A Contracts-based Approach for Spectrum Sharing Among Cognitive Radios

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***Abstract*—Development of dynamic spectrum access and allocation techniques recently have made feasible the vision of cognitive radio systems. However, a fundamental question arises: Why would licensed primary users of a spectrum band allow secondary users to share the band and degrade performance for them? This incentive issue has been sought to be addressed by designing incentive-compatible auction mechanisms [3]. This, however, does not solve the problem as the auctioneer (usually the primary user) is also an interested party. So, why would the secondary user trust the primary user to not manipulate the auction. We propose that a more appropriate mechanism to solve this incentive problem is a contractual mechanism. In this paper, we consider a simple setting: A single primary transmitter-receiver pair and a single secondary transmitter-receiver pair with a Gaussian interference channel between them. We consider the setting of complete information when channel attenuation coefficients and noise levels at the receivers are common knowledge. We consider when receivers cooperate to do successive-interference cancellation. In contrast to the results of [4] for unlicensed bands, we show that it is possible to achieve socially optimal rate allocations with contract mechanisms in licensed bands.**

I. INTRODUCTION

The scarcity of spectrum is becoming an impediment to the growth of more capable wireless networks. Several measures are sought to address this problem: Freeing up unused spectrum, sharing of spectrum through new paradigms such as cognitive radio sensing, as well as sophisticated information theoretic schemes and network coding methods. Nearly all such methods presume perfect user cooperation. This, however, is an unjustified assumption. And with non-cooperative, selfish users who act strategically, network (sum-rate) capacity can be arbitrarily bad as shown for the single-hop Gaussian interference channel in unlicensed bands [4].

For licensed bands, one of the key challenges is when users have cognitive radio capability, *why would primary users give up their ownership rights over their spectrum and share it with secondary users at the cost of performance degradation to themselves?* FCC mandates are not going to solve the problem as primary users can always transmit junk to keep channels busy and deter secondary users [12], [1].

This incentive issue has been sought to be addressed by guaranteeing the primary user a payment in lieu of sharing his spectrum and suffering some performance degradation. This has been sought to be implemented in various ways: dynamic competitive pricing [13], [9], [11], and spectrum auctions [7], [6], [3] . While competitive pricing is usually not incentive-compatible and not robust to manipulation by strategic users, carefully designed auctions can potentially be strategy-proof and yield socially optimal outcomes. In many scenarios, we can even operate them as double-sided auctions or markets when there are both buyers and sellers. Unfortunately, for auctions to be practical, they must be operated by a neutral, disinterested party as an auctioneer. Otherwise, the auctioneer can manipulate the auctions to his advantage. This situation is unlikely to arise in most cognitive radio systems. The spectrum sharing and allocation must happen as a direct result of interaction between a primary user and one or more secondary users. Thus, a *principal-agent model* is more appropriate for such a scenario [10], [2]. One user (possibly the primary) acts as a principal, and offers several contracts to the agent(s) (possibly the secondary user(s)). The agent(s) then picks one of the possible contracts or may reject all of them. We specify the class of contracts that a primary user can offer such

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that both the primary and the secondary users are able to maximize their individual utilities while still achieving a social welfare objective. The principal-agent model has been used in solving some problems in communication networks such as network formation [8] and wireless multihop routing [5]. The principal-agent model and the contractual mechanism approach to spectrum sharing is new.

II. Model and Related Work

Suppose there are M transmitter-receiver pairs that share the Gaussian Interference Channel (GIC) of bandwidth W. We can model it as: $y_i[n] = \Sigma_{j=1}^{M} h_{j,i} x_j[n] + z_i[n]; i = 1, \ldots M$ where x_i is the signal from the transmitter i, y_i is the signal received at the receiver i, $h_{j,i}$ is the channel attenuation coefficient from transmitter j to receiver i, and the noise process z_i are i.i.d. over time with distribution $\mathcal{N}(0, N_0)$. Each user could treat the signal from other users as interference. We also assume that the users use random Gaussian codebooks for transmission. Then, the maximum rate that the system can achieve is given by $R_i = W \log\left(1 + \frac{c_{i,i} P_i}{N_0 W + \Sigma_{j \neq i} c_{j,i} P_j}\right)$, $\forall i$ where P_i is the transmitted power of the user i, and $c_{i,j} = |h_{i,j}|^2$. Also, due to power constraints, P_i must satisfy $P_i \leq \bar{P}_i$ for all i.

