Joint Waveform Adaptation and Power Control in Multi-Cell Wireless Data Networks: A Game-Theoretic Approach

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***Abstract*—The problem of non-cooperative spreading code allocation, linear receiver design, and transmit power control for multicell wireless networks is considered in this paper. Several utility functions to be maximized are considered, and, among them, we cite the received SINR, and the transmitter energy efficiency, which is measured in bit/Joule, and represents the number of successfully delivered bits for each energy unit used for transmission. Resorting to the theory of potential games, non-cooperative games admitting Nash equilibria in multi-cell networks regardless of the channel coefficient realizations are designed. Computer simulations confirm that the considered games are convergent, and show the huge benefits that resource allocation schemes can bring to the performance of wireless data networks.**

***Index Terms*—Multicell networks, potential games, energy-efficiency, waveform adaptation, power control.**

I. Introduction

Game theoretic tools are nowadays widely used to design and analyze resource allocation procedures in wireless networks. Indeed, game theory, a branch of mathematics studying the interactions among several autonomous subjects with contrasting interests, can be well used in multiuser wireless networks to model the interactions between selfish active users, who are indeed in mutual competition for the available bandwidth and, in general, for the shared available network resources [1].

This paper is concerned with a multipoint-to-multipoint wireless data network using a nonorthogonal multiple access strategy such as code division multiple access (CDMA). The considered model is general enough to model a standard multi-cell wireless data network, an ad-hoc network with multiple peer-to-peer links, a wireless network with femtocells, and, finally, a cognitive radio system with primary and secondary users. While several studies and abundance of results are available on non-cooperative resource allocation procedures for single-cell data networks (see [2]–[5] and references therein for a non-exhaustive list), the case of multi-cell networks has been instead much less investigated, also due to the fact that several non-cooperative resource allocation games conceived for single-cell systems appear to be no longer convergent (i.e., to have no equilibrium) in systems with multiple receivers (i.e., in multi-cell networks). As notable exceptions, we cite here the work [6], wherein a non-cooperative power control game for energy-efficiency maximization is proposed, and the recent paper [7], wherein, resorting to the theory of potential games [8], a non-cooperative spreading code allocation algorithm has been proposed, under the assumption that a simple matched filter is used at the receiver. Roughly speaking, in a potential game each change in the utility enjoyed by a given player due to an *unilateral* change of strategy by that player is paired by a similar change in a global function called the potential function. In a potential game, the best response strategy always leads to a Nash equilibrium (NE). Using [7] as our departure point, in this paper we make the following contributions:

- We propose and analyze several non-cooperative games for joint transmitter and receiver optimization, and aimed at maximizing utility functions strictly related to the signal-to-interference plus noise ratio (SINR). For such games we analytically prove the existence of Nash equilibria.
- We propose a non-cooperative joint transceiver optimization and transmit power control game aimed at maximization of the energy efficiency of each active user. Energy efficiency, measured in bit/Joule, represents the number of bits that are *successfully* delivered at the receiver for each energy unit taken from the battery and used for transmission. Unfortunately, for such a game the existence of an NE is shown only through numerical evidence, since we were not able to obtain an analytical proof.

We give extensive numerical simulations showing the huge benefits that the considered resource allocation schemes bring to the performance of a multi-cell wireless network.

This paper is organized as follows. Next section contains a description of the considered multi-cell system model, while section III is devoted to the exposition of transmitter and receiver non-cooperative adaptation games for performance maximization. Section IV considers the problem of energy-efficient non-cooperative power control and transmitted waveform adaptation. Numerical results are discussed in Section V, while brief concluding remarks are given is Section VI.

II. System model for a general multi-cell network

Let us thus consider the uplink of a multi-cell direct-sequence CDMA wireless data network. Denote by B the number of access points, and let $h_{i,j}$ be the real channel gain between the i-th user and the j-th AP; moreover, denote by

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$a(i)$ the index of the AP assigned to the i-th user[1]. After chip-matched filtering and chip-rate sampling, the N-dimensional received data vector at the ℓ-th AP, say $\boldsymbol{r}_\ell$, can be written as

$$\boldsymbol{r}_\ell = \sum_{k=1}^{K} \sqrt{p_k} h_{k,\ell} b_k \boldsymbol{s}_k + \boldsymbol{n}_\ell \ , \quad \ell = 1, \ldots, B \ . \qquad (1)$$

