes the expectation of length of the tour. In this study of the small PTSP under the strategy a priori, we will use a method named: method of gumming. This method illustrate the strategy a priori for TSP. Below an outline on this method.

We give ourselves a unit V of n cities to be visited. Being given a tour a priori T through n cities, and for each instance S of the problem, the method of gumming, to generate a tour through S, consists in "gumming" the cities absent from the tour a priori T .

Basing on research of Bellalouna relating to the small PTSP [24], we will expose in this paragraph our study of the problem. This one, will be directed along two major axes:

- First, to see whether there are conditions under-which small TSP is equivalent to the small PTSP, in order to analyze the stableness of the problem.
- Then, to seek the optimal tour with the probabilistic direction, which we call : T_{PTSP} .

We will start this study by giving four definitions which we will use in our following propositions:

- A tour is a permutation between the n cities, it gives the order of visit of these cities. A tour is represented by T and we have:
- $T[i]$ is the i^{th} city of the tour T, $i \in \{1, ..., n\}$
- $T[i-1]$ is the predecessor of $T[i]$
- $T[i+1]$ is the successor of $T[i]$

Proposition 1 *Let C be a small matrix with distinct values, we suppose that $D_2 = \{1\}, D_0 = \{n\}$ et $D_b = \emptyset$. Without loss of general information, we suppose that:$a_1 < a_2 < ... < a_{n-1} < a_n$. Then:*

$$PTSP \equiv TSP \text{ if and only if } a_{n-1} < b_1 \text{ and } a_n < \min_{2 \le i \le n-1} \{b_i\}$$

Proposition 2 *Let C be a small matrix with distinct values, we suppose that $D_2 = \{1\}, D_0 = \{n\}, D_b = \{\}$. Without loss of general information, we suppose that : $a_1 < a_2 < ... < a_{n-1} < a_n$. Moreover, we suppose that : $b_2 < ... < b_{n-1}$. Then:*

$$T_{PTSP} = T^*_{TSP}$$

4 Conclusion

The objective of this communication was the study of the probabilistic extension of well-solved special cases of traveling salesman problem. We aimed essentially the notion of stability. We recall for this purpose, that the study of the stability of PCO represents the theoretical motivation of the introduction of the probabilities into these problems. This case is named Small TSP, in this last, we arrived to very interesting results. Firstly, under conditions necessary and sufficient, we could show that TSP was equivalent to the PTSP. Thus the introduction of the probabilities to this problem did not change its complexity : The Small PTSP remains easy. Secondly, we found a result convincing within the framework of the study of stability. Under few assumptions, only one optimal tour among all the deterministic optimal tours is a probabilistic optimal tour : the tour induced by the method of modification of the stable optimal tour a priori is optimal.

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Summary

The Probabilistic Traveling Salesman Problem is in general *NP-Hard,* the objective of this paper is to look for particular cases which can be solved in an easy way. First we give the relation between the two problems deterministic and probabilistic. Second, we study the stability of the deterministic problem. We prove that in the small traveling salesman problem, several tours were optimal, but there is only one stable among them : The absence of certain cities does not disturb the tour induced by the method of modification of the optimal tour a priori.

Résumé

Le problème du voyageur de commerce est généralement NP-Hard. Notre but est de chercher des cas qui sont "Faciles", càd résolubles en temps polynomial. Le Problème du Voyageur de Commerce Petit illustre bien ces cas, notre étude comporte deux volets, d'abord nous donnons la relation entre le problème déterministe et son homologue probabiliste, ensuite nous montrons un résultat remarquable, à savoir : ce problème est stable : la perturbation par l'absence de certaines données n'influe pas l'optimalité de la solution.