The spectrum sharing problem is to determine a set of power allocations $P = (P_1, \cdots, P_M)$ that maximize a given global utility function (such as the achievable sum-rate $\sum_i R_i$) while satisfying the power constraints. However, users are selfish and may not cooperate with each wanting to maximize its' own rate. Thus, they pick their power allocations P each wanting to maximize their own rate and leading to a spectrum sharing game between them. To predict the outcome of such a game, we look at its' Nash equilibrium P^* such that given the power allocations of all the other users P^*_{-i}, user i's rate is maximized at P^*_i, i.e., $R_i(P^*_i, P^*_{-i}) \geq R_i(P_i, P^*_{-i}), \ \ \forall P_i \leq \bar{P}_i$. In [4], it was shown that in a flat fading GIC a Nash equilibrium (NE) exists, all NE are pure strategy equilibria, and under certain conditions, full-spread power allocation is a NE. Moreover they have shown that under some conditions, full-spread is the unique NE. However, in most cases, the set of rates that result from the full-spread NE is not Pareto efficient. So there may be a significant performance loss if the M users operate at this point due to lack of cooperation. In fact, in many cases this inefficient outcome is the only possible outcome of the game. For the more general parallel GIC, existence of Nash equilibrium was proved in [14].

The above discussion assumed that users did not use cooperative schemes from multi-user information theory. A particular scheme of relevance is *Successive Interference Cancellation (SIC)* which works as follows. Suppose user 1 decodes his own signal by treating interference from all other users as noise, then he can achieve a rate $R_1 = \log\left(1 + \frac{c_{1,1} P_1}{N_0 W + \Sigma_{j>1} c_{j,1} P_j}\right)$. Now, user 2 can do the same and decode user 1's signal first as above. Then, he can subtract this signal from the received signal, and decode his own signal by treating all other users as noise and achieve a rate $R_2 = \log\left(1 + \frac{c_{2,2} P_2}{N_0 W + \Sigma_{j>2} c_{j,2} P_j}\right)$, which is greater than what he could have received if he had treated user 1's signal as noise as well. Proceeding in this way, user M then achieves a rate $R_M = \log\left(1 + \frac{c_{M,M} P_M}{N_0 W}\right)$.

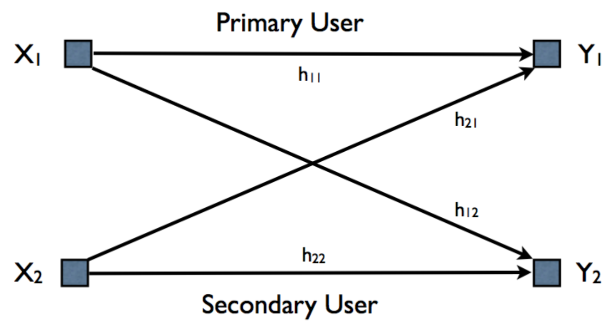

Fig. 1. Gaussian Interference channel with a primary and secondary user

When there are only two users, we will call user 2 as a *dominant user*, and the other as a *non-dominant user*. If we proceed according to a different permutation on the users, we will achieve a different rate vector R'. By time-sharing between the various achievable rate vectors, we get an achievable rate region $\mathcal{R}_{SIC}$ that is strictly larger than that obtained with the naive non-cooperative scheme $\mathcal{R}_{naive}$. Thus, all users could potentially gain if we could find a way for them to cooperate. This would be successful if their incentives are aligned for cooperation.

III. Contract Designs for Spectrum Sharing

Spectrum sharing with naive coding, i.e., treating interference from other users as noise can lead to inefficient outcomes as Nash equilibria. Thus, a question arises whether it is possible to alleviate this inefficiency by introducing an incentive alignment mechanism. Our focus in this paper is a licensed band setting with cognitive radios where there is a *primary user* who owns the spectrum band and a *secondary user* who wants to share the spectrum with the primary user. When there are only two users sharing a Gaussian interference channel, if they both agree to cooperate in doing successive interference cancellation, the *dominant user* suffers no performance degradation at all. His achievable rate is the same as if the other user were not present. However, the primary user still may not want to share spectrum as sharing spectrum by doing more sophisticated coding (e.g., SIC) can entail a *coding complexity cost*, thus pointing out to a clear need for an incentive alignment scheme for sharing spectrum using advanced cooperative communication schemes such as SIC.