Assuming that a linear detector is used at the receiver, so that the symbol b_k is detected according to the rule $\widehat{b}_k = \text{sign}\left\{\boldsymbol{d}_k^T \boldsymbol{r}_{a(k)}\right\}$, the SINR for the k-th user is expressed as

$$\gamma_k = \frac{p_k h_{k,a(k)}^2 (\boldsymbol{d}_k^T \boldsymbol{s}_k)^2}{\boldsymbol{d}_k^T \left(\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T\right) \boldsymbol{d}_k} \qquad (2)$$

III. Spreading code allocation

We consider now the problem of spreading code allocation for multi-cell system, thus reviewing some of the existing non-cooperative approaches, and proposing two new procedures.

A. Greedy spreading code allocation with LMMSE reception [5]

Consider the case that an linear minimum mean square error (LMMSE) filter is used at the receiver. In this case the k-th user SINR can be expressed as

$$\gamma_k = p_k h_{k,a(k)}^2 \boldsymbol{s}_k^T \left(\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T\right)^{-1} \boldsymbol{s}_k \ . \quad (3)$$

Given the above expression, it is trivially shown that the SINR-maximizing spreading code for the k-th user is the eigenvector associated to the minimum eigenvalue of the matrix

$$\left(\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T\right) \ ,$$

which is indeed the covariance matrix of the overall interference suffered by the k-th user. The non-cooperative game wherein users cyclically update their spreading code in order to maximize the SINR in Eq. (3) is widely known as greedy interference avoidance procedure [5]. Such a procedure, while being always convergent in single-cell systems, does not always converge in multi-cell systems, and is thus not suited to our scenario; for comparison purposes, however, in the following we will include performance results for this technique as well.

B. Minimization of the individual MSE [4], [9]

As an alternative optimization criterion, we can consider minimization of the individual MSE. The MSE for the k-th user, say ϵ_k^2, is expressed as

$$\begin{aligned}\epsilon_k^2 = E\left\{(b_k - \boldsymbol{d}_k^T \boldsymbol{r}_{a(k)})^2\right\} = 1 - 2\sqrt{p_k} h_{k,a(k)} \boldsymbol{d}_k^T \boldsymbol{s}_k - \\ -\frac{\mathcal{N}_0}{2}\|\boldsymbol{d}_k\|^2 + \boldsymbol{d}_k^T \left(\sum_{j=1}^{K} p_j h_{j,a(k)}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T\right) \boldsymbol{d}_k \ .\end{aligned} \qquad (4)$$

Following [4], [9], it is easily seen that the minimizer of ϵ_k^2 can be obtained as the unique stable fixed point of the following iterations:

$$\begin{cases} \boldsymbol{d}_k = \sqrt{p_k} h_{k,a(k)} \boldsymbol{M}_{\boldsymbol{r}_{a(k)}}^{-1} \boldsymbol{s}_k \ , \\ \boldsymbol{s}_k = \boldsymbol{d}_k / \|\boldsymbol{d}_k\| \ , \end{cases} \qquad (5)$$

for any $k = 1, \ldots, K$. In the above equation $\boldsymbol{M}_{\boldsymbol{r}_{a(k)}} = E\left\{\boldsymbol{r}_{a(k)} \boldsymbol{r}_{a(k)}^T\right\}$ is the covariance matrix of the data vector received at the $a(k)$-th AP. Now, while it is well known that iterations (5) are always convergent in a single-cell system, in a multi-cell system convergence is not always guaranteed. Again, in the following we will be including performance results also for this technique for comparison purposes.

C. Maximization of the sum of inverse SINR [7]

As previously discussed, non-cooperative maximum SINR game with respect to the spreading code and uplink receiver [5] is not always convergent. In [7], instead, based on the theory of potential games, a modification to the utility function to be considered has been introduced, so as to have a guaranteed convergence for any channel realizations. Let us thus assume that a matched filter (MF) is used at the receiver and consider the sum of the inverse SINR, i.e.:

$$V = \sum_{k=1}^{K} \frac{1}{\gamma_k} \ . \qquad (6)$$

Pointing out the dependence on the k-th spreading code $\boldsymbol{s}_k$, V can be expressed as

