



Test de comparaison du paramètre de longue mémoire

Frédéric Lavancier, Anne Philippe, Donatas Surgailis

► **To cite this version:**

Frédéric Lavancier, Anne Philippe, Donatas Surgailis. Test de comparaison du paramètre de longue mémoire. 42èmes Journées de Statistique, 2010, Marseille, France, France. 2010. <inria-00494803>

HAL Id: inria-00494803

<https://hal.inria.fr/inria-00494803>

Submitted on 24 Jun 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A TWO-SAMPLE TEST FOR COMPARISON OF LONG MEMORY PARAMETERS

Frédéric Lavancier^a & Anne Philippe^a & Donatas Surgailis^b

^a *Laboratoire de Mathématiques Jean Leray, Université de Nantes, France*

^b *Institute of Mathematics and Informatics, Vilnius, Lithuania*

Mots clés :

1. Processus
2. Statistique mathématique

1 Introduction

We propose a procedure for testing the null hypothesis $d_1 = d_2$ that long memory parameters $d_i \in [0, 1/2)$ of two samples of length n , taken from respective stationary processes X_i , $i = 1, 2$, are equal, against the alternative $d_1 > d_2$. A natural extension of one-sample test developed in [1] about unknown long memory parameter d is to consider test statistic, T_n of the form

$$T_n = \frac{V_1/S_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{V_1/S_{11,q}}, \quad (1)$$

where $V_i/S_{ii,q}$ is computed from sample $(X_i(1), \dots, X_i(n))$ ($i = 1, 2$). Here, V_i is the empirical variance of partial sums of X_i

$$V_i = n^{-2} \sum_{k=1}^n \left(\sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2 - n^{-3} \left(\sum_{k=1}^n \sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2. \quad (2)$$

$S_{ii,q}$ is the Newey-West or HAC estimator of the long-run variance of X_i (see [3])

$$S_{ij,q} = \sum_{h=-q}^q \left(1 - \frac{|h|}{q+1} \right) \hat{\gamma}_{ij}(h), \quad (3)$$

where $\gamma_{ij}(h)$ are the empirical cross covariances of samples $(X_1(1), \dots, X_1(n))$ and $(X_2(1), \dots, X_2(n))$.

From [1], one easily derives the asymptotic null distribution T of the statistic T_n under the condition that the two samples are independent. It is also easy to show that

for $d_1 \neq d_2$, one of the ratios in (1) tends to infinity and the other one to zero, meaning that the test is consistent against the alternative $d_1 \neq d_2$.

However, independence of the two samples is too restrictive and may be unrealistic in many applications. In order to eliminate the eventual dependence between samples, a modification \tilde{T}_n of (1) is proposed, which uses residual observations $(\tilde{X}_1(1), \dots, \tilde{X}_1(n))$, obtained by regressing partial sums of X_1 on partial sums of X_2 .

$$\tilde{X}_1(t) = X_1(t) - (S_{12,q}/S_{22,q})X_2(t), \quad t = 1, \dots, n. \quad (4)$$

The statistics \tilde{T}_n is defined as follows

$$\tilde{T}_n = \frac{\tilde{V}_1/\tilde{S}_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{\tilde{V}_1/\tilde{S}_{11,q}}. \quad (5)$$

$\tilde{V}_1, \tilde{S}_{11,q}$ are the statistics in (2), (3), respectively, where $X_1(t), t = 1, \dots, n$ is replaced by $\tilde{X}_1(t), t = 1, \dots, n$.

2 Asymptotic properties of \tilde{T}_n

Consider the class of bivariate linear models $((X_1(t), X_2(t)), t \in \mathbf{Z})$ given by

$$X_i(t) = \sum_{k=0}^{\infty} \psi_{i1}(k)\xi_1(t-k) + \sum_{k=0}^{\infty} \psi_{i2}(k)\xi_2(t-k), \quad i = 1, 2, \quad (6)$$

where

- $\psi_{ij}(k) \sim |k|^{d_{ij}-1}$ ($k \rightarrow \infty$), and $d_{ij} \in (0, 1/2)$.
- $(\xi_1(t), \xi_2(t)), t \in \mathbf{Z}$ is a bivariate (weak) white noise with nondegenerate covariance matrix

$$\begin{pmatrix} 1 & \rho_\xi \\ \rho_\xi & 1 \end{pmatrix}. \quad (7)$$

Let $q = q_n \rightarrow \infty$ and $n/q_n \rightarrow \infty$ as $n \rightarrow \infty$.

Under these conditions, the statistic \tilde{T}_n satisfies the following convergence results :

- (i) If $d_1 = d_2 = d$ then

$$\tilde{T}_n \xrightarrow{\text{law}} \tilde{T}, \quad (8)$$

where the distribution of \tilde{T} only depends on $(d1, d2)$.

- (ii) If $d_1 > d_2$ then

$$\tilde{T}_n \xrightarrow{p} \infty. \quad (9)$$

3 Testing procedure

Let $t_\alpha(d)$ denote the upper α -quantile of the r.v. \tilde{T} defined in (8) when $\rho = 0$), viz.

$$\alpha = P(\tilde{T} > t_\alpha(d)), \quad \alpha \in (0, 1). \quad (10)$$

Let

$$\hat{d} = (\hat{d}_1 + \hat{d}_2)/2, \quad (11)$$

where \hat{d}_i is an estimator of d_i satisfying

$$\hat{d}_i - d_i = o_p(1/\log n) \quad (i = 1, 2). \quad (12)$$

Similarly to [1], it can be proved that the quantile function $t_\alpha(d)$ is continuous in $d \in [0, 1/2)$ for any $\alpha \in (0, 1)$. Therefore, the estimated quantile $t_\alpha(\hat{d}) \rightarrow_p t_\alpha(d)$ as $n \rightarrow \infty$ and the asymptotic level of the tests associated to the critical regions in (13) is preserved by replacing $t_\alpha(d)$ by $t_\alpha(\hat{d})$.

Testing the equality of the memory parameters in the case of possibly dependent samples. We wish to test the null hypothesis $d_1 = d_2$ against the alternative $d_1 > d_2$ in the general case when X_1 and X_2 are possibly dependent. The decision rule at α -level of significance is the following: we reject the null hypothesis when

$$\tilde{T}_n > t_\alpha(\hat{d}). \quad (13)$$

The consistency of this test is ensured by the result given in section 2.

Bibliographie

- [1] Giraitis L., Leipus, R., Philippe, A., 2006. A test for stationarity versus trends and unit roots for a wide class of dependent errors. *Econometric Theory* 21, 989–1029.
- [2] Lavancier, F., Philippe, A., Surgailis, D., 2009. Covariance function of vector self-similar process. *Statist. Probab. Letters* 79, 2415–2421.
- [3] Abadir, K., Distaso, W., Giraitis, L., 2009. Two estimators of the long-run variance: beyond short memory. *Journal of Econometrics* 150, 56–70.