We now introduce the *principal-agent model* [10][2] where the principal offers one or more contracts to the agent. The agent then selects one or rejects all. Denote the power used by the principal as P_p and that used by the agent as P_a. Let the utility of the principal be its rate $R_p(P_p, P_a)$ and utility of the agent be its rate $R_a(P_a, P_p)$. Denote by $\lambda(P_p, P_a)$ the payment that the agent makes to the principal. This can be positive or negative.

We define a *spectrum contract* to be the tuple $(\lambda(P_p, P_a), P_p, P_a)$ such that the operating point is (P_p, P_a) and the payment is $\lambda(P_p, P_a)$. The principal wants to design a contract payment function $\lambda(P_p, P_a)$ that maximizes his payoff $R_p + \lambda$ once the agent has accepted and picked the operating point (P_p, P_a). Let $\bar{U}_p$ and $\bar{U}_a$ denote the reservation utilities for the principal and the agent that they can derive if the contract is not accepted. The agent will accept a contract only if he can find a *feasible* operating point at which his payoff is at least as large as his reservation utility $\bar{U}_a$. This is called an *individual rationality (IR)* constraint, i.e., it has to be *rational* for the agent to participate. Furthermore, among the IR and IC operating points, he will pick one that maximizes his payoff. This is called *incentive compatibility (IC)*. Thus, in designing the contracts, the principal should take both the IR and IC constraints into account. The principal's problem is then given by the following optimization problem.

CD-OPT:

$$\begin{aligned} \max_{P_p, P_a, \lambda} \quad & & R_p(P_p, P_a) + \lambda(P_p, P_a) \\ s.t. \quad & \textbf{[IR]:} & R_a(P_a, P_p) - \lambda(P_p, P_a) \geq \bar{U}_a \\ & \textbf{[IC]:} & R_a(P_s, P_p) - \lambda(P_p, P_a) \geq \\ & & R_a(P_a^{'}, P_p) - \lambda(P_p, P_a^{'}), \forall {P_a}' \leq \bar{P}_a. \end{aligned}$$

The rate functions R_p, R_a above are given by $R_1(P_1, P_2) = W \log(1 + \frac{c_{1,1} P_1}{N_0 W + c_{2,1} P_2})$. The optimization problem **CD-OPT** above is a non-convex, variational problem, solving which, in general, is difficult. The existence of a solution $(\lambda^*, P_P^*, P_a^*)$ will be established by construction in subsequent discussion.

We assume a complete information setting. The primary and the secondary user, if they agree to a contract, use successive interference cancellation (SIC). Either the primary or the secondary user can act as a dominant user. Furthermore, either of the dominant or the non-dominant user can be the principal, i.e., the one who proposes the contract. We denote the transmission powers of the dominant and non-dominant users as P_d and P_{nd} respectively with power constraints $P_d \leq \bar{P}_d$ and $P_{nd} \leq \bar{P}_{nd}$. Without loss of generality, we will assume $W = 1$ and $N_0 = 1$. We will assume the utilities of the two users to be equal to their rates, i.e., $u_d = R_d(P_d, P_{nd}) = \log(1 + P_d)$ and $u_{nd} = R_{nd} = \log\left(1 + \frac{P_{nd}}{1+P_d}\right)$.

We define a *social welfare function* as $S(P_d, P_{nd}) = u_d(P_d, P_{nd}) + u_{nd}(P_{nd}, P_d)$. We say that a rate allocation (R_d^{**}, R_{nd}^{**}) is *socially optimal* if it is achieved by a power allocation (P_d^{**}, P_{nd}^{**}) that maximizes the social welfare function subject to the power constraints. We will say that a *spectrum contract is optimal* if it achieves a socially optimal rate allocation.