$$\begin{aligned} V = \ & \boldsymbol{s}_k^T \left[\frac{\sigma_n^2}{p_k h_{k,a(k)}^2} \boldsymbol{I} + \right. \\ & \left. + \sum_{j \neq k} \left(\frac{p_j h_{j,a(k)}^2}{p_k h_{k,a(k)}^2} + \frac{p_k h_{k,a(j)}^2}{p_j h_{j,a(j)}^2}\right) \boldsymbol{s}_j \boldsymbol{s}_j^T\right] \boldsymbol{s}_k + D \ , \end{aligned} \qquad (7)$$

with D an additive term independent of $\boldsymbol{s}_k$. It is thus clear that a non-cooperative game wherein the utility function for the k-th user is

$$\begin{aligned} u_k = \ & \boldsymbol{s}_k^T \left[\frac{\sigma_n^2}{p_k h_{k,a(k)}^2} \boldsymbol{I} + \right. \\ & \left. + \sum_{j \neq k} \left(\frac{p_j h_{j,a(k)}^2}{p_k h_{k,a(k)}^2} + \frac{p_k h_{k,a(j)}^2}{p_j h_{j,a(j)}^2}\right) \boldsymbol{s}_j \boldsymbol{s}_j^T\right] \boldsymbol{s}_k \ , \end{aligned} \qquad (8)$$

is a potential game with potential function V and thus admits an NE.

[1]Note that we are assuming here that each user is assigned to a certain AP, i.e. AP assignments have already taken place.

$$Q = \sum_{m=1}^{K} \rho_m = \underbrace{s_k^T \left[p_k h_{k,a(k)}^2 \sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_k p_j h_{k,a(k)}^2 h_{j,a(k)}^2 s_j s_j^T + \sum_{j \neq k} p_k p_j h_{k,a(j)}^2 h_{j,a(j)}^2 s_j s_j^T \right] s_k}_{\text{depends on } s_k} + \underbrace{\sum_{j=1, j\neq k}^{K} p_j h_{j,a(j)}^2 s_j^T \left(\sigma_n^2 \boldsymbol{I} + \sum_{\ell \neq k,j} p_l h_{\ell,a(j)}^2 s_\ell s_\ell^T \right) s_j}_{\text{does not depend on } s_k} . \tag{10}$$

$$\sum_{m=1}^{K} \epsilon_m^2 = \underbrace{1 - 2\sqrt{p_k} h_{k,a(k)} \boldsymbol{d}_k^T s_k + \frac{\mathcal{N}_0}{2} \|\boldsymbol{d}_k\|^2 + \boldsymbol{d}_k^T \left(\sum_{j=1}^{K} p_j h_{j,a(k)}^2 s_j s_j^T \right) \boldsymbol{d}_k + \sum_{\ell \neq k} \boldsymbol{d}_\ell^T \left(p_k h_{k,a(l)}^2 s_k s_k^T \right) \boldsymbol{d}_\ell}_{\text{depends on } s_k} + \underbrace{(K-1) + \sum_{\ell \neq k} \boldsymbol{d}_\ell^T \left(\sum_{j \neq k} p_j h_{j,a(\ell)}^2 s_j s_j^T \right) \boldsymbol{d}_\ell - 2 \sum_{\ell \neq k} \sqrt{p_\ell} h_{\ell,a(\ell)} \boldsymbol{d}_\ell^T s_\ell + \sum_{\ell \neq k} \frac{\mathcal{N}_0}{2} \boldsymbol{d}_\ell}_{\text{does not depend on } s_k} . \tag{12}$$

D. Greedy interference avoidance revisited

We now propose a new non-cooperative game which will be shown to achieve much superior performance levels than the previously discussed solutions.

Since the greedy interference avoidance procedure is not always convergent in multi-cell systems, we resort to the theory of potential games in order to come up with a modified utility function whose non-cooperative maximization leads to an NE. Assume that an LMMSE detector is user at the receiver, so that the k-th user SINR can be shown to be written as

$$\gamma_k = p_k h_{k,a(k)}^2 s_k^T \left(\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 s_j s_j^T \right)^{-1} s_k .$$

Considering the minimization of the sum of the inverse SINRs (as done in [7] for the case of a matched filter receiver) reveals to be a complicated task in this case, and, also, maximization of the sum of the SINRs turns out to be complicated as well. We consider instead the following quantity

$$Q = \sum_{k=1}^{K} \rho_k = \sum_{k=1}^{K} p_k h_{k,a(k)}^2 s_k^T \left(\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 s_j s_j^T \right) s_k . \tag{9}$$

