Les dimensions Fractals : Mythes et réalités écologiques

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Français

La notion de fractal proposée par Mandelbrot a séduit la communauté écologique pour les notions d'indépendance d'échelle et de transfert d'échelle qu'elle recèle. L'objectif de ce travail est de préciser qu'en réalité, la notion de fractal cache deux concepts différents de rugosité d'un part et d'auto-homothétie d'autre part. Les cas où ces notion se rejoignent sont très particuliers (processus Gaussiens) et rarement rencontrés dans la nature. En s'appuyant sur la définition rigoureuse de la dimension fractal qui repose sur la dimension de Haussdorf-Besicovitch, on montre que la dimension fractal quantifie une propriété locale des processus à savoir la rugosité. De plus, lorsque son estimation est basée sur le variogramme, on montre que la formule retenue par la bibliographie écologique qui utilise la demie-pente du log-variogramme est erronée. L'utilisation de la pente du log-madogramme est recommandée. La différence est illustrée à l'aide de données réelles.
La dimension d'auto-homothétie doit être distinguée de la dimension fractal. On montre des cas où la dimension fractal est non entière alors que le processus n'est pas auto-homothétique. Il est donc important, dans la pratique, de ne pas confondre ces deux notions souvent cachées derrière le même vocable de dimension fractal.
Enfin, parce que précisément la dimension fractal peut être associée à des indépendances d'échelles, il importe de préciser l'effet du support d'information, qui ici, comme dans l'ensemble des statistiques spatiales existe.

Anglais

Fractals have been widely used in ecology. Among other, their scale independence seems interesting for describing scale transfer in ecology. We show here that the original definition based on the Haussdorf-Besicovitch dimension (DHB), as well as the estimators based on Minkowski-Boulingand dimension (DMB) actually quantify a local property called roughness. As a consequence, the scale independence property of fractals, which is very attractive and powerful in ecological modelling, is left out by the mathematical definition which refers to a local property, the roughness. The primary objective of the present paper is to clarify the dual aspect of fractals and to show that those two properties may be quantified by a single dimension (DHB) only in some specific cases. We show that for natural objects in ecology, a non-integer DHB quantifies per se only the roughness and self-similarity needs to be evidenced or postulated by other means. Second, we revisit some aspects of the practical estimation of DHB. We show that when geostatistics is used, the formula based on the slope at the origin of the variogram, widely found in the literature, indeed only holds for Gaussian situations. We recommend the use of a more general formula based on the first order variogram (madogram) which holds for any type of variable distribution. We discuss then other aspects related to the estimation of DHB on non self-similar objects, its simplification for 2D fields, and its possible relationship with self-similarity. In resonance with earlier works, we point out the problem of scale and resolution when estimating a fractal dimension. Some of our considerations and developments are illustrated with field data recorded during a scientific acoustic survey on the North Sea herring. I conclude on a synthesis of practical

recommendations and warnings to ecologists when using fractal dimension..

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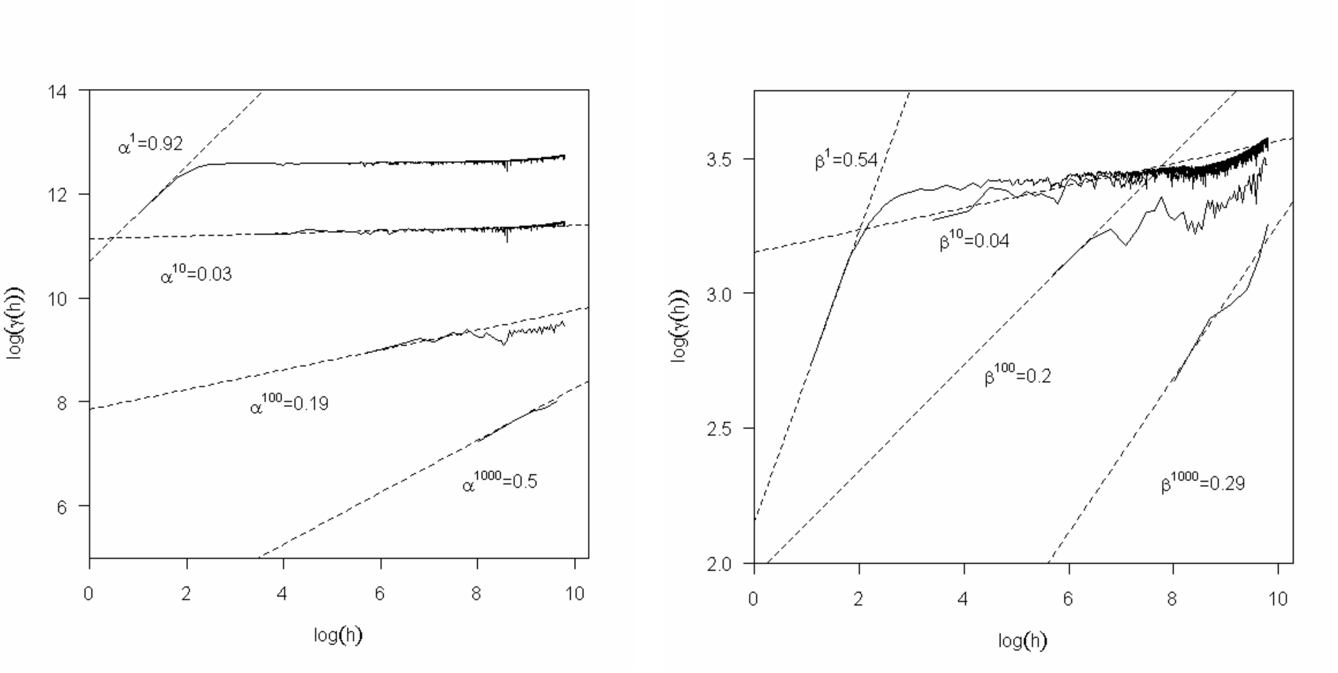

Field acoustic data collected along one scientific survey transect in the North Sea. These data were available at the smallest possible resolution (i.e. one ping ~ 3m) up to any concatenation of pings: 10 pings ~30 m, 100 pings~300 m, and 1000 pings~3000 m. They should be considered as exhaustive along the survey transect. For each of the four resolutions, we computed the empirical madogram (right panel) and variogram (left panel) and considered their log-log representations together with their linear regression for small distances. The slopes at the origin of the log-variogram are not twice the slopes of the log-madogram. In addition , their vary with the support indicating a modification of the fractal dimension with the support.