IV. Optimal Contracts with Asymmetric Channel Gains

In an earlier paper [15], we have considered the setting when the channels gains are symmetric, i.e., when $c_{i,i} = c_{i,j}$. In this paper, we generalize those results. We denote $P_{ij} = c_{i,j} P_i$ to be the received power from the i^{th} transmitter at the j^{th} receiver. Thus, P_{ps} will denote the power from the primary's transmitter at the secondary's receiver with similar interpretations for P_{pp}, P_{sp}, P_{ss}. We would like to specify the conditions under which (i) an optimal contract exists, (ii) the optimal contract results in a Pareto-optimal operating point, and (iii) the optimal contract results in a socially optimal operating point.

The dominant user's rate depends only on its received power at its own receiver. For example, when the primary acts as the dominant user, its transmission rate will be $\log(1 + P_{p,p})$. For the SIC to be successful, the transmission rate of the non-dominant user must be less than a maximum value. This maximum rate will depend only on the power levels at the dominant user's receiver, not on the power levels at the non-dominant user's receiver. Thus, the rate of the non-dominant user must be less than this *maximum allowed rate*, R_{allowed}. For example, when the primary acts as the dominant user, the R_{allowed} for the secondary is $R_{nd}(P_{s,p}, P_{p,p}) = \log\left(1 + \frac{P_{s,p}}{1+P_{p,p}}\right)$. However, the maximum rate at which the non-dominant user can transmit to its own receiver depends only on its received power and the interference power level at its own receiver, not on the power levels at the dominant user's receiver. We denote this as the *maximum achievable rate*, R_{achieve}. For example, when the primary is the dominant user, the R_{achieve} for the secondary is $R_{nd}(P_{s,s}, P_{p,s}) = \log\left(1 + \frac{P_{s,s}}{1+P_{p,s}}\right)$. So, when R_{achieve} is greater than the R_{allowed}, the non-dominant user can transmit at the rate R_{allowed}. But, when R_{achieve} is less than R_{allowed}, the secondary's receiver cannot decode a data stream which is transmitted at a rate R_{allowed}, since it is greater than the capacity R_{achieve}. So, the best that the non-dominant user can do is to transmit at the maximum achievable rate R_{achieve}. Since the contract design optimization problem depends on the actual transmission rate, the solution also depends on the relative values of R_{allowed} and R_{achieve}.

A. Optimal Contracts Without Time-Sharing

We can address the optimal contract design problem by considering a particular case of principal-agent model spectrum sharing game in a two user cognitive radio setting. The results obtained in this case can be generalized to other cases very easily.

Case (1): Primary user is the dominant user and the principal : Since the primary is acting as the dominant user, it can transmit at its maximum rate $R_d(P_{p,p}, P_{s,p}) = \log(1 + P_{p,p})$. The secondary, which is the non-dominant user, is allowed to transmit at maximum rate $R_{\text{allowed}} = R_{nd}(P_{s,p}, P_{p,p}) = \log\left(1 + \frac{P_{s,p}}{1+P_{p,p}}\right)$. The maximum rate that the secondary user can successfully transmit to its own receiver is given by $R_{\text{achieve}} = R_{nd}(P_{s,s}, P_{p,s}) = \log\left(1 + \frac{P_{s,s}}{1+P_{p,s}}\right)$. We solve the optimization problem by considering two sub cases separately.

Case (1.a): When $R_{\text{achieve}} > R_{\text{allowed}}$: Here, the primary being the dominant user, will transmit at the rate $R_d(P_{p,p}, P_{s,p})$. The secondary user will transmit at the rate R_{allowed}. So, the primary user's optimization problem is as follows,

$$\begin{aligned} \max_{P_p,P_s,\lambda} \quad & R_d(P_{p,p}, P_{s,p}) + \lambda(P_p, P_s) \qquad s.t. \\ \text{[IR]:} \quad & R_{nd}(P_{s,p}, P_{p,p}) - \lambda(P_p, P_s) \geq 0 \\ \text{[IC]:} \quad & R_{nd}(P_{s,p}, P_{p,p}) - \lambda(P_p, P_s) \geq \\ & R_{nd}(P_{s,p}^{'}, P_{p,p}) - \lambda(P_p, P_s^{'}), \forall {P_s}' \leq \bar{P}_s. \end{aligned}$$