Note that the above quantities is directly tied to the SINRs enjoyed by the active users in the network, since it is easy to show that Q is a decreasing function of the SINR of each user. Upon straightforward algebraic manipulation, we find Eq. (10) shown at the top of the page. Accordingly, a non-cooperative game wherein each user aims at maximizing the utility

$$u_k = -s_k^T \left[\sigma_n^2 \boldsymbol{I} + \sum_{j \neq k} p_j h_{j,a(k)}^2 s_j s_j^T + \sum_{j \neq k} p_j \frac{h_{k,a(j)}^2}{h_{k,a(k)}^2} h_{j,a(j)}^2 s_j s_j^T \right] s_k , \tag{11}$$

is a potential game whose potential function is $-Q$. Accordingly, such a non-cooperative game always admits an NE.

E. Non-cooperative minimization of the TMSE

Since non-cooperative minimization of the individual MSE is not always convergent in a multi-cell scenario, we can again resort to the theory of potential games to obtain a convergent non-cooperative game in this case too. Let us thus consider the total MSE, defined as $\sum_{k=1}^{K} \epsilon_k^2$. Upon some straightforward algebraic manipulations, we have Eq. (12), shown at the top of this page. It is easy to realize that the part dependent on s_k, say $L(s_k)$, may be written as

$$L(s_k) = \epsilon_k^2 + \sum_{\ell \neq k} \boldsymbol{d}_\ell^T \left(p_k h_{k,a(l)}^2 s_k s_k^T \right) \boldsymbol{d}_\ell , \tag{13}$$

thus implying that the latter summand in the right-hand-side of the above equation is the correcting term that needs to be added to the MSE for the k-th user to make the non-cooperative game convergent. Summing up, we thus consider the following game:

$$\min_{s_k, \boldsymbol{d}_k} L(s_k) , \text{ subject to: } \|s_k\| = 1 . \tag{14}$$

Using standard Lagrangian optimization techniques, we have that the solution to (13) is written as

$$s_k = \sqrt{p_k} h_{k,a(k)} \left(\lambda \boldsymbol{I} + \sum_{\ell=1}^{K} p_k h_{k,a(\ell)}^2 \boldsymbol{d}_\ell \boldsymbol{d}_\ell^T \right)^{-1} \boldsymbol{d}_k , \tag{15}$$

where λ, the Lagrange multiplier, is such that $\|s_k\| = 1$.

IV. A NON-COOPERATIVE GAME FOR ENERGY EFFICIENT COMMUNICATIONS

Let us now consider the case that each transmitter is interested in maximizing its energy-efficiency, i.e. the number of data bits successfully delivered to the receiver for each energy unit taken from the battery and used for transmission. Following [3], [6], the following utility function should be considered for the k-th user

$$u_k = R \frac{L}{M} \frac{f(\gamma_k)}{p_k} , \tag{16}$$

with R the transmit data rate, L/M the ratio between the payload length and the total length of each data packet, and $f(\cdot)$ the efficiency-function, which is usually expressed as $f(\gamma_k) = (1 - e^{-\gamma_k})^M$. We are here interested in the non-cooperative maximization of u_k with respect to p_k, $\boldsymbol{s}_k$ and $\boldsymbol{d}_k$. Note that

$$\max_{p_k, \boldsymbol{s}_k, \boldsymbol{d}_k} \frac{f(\gamma_k)}{p_k} = \max_{p_k} \frac{f\left(\max\limits_{\boldsymbol{s}_k \boldsymbol{d}_k} \gamma_k\right)}{p_k} . \tag{17}$$

Otherwise stated, each user can take care first of SINR maximization with respect to $\boldsymbol{s}_k$ and $\boldsymbol{d}_k$, and then of the maximization of its energy-efficiency. While things are easy in a single-cell system, and indeed results for this scenario are reported in [3], in multi-cell systems unfortunately the SINR maximization is well known to be not always convergent, thus implying that some approximations and modifications are to be considered in order to obtain a game admitting an NE. On the other hand, maximization of the energy efficiency with respect to the transmit power only leads to an NE also in a multi-cell network, and indeed this problem has been thoroughly analyzed in [6], for the case in which a plain matched filter is used at the receiver. Following the same approach as in [6], it is easy to show that non-cooperative energy-efficiency maximization with respect to the transmit power converges to an NE also in the case in which an LMMSE multiuser detector is used at the receiver.

