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# A TWO-SAMPLE TEST FOR COMPARISON OF LONG MEMORY PARAMETERS

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**Mots clés :**

1. Processus
2. Statistique mathématique

## 1 Introduction

We propose a procedure for testing the null hypothesis  $d_1 = d_2$  that long memory parameters  $d_i \in [0, 1/2)$  of two samples of length  $n$ , taken from respective stationary processes  $X_i$ ,  $i = 1, 2$ , are equal, against the alternative  $d_1 > d_2$ . A natural extension of one-sample test developed in [1] about unknown long memory parameter  $d$  is to consider test statistic,  $T_n$  of the form

$$T_n = \frac{V_1/S_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{V_1/S_{11,q}}, \quad (1)$$

where  $V_i/S_{ii,q}$  is computed from sample  $(X_i(1), \dots, X_i(n))$  ( $i = 1, 2$ ). Here,  $V_i$  is the empirical variance of partial sums of  $X_i$

$$V_i = n^{-2} \sum_{k=1}^n \left( \sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2 - n^{-3} \left( \sum_{k=1}^n \sum_{t=1}^k (X_i(t) - \bar{X}_i) \right)^2. \quad (2)$$

$S_{ii,q}$  is the Newey-West or HAC estimator of the long-run variance of  $X_i$  (see [3])

$$S_{ij,q} = \sum_{h=-q}^q \left( 1 - \frac{|h|}{q+1} \right) \hat{\gamma}_{ij}(h), \quad (3)$$

where  $\gamma_{ij}(h)$  are the empirical cross covariances of samples  $(X_1(1), \dots, X_1(n))$  and  $(X_2(1), \dots, X_2(n))$ .

From [1], one easily derives the asymptotic null distribution  $T$  of the statistic  $T_n$  under the condition that the two samples are independent. It is also easy to show that

for  $d_1 \neq d_2$ , one of the ratios in (1) tends to infinity and the other one to zero, meaning that the test is consistent against the alternative  $d_1 \neq d_2$ .

However, independence of the two samples is too restrictive and may be unrealistic in many applications. In order to eliminate the eventual dependence between samples, a modification  $\tilde{T}_n$  of (1) is proposed, which uses residual observations  $(\tilde{X}_1(1), \dots, \tilde{X}_1(n))$ , obtained by regressing partial sums of  $X_1$  on partial sums of  $X_2$ .

$$\tilde{X}_1(t) = X_1(t) - (S_{12,q}/S_{22,q})X_2(t), \quad t = 1, \dots, n. \quad (4)$$

The statistics  $\tilde{T}_n$  is defined as follows

$$\tilde{T}_n = \frac{\tilde{V}_1/\tilde{S}_{11,q}}{V_2/S_{22,q}} + \frac{V_2/S_{22,q}}{\tilde{V}_1/\tilde{S}_{11,q}}. \quad (5)$$

$\tilde{V}_1, \tilde{S}_{11,q}$  are the statistics in (2), (3), respectively, where  $X_1(t), t = 1, \dots, n$  is replaced by  $\tilde{X}_1(t), t = 1, \dots, n$ .

## 2 Asymptotic properties of $\tilde{T}_n$

Consider the class of bivariate linear models  $((X_1(t), X_2(t)), t \in \mathbf{Z})$  given by

$$X_i(t) = \sum_{k=0}^{\infty} \psi_{i1}(k)\xi_1(t-k) + \sum_{k=0}^{\infty} \psi_{i2}(k)\xi_2(t-k), \quad i = 1, 2, \quad (6)$$

where

- $\psi_{ij}(k) \sim |k|^{d_{ij}-1}$  ( $k \rightarrow \infty$ ), and  $d_{ij} \in (0, 1/2)$ .
- $(\xi_1(t), \xi_2(t)), t \in \mathbf{Z}$  is a bivariate (weak) white noise with nondegenerate covariance matrix

$$\begin{pmatrix} 1 & \rho_\xi \\ \rho_\xi & 1 \end{pmatrix}. \quad (7)$$

Let  $q = q_n \rightarrow \infty$  and  $n/q_n \rightarrow \infty$  as  $n \rightarrow \infty$ .

Under these conditions, the statistic  $\tilde{T}_n$  satisfies the following convergence results :

- (i) If  $d_1 = d_2 = d$  then

$$\tilde{T}_n \xrightarrow{\text{law}} \tilde{T}, \quad (8)$$

where the distribution of  $\tilde{T}$  only depends on  $(d1, d2)$ .

- (ii) If  $d_1 > d_2$  then

$$\tilde{T}_n \xrightarrow{p} \infty. \quad (9)$$

### 3 Testing procedure

Let  $t_\alpha(d)$  denote the upper  $\alpha$ -quantile of the r.v.  $\tilde{T}$  defined in (8) when  $\rho = 0$ ), viz.

$$\alpha = P(\tilde{T} > t_\alpha(d)), \quad \alpha \in (0, 1). \quad (10)$$

Let

$$\hat{d} = (\hat{d}_1 + \hat{d}_2)/2, \quad (11)$$

where  $\hat{d}_i$  is an estimator of  $d_i$  satisfying

$$\hat{d}_i - d_i = o_p(1/\log n) \quad (i = 1, 2). \quad (12)$$

Similarly to [1], it can be proved that the quantile function  $t_\alpha(d)$  is continuous in  $d \in [0, 1/2)$  for any  $\alpha \in (0, 1)$ . Therefore, the estimated quantile  $t_\alpha(\hat{d}) \rightarrow_p t_\alpha(d)$  as  $n \rightarrow \infty$  and the asymptotic level of the tests associated to the critical regions in (13) is preserved by replacing  $t_\alpha(d)$  by  $t_\alpha(\hat{d})$ .

*Testing the equality of the memory parameters in the case of possibly dependent samples.* We wish to test the null hypothesis  $d_1 = d_2$  against the alternative  $d_1 > d_2$  in the general case when  $X_1$  and  $X_2$  are possibly dependent. The decision rule at  $\alpha$ -level of significance is the following: we reject the null hypothesis when

$$\tilde{T}_n > t_\alpha(\hat{d}). \quad (13)$$

The consistency of this test is ensured by the result given in section 2.

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