The [IR] constraint implies that the maximum payment λ that the primary user can get is $R_{nd}(P_{s,p}, P_{p,p})$. Thus, the principal can offer a contract $\lambda_d^*(P_p, P_s) = R_{nd}(P_{s,p}, P_{p,p})$. So the optimization problem reduces to

$$\max_{P_p \leq \bar{P}_p, P_s \leq \bar{P}_s} R_d(P_{p,p}, P_{s,p}) + R_{nd}(P_{s,p}, P_{p,p})$$

which can be reduced further to

$$\max_{P_p \leq \bar{P}_p, P_s \leq \bar{P}_s} \log\left(1 + P_{p,p} + P_{s,s}\right).$$

The solution for this is $P_p^* = \bar{P}_p, P_s^* = \bar{P}_s$. Thus, when $R_{\text{achieve}} > R_{\text{allowed}}$, we can find an optimal contract, and the operating point is on the Pareto-optimal boundary of the capacity region. The issue of social optimality is deferred to the last subsection.

Case (1.b): When $R_{\text{achieve}} < R_{\text{allowed}}$: In this case, the primary user will transmit at its maximum rate $R_d(P_{p,p}, P_{s,p})$. As the non-dominant user, the secondary user can transmit at a rate R_{allowed}. However, since this rate is greater than its achievable rate, secondary user's receiver cannot decode the data stream at this rate. So, the best that the secondary user can do is to transmit at its achievable rate $R_{\text{achieve}} = R_{nd}(P_{s,s}, P_{p,s}) = \log\left(1 + \frac{P_{s,s}}{1+P_{p,s}}\right)$. Then, the primary user's optimization problem is the following:

$$\begin{aligned} \max_{P_p,P_s,\lambda} \quad & R_d(P_{p,p}, P_{s,p}) + \lambda(P_p, P_s) \quad s.t. \\ \text{[IR]:} \quad & R_{nd}(P_{s,s}, P_{p,s}) - \lambda(P_p, P_s) \geq 0 \\ \text{[IC]:} \quad & R_{nd}(P_{s,s}, P_{p,s}) - \lambda(P_p, P_s) \geq \\ & R_{nd}(P_{s,s}^{'}, P_{p,s}) - \lambda(P_p, P_s^{'}), \forall {P_s}' \leq \bar{P}_s. \end{aligned}$$

Again, the [IR] constraint implies that the maximum payment λ that the primary user can get is $R_{nd}(P_{s,s}, P_{p,s})$. Thus, the principal can offer a contract $\lambda_d^*(P_p, P_s) = R_{nd}(P_{s,s}, P_{p,s})$. So, the optimization problem reduces to

$$\max_{P_p \leq \bar{P}_p, P_s \leq \bar{P}_s} \log\left(\frac{(1 + P_{p,p})(1 + P_{s,s} + P_{p,s})}{(1 + P_{p,s})}\right).$$

Since $P_{s,s} = c_{s,s}P_s$, the above expression is monotonically increasing in P_s. So, one part of the solution is $P_s^* = \bar{P}_s$. Now, when $c_{p,p} > c_{p,s}$, the expression $\frac{(1+P_{p,p})}{(1+P_{p,s})} = \frac{(1+c_{p,p}P_p)}{(1+c_{p,s}P_p)}$ is monotonically increasing in P_p. So, we are looking for the solution of a monotonically increasing function in P_p. Thus, the obvious solution for the optimization problem stated above is $P_p^* = \bar{P}_p, P_s^* = \bar{P}_s$.

So, when the direct channel gain $c_{p,p}$ of the dominant user is greater than or equal to its cross channel gain $c_{p,s}$, we have an optimal contract, and the operating point is on the Pareto-optimal boundary. Remarkably, the result depends only on the channel gain of the dominant user. However, when $c_{p,p} < c_{p,s}$, we are no longer trying to solve the optimization problem for a monotonically increasing sequence. We can rewrite the optimization problem as:

$$\max_{P_p \leq \bar{P}_p} \log\left(\frac{(1 + c_{p,p}P_p)(1 + c_{s,s}\bar{P}_s + c_{p,s}P_p)}{(1 + c_{p,s}P_p)}\right).$$

Since log is a monotonically increasing function, this is equivalent to

$$\max_{P_p \leq \bar{P}_p} \frac{(1 + c_{p,p}P_p)(1 + c_{s,s}\bar{P}_s + c_{p,s}P_p)}{(1 + c_{p,s}P_p)}.$$