Here, instead, we propose to consider the concatenation of two different games, namely

a) for fixed transmit powers, the non-cooperative minimization of the TMSE with respect to the users' spreading codes, assuming that an LMMSE receiver is used at the receiver; and
b) for fixed spreading codes, the maximization of the energy-efficiency (16) with respect to the transmit powers.

More precisely, we assume that users continuously switch between games a) and b), until convergence is reached. Unfortunately, we are not able to provide an analytical proof that the proposed alternative strategy always converges to an NE, and indeed this is the object of current investigation; however, we point out that extensive numerical simulations have shown that an NE always exists; the remarkable performance advantage that the proposed strategy brings with respect to the case in which spreading code adaptation is not carried out are discussed in the forthcoming section.

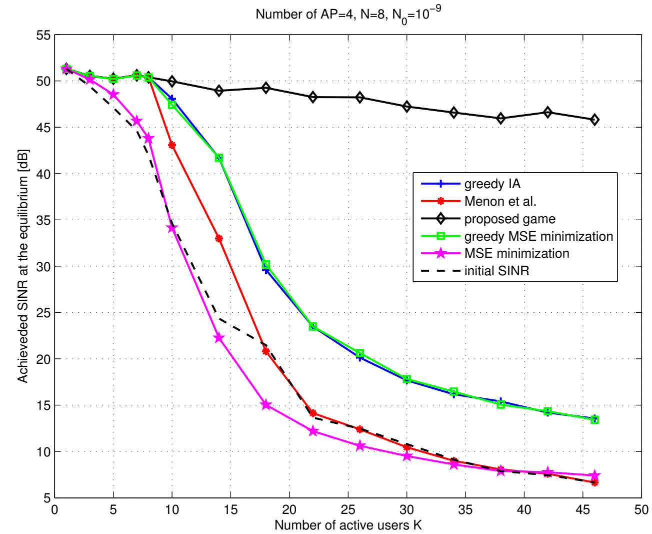

Figure 1. Achieved SINR at the equilibrium for the considered spreading code allocation procedures versus the number of active users.

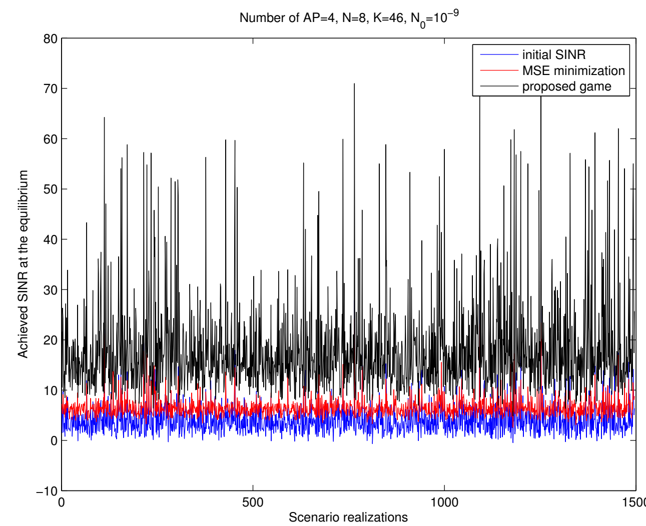

Figure 2. Achieved SINR at the equilibrium across the 1500 random realizations for a system with $K = 46$ active users.

V. NUMERICAL RESULTS

We have considered a system with processing gain $N = 8$ and $B = 4$ APs placed in a square of 10^6 sq. meters. Users' location have been randomly generated inside this area, while the channel coefficients $h_{i,j}^2$ have been generated according to an exponential distribution with mean equal to $d_{i,j}^{-2}$, with $d_{i,j}$ the distance between the i-th user and the j-th access point. It is assumed that each user's data are decoded at the AP with the largest channel coefficient, namely $a(k) = \arg \max\limits_{\ell=1,\ldots,B} \left(h_{k,\ell}^2\right)$. The curves here shown come from an average over 1000 independent realizations of the channel coefficients, users' locations, and starting set of spreading codes.