To simplify the optimization problem, we can make a variable change as follows. We define $P_p^{'} = c_{p,p}P_p$ and $\bar{P}_p^{'} = c_{p,p}\bar{P}_p$. Since $(1 + c_{s,s}\bar{P}_s)$ is a constant in this optimization problem, we write $K = 1 + c_{s,s}\bar{P}_s$. Also, we define, $\alpha = \frac{c_{p,s}}{c_{p,p}}$ and from the condition $c_{p,p} < c_{p,s}$, we see that $\alpha > 1$. So, the optimization problem is:

$$\max_{P_p^{'} \leq \bar{P}_p^{'}} \frac{(1 + P_p^{'})(K + \alpha P_p^{'})}{(1 + \alpha P_p^{'})}.$$

Treating it as an unconstrained optimization problem, and differentiate with respect to $P_p^{'}$ to find critical points, we obtain the equation

$$\frac{(\alpha^2){P_p^{'}}^2 + (2\alpha)P_p^{'} + (K + \alpha - \alpha K)}{(1 + \alpha P_p^{'})^2} = 0$$

The above equation will have a real solution only if the discriminant $4\alpha^2 - 4\alpha^2(K + \alpha - \alpha K)$ is non-negative. This implies that at the candidate maxima/minima, $(K + \alpha - \alpha K) < 1$. To check whether this is a maxima or minima, we have to look at the second derivative of the objective function, which is equal to $\frac{(2\alpha - 2\alpha(\alpha + K - K\alpha))}{(1 + \alpha P_p^{'})^3}$. Since $(K + \alpha - \alpha K) < 1$ at the candidate maxima/minima, the second derivative is always positive and these are the minima of the objective function. Thus, we can conclude that the maximum of the objective function is achieved at either of the two boundary points where $P_p^{'} = \bar{P}_p^{'}$, or at $P_p^{'} = 0$. If we do away with the variable change we applied, this is equal to $P_p = \bar{P}_p$, or $P_p = 0$. So, the solution of the optimization problem is $P_p^* = \bar{P}_p, P_s^* = \bar{P}_s$ or, $P_p^* = 0, P_s^* = \bar{P}_s$. The exact solution depends on the channel gains, and the maximum power constraints of each user. Interestingly, the solution $P_p^* = 0, P_s^* = \bar{P}_s$ suggest that the best possible strategy for the primary user is not to transmit anything, which is not a Pareto-optimal solution. In short, when the direct channel gain $c_{i,i}$ of the dominant user is less than its cross channel gain $c_{i,j}$, a Pareto-optimal equilibrium contract does not exist.

B. *Moral Hazards*

When the channels are asymmetric and $R_{\text{achieve}} < R_{\text{allowed}}$, the non-dominant user can only transmit at R_{achieve} and he has no incentive to deviate from an agreement. However, when $R_{\text{achieve}} > R_{\text{allowed}}$, the non-dominant user can increase his rate from the agreed upon R_{allowed} to a higher rate R_{achieve} and thus increase his payoff. There is a *moral hazard* problem due to hidden action by the non-dominant user. Thus, if a contract mechanism is not robust to this moral hazard problem, such a mechanism will fail. From the previous discussion, it is clear that at an equilibrium, the dominant user transmits at the maximum possible rate and has no incentive to deviate. Thus, the moral hazard problem is due only to the hidden action by the non-dominant user. We now propose mechanisms to avoid such a problem.

When the primary user is a non-dominant user, the secondary user transmits at the dominant user rate $R_d(P_{s,s}, P_{p,s})$, and makes a non-zero payment $\lambda^*(P_p, P_s)$ to the primary user for sharing the spectrum. The primary user can accept such a payment but then deviate from the agreed operating point causing the secondary user to obtain a lower rate. Such a hidden action by the primary user can be prevented by a *deferred payment* by the secondary user. The payment is made after accessing the channel at the agreed rate. With this simple modification, if the primary user deviates, the secondary user simply does not pay the primary user whose utility will then fall below his reservation utility.