Fig. 1 shows the achieved SINR at the equilibrium for the illustrated spreading code allocation procedures versus the number of active users, while Fig. 2 reports the achieved SINR at the equilibrium across the 1500 random realizations for a

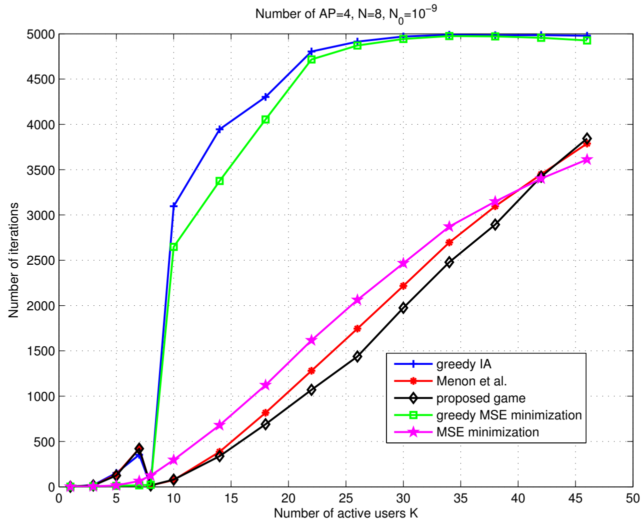

Figure 3. Number of iterations needed to reach the equilibrium for the considered spreading code allocation procedures versus the number of active users. A maximum of 5000 iterations has been included in the simulation program.

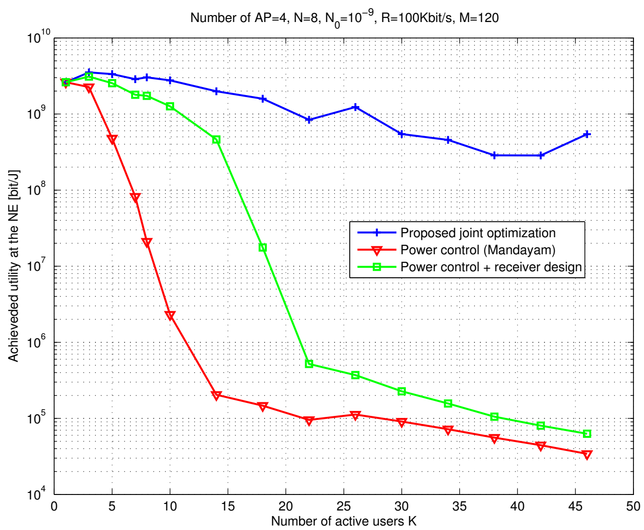

Figure 4. Achieved energy-efficiency (bit/Joule) at the equilibrium versus the number of active users for three different non-cooperative games, i.e. (a) power control with a matched filter at the receiver [6], (b) joint power control and uplink receiver design , and (c) joint power control, spreading code allocation and uplink receiver design.

system with $K = 46$ users. It is seen that the newly proposed procedure in section III.D (greedy interference avoidance revisited) largely outperforms the other games. In Fig. 3, we report the average number of iterations needed to achieve convergence. A maximum of 5000 iterations has been included in the simulation program, and it is seen that greedy interference avoidance [5] and individual MSE minimization [9] games at some point reach such a maximum value, thus confirming the not-always convergent behavior of these procedures.

Finally, in Fig. 4 we report the achieved energy-efficiency (bit/Joule) at the equilibrium (which, we recall, has been reached in all the randomly generated scenarios) versus the number of active users for three different non-cooperative games, i.e. (a) power control with a matched filter at the receiver [6], (b) joint power control and uplink receiver design, and (c) joint power control, spreading code allocation and uplink receiver design. Again, we see that the newly proposed joint procedure greatly outperforms the competing alternatives: indeed, for a fully loaded system (i.e. $K = 46$ users), the proposed game provides at the NE an energy efficiency that is 10^4 times larger than that achieved by the alternatives, which means that, for a given amount of energy to be used for transmission, 10.000 times more data can be reliably transmitted.

VI. Conclusions

This paper has considered the problem of joint transmitter waveform adaptation and power control in a multi-cell multiuser wireless data network. Leveraging on the study [7], wherein it has been revealed that the theory of potential games can be used to obtain convergent non-cooperative resource allocation games in multi-cell networks, we have proposed a transmitter waveform adaptation game that has been shown to outperform competing alternatives. Additionally, we also considered the issue of energy-efficiency in a multi-cell network, and a new joint power control and transmit waveform adaptation game has been proposed for its maximization. Unfortunately, the existence of an NE for such a game has been shown only through numerical evidence, while an analytical proof is still missing and forms the object of current research. It should be also noted that the considered framework may be used to assess the impact on the system performance of the introduction of femtocells in wireless data networks. Indeed, wireless networks using femtocells are just an instance of a general multi-cell network: the application of the results of this paper to systems equipped with femtocell is indeed a further object of the author's current research.

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