When the secondary user acts as the non-dominant user, a deferred payment mechanism does not help to solve the moral hazard problem. The secondary user can make the payment, deviate from the agreed operating point, derive a positive payoff and leave the game. A rational primary user can anticipate that such a deviation by the secondary user will cause his payoff to fall below his (positive) reservation utility and thus not participate in the mechanism. To prevent such a hidden action by the secondary user, the primary can be paid a *refundable access fee* equal to his reservation utility in addition to the usage payment. If the secondary user deviates, the primary user forfeits the secondary user's access fee else it is refunded.

Thus, through simple payment mechanisms that exploit timing, the moral hazard problem in spectrum sharing contract mechanisms can be alleviated.

C. Socially Optimal Contracts

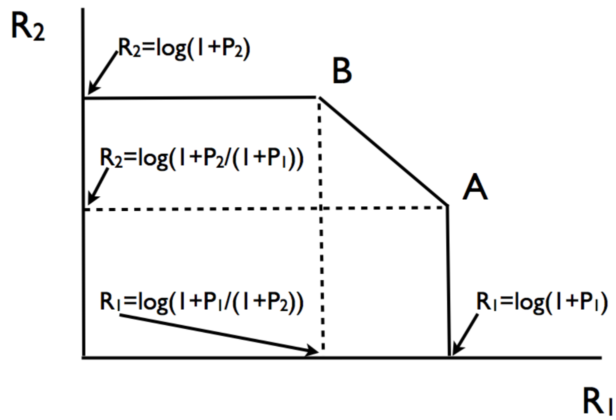

Fig. 2. Capacity region of the Gaussian Interference Channel

When the channel coefficients are symmetric, the capacity region of a two user GIC with SIC is defined by the constraints $R_1 \leq \log(1+\bar{P}_1), R_2 \leq \log(1+\bar{P}_2), R_1+R_2 \leq \log(1+\bar{P}_1+\bar{P}_2)$. Thus, the Pareto-optimal boundary of the capacity region is a straight line with slope -1 and all the points on the Pareto-optimal boundary are socially optimal (i.e., the sum rate is maximum).

When the channel coefficients are asymmetric, this is no longer the case. When the primary user acts as a dominant user, it can transmit at a maximum rate $R_d = \log(1+P_{p,p})$, whereas the secondary user which acts as the non-dominant user is allowed to transmit at a maximum rate $R_{nd} = \log\left(1+\frac{P_{s,p}}{1+P_{p,p}}\right)$. Thus, the sum rate at the operating point A is $R_{sum,A} = \log(1+P_{p,p}+P_{s,p})$. Similarly, when the secondary user acts as the dominant user, it will transmit at a maximum rate $R_d = \log(1+P_{s,s})$ and the primary which acts as non-dominant user is allowed to transmit at a maximum rate $R_{nd} = \log\left(1+\frac{P_{p,s}}{1+P_{s,s}}\right)$. So, the sum rate at the operating point B is $R_{sum,B} = \log(1+P_{s,s}+P_{p,s})$. Any operating point on the line segment AB in Figure 2 can be achieved by appropriate time sharing between the primary and the secondary user. However, since the slope is different from -1, the maximum value of the sum rate is achieved either at A (when $R_{sum,A} > R_{sum,B}$) or at B (when $R_{sum,A} < R_{sum,B}$). So the socially optimal operating point is either A or B. However the operating point is decided based not on this optimality condition, but based on the preference of the users. So, if the contractual mechanism selects the non-optimal operating point, the efficiency loss can be characterized by price of anarchy.

However, the question we now need to address is, is there an optimal contract in which the player's preferred operating point coincides with the socially optimal operating point? We can show that if we allow the option of time sharing, the operating point resulting from the optimization problem will coincide with the socially optimal operating point.

D. Optimal Contracts with Time-Sharing

We can set up the problem of time sharing between the primary and secondary user as follows. Denote by α the time-sharing variable. The principal acts as dominant user α fraction of the time, and as a non-dominant user $\bar{\alpha} = 1-\alpha$ fraction of the time with $\alpha \in [0,1]$. When the primary user acts as the principal, his optimization problem is:

$$\max_{P_p,P_s,\alpha,\lambda} \alpha R_d(P_{p,p},P_{s,p}) + \bar{\alpha}R_{nd}(P_{p,s},P_{s,s}) + \lambda(P_p,P_s,\alpha)$$
$$\textbf{[IR]: } \alpha R_{nd}(P_{s,p},P_{p,p}) + \bar{\alpha}R_d(P_{s,s},P_{p,s}) - \lambda(P_p,P_s,\alpha) \geq 0$$
$$\textbf{[IC]: } \alpha R_{nd}(P_{s,p},P_{p,p}) + \bar{\alpha}R_d(P_{s,s},P_{p,s}) - \lambda(P_p,P_s,\alpha) \geq$$
$$\alpha R_{nd}(P^{'}_{s,p},P_{p,p}) + \bar{\alpha}R_d(P^{'}_{s,s},P_{p,s}) - \lambda(P_p,P^{'}_s,\alpha), \forall P^{'}_s \leq \bar{P}_s.$$

The [IR] constraint implies that the maximum payment λ that the principal can get is $\alpha R_{nd}(P_{s,p},P_{p,p}) + \bar{\alpha}R_d(P_{s,s},P_{p,s})$. Thus, the principal can offer a contract

$$\lambda^*_p(P_p,P_s,\alpha) = \alpha R_{nd}(P_{s,p},P_{p,p}) + \bar{\alpha}R_d(P_{s,s},P_{p,s}) = \log\left(\frac{(1+P_{s,p}+P_{p,p})^{\alpha}(1+P_{s,s})^{\bar{\alpha}}}{(1+P_{s,s})^{\alpha}}\right),$$

which satisfies the [IR] constraint with equality. The [IC] constraint is now trivially satisfied. With this, the principal's optimization problem reduces to

$$\max_{P_p,P_a,\alpha} \alpha R_d(P_{p,p},P_{s,p}) + \bar{\alpha}R_{nd}(P_{p,s},P_{s,s}) + \alpha R_{nd}(P_{s,p},P_{p,p}) + \bar{\alpha}R_d(P_{s,s},P_{p,s})$$

which further reduces to

$$\max_{P_p,P_s,\alpha} \log\left(\frac{(1+P_{p,p}+P_{s,p})^{\alpha}(1+P_{s,s}+P_{p,s})}{(1+P_{s,s}+P_{p,s})^{\alpha}}\right).$$

It can be seen easily that for any power allocation, the optimum value for α is either 0 or 1. The principal chooses $\alpha = 1$ when $(1+\bar{P}_{p,p}+\bar{P}_{s,p}) > (1+\bar{P}_{s,s}+\bar{P}_{p,s})$, or equivalently when $R_{sum,A} > R_{sum,B}$. But, $\alpha = 1$ implies that primary is acting as the dominant user and hence the operating point is at A in Figure 2. At the same time, since

$R_{sum,A} > R_{sum,B}$, the socially optimal operating point is also A. Thus, the equilibrium operating point coincides with the socially optimal operating point. Similarly, we can see that when $R_{sum,A} < R_{sum,B}$, the primary user will choose $\alpha = 0$ and thus his preferred operating point coincides with the socially optimal operating point. Thus, allowing the option of time sharing in the contract design results in equilibrium outcomes that are socially optimal.

V. Discussion and Further Work

This paper presents a new approach to incentivized spectrum sharing for licensed bands in cognitive radio systems. One of the impediments is that such schemes require cooperation between various users each of whom is an independent and selfish user. There is little justification in assuming that users will expend their resources and particularly battery power to aid communications of other users. Here, we have proposed an incentive mechanism approach that not only enables deployment of sophisticated cooperative communication schemes (such as SIC) but also is natural and easy to implement. In most cases, the contract mechanism yields social welfare maximizing rate allocations. Also, we have showed that we can design contract mechanisms that are robust to possible hidden actions by the primary or secondary user.

One issue with using cooperative communication schemes such as the successive interference cancellation is that it requires codebook exchange, and hence has set-up high overhead. Thus, it can be difficult to implement the proposed solution on a per connection basis. The proposed approach is likely more suitable where there is interaction on a longer time-scale.

The setting we have considered in this paper is that of complete information, i.e., channel gains and power budgets are common knowledge. However, the asymmetric information scenario when channel gains of other users are not known, and furthermore, the actions (i.e., the transmit powers used) by the other players cannot be observed, is a lot more realistic and interesting. This can be viewed as a *double sided moral-hazard adverse selection problem* that to our knowledge has not been solved even in the game theory literature [10], [2]. As part of further work, we will address these issues.

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