Competitive Interference-aware Spectrum Access in Cognitive Radio Networks

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Abstract—**Cognitive radio networks provide the capability to share the wireless channel with licensed (primary) users in an opportunistic manner. Primary users have a license to operate in a certain spectrum band; their access can only be controlled by the Primary Operator and is not affected by any other unlicensed (secondary) user. On the other hand, secondary users (SUs) have no spectrum license, and they attempt to exploit the spectral gaps left free by primary users.**

This work studies the spectrum access problem in cognitive radio networks from a game theoretical perspective. The problem is modeled as a non-cooperative spectrum access game where secondary users access simultaneously multiple spectrum bands left available by primary users, optimizing their objective function which takes into account the congestion level observed on the available spectrum bands.

As a key innovative feature with respect to existing works, we model accurately the interference between SUs, capturing the effect of spatial reuse. We demonstrate the existence of the Nash equilibrium, and derive equilibrium flow settings. Finally, we provide numerical results of the proposed spectrum access game in several cognitive radio scenarios, and study the impact of the interference between SUs on the game efficiency. Our results indicate that the congestion cost functions we propose in this paper lead to small gaps between Nash equilibria and optimal solutions in all the considered network scenarios, thus representing a starting point for designing pricing mechanisms so as to obtain a socially optimal use of the network.

Index Terms: **- Cognitive Radio Networks, Spectrum access, Game Theory, Price of Anarchy.**

I. Introduction

Even though the frequency spectrum is the scarcest resource for wireless communications, it results generally underutilized: in fact, actual spectrum usage measurements performed by the FCC's Spectrum Policy Task Force [1] reveal that at any given time and location, much of the prized spectrum lies idle. Such underutilization has stimulated a huge research effort in several domains (e.g., engineering, economics, and regulation communities) to propose better spectrum management policies and techniques. For this reason, several dynamic spectrum access techniques have been recently proposed to better utilize the available spectrum, reducing its wastage.

Cognitive radio networks (CRNs) are envisioned to deliver high bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques [2]. Such networks provide the capability to share the wireless channel with primary users in an opportunistic manner. In CRNs, a *primary* (or licensed) user has a license to operate in a certain spectrum band; his access is generally controlled by the Primary Operator (PO) and should not be affected by the operations of any other unlicensed user. On the other hand, unlicensed (*secondary*) users have no spectrum license, and they implement additional functionalities to share the licensed spectrum band without interfering with primary users.

In this work, we focus on the dynamic spectrum access problem in cognitive radio networks from a game theoretical perspective. We consider multiple secondary users (SUs) competing in a non-cooperative way for a limited set of frequencies left available by primary users. As a consequence, game theory is the natural framework to study the interactions among such users.

Non-cooperative games for competitive spectrum access in cognitive radio networks have been recently considered in [3], [4], [5], [6], [7]. The works in [6], [7], which propose static and dynamic spectrum sharing schemes as well as spectrum pricing techniques, are somehow close to our work, but they do not model explicitly the interference between secondary users.

This paper overcomes this limitation by proposing a novel game theoretic model that solves the spectrum access problem in cognitive radio networks considering multiple POs and a given set of secondary users. More specifically, we consider a *non-cooperative spectrum access game* where secondary users access simultaneously multiple spectrum bands left available by primary users, optimizing their objective function which depends both on the total flow transmitted on each link (congestion cost) and the amount of flow that such user transmits on it.

As a key innovative feature with respect to existing works, our spectrum access game models explicitly the interference between secondary users as well as the spatial reuse of frequencies. This is achieved introducing user-specific parameters that specify, for each available spectrum band, who are the interferers that contribute to the perceived link congestion.

We demonstrate the existence of the Nash Equilibrium Point (NEP), and derive equilibrium spectrum access settings. Furthermore, we perform a thorough numerical analysis of the proposed game in several CRN scenarios, measuring the efficiency of the equilibria of our game and discussing the causes that lead to efficiency loss. More in detail, we investigate systematically the impact of several parameters (like the number of SUs and wireless channels, as well as the interference between SUs) on the system performance, determining the *Price of Anarchy* (PoA) of the proposed

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spectrum access game. The PoA quantifies the loss of efficiency as the ratio between the cost of the worst Nash equilibrium and that of the optimal solution, which could be designed by a central authority. The PoA, therefore, indicates the maximum degradation due to distributed secondary users decisions (anarchy) [8].

Numerical results indicate that the congestion cost functions we propose in this paper lead to small gaps between Nash equilibria and optimal solutions in all the considered network scenarios, thus representing a starting point for designing pricing mechanisms that foster a socially optimal use of cognitive radio networks.

The paper is structured as follows: Section II introduces the network model, including users' objective functions, as well as the proposed cost functions. Section III demonstrates the existence of at least one Nash Equilibrium Point (NEP) and illustrates a procedure to compute all equilibria. Section IV analyzes and discusses numerical results for the proposed model in several CRN scenarios. Finally, Section V concludes this paper.

II. The Interference-aware Spectrum Access Game

We consider a cognitive radio wireless system with a set $V = \{1, \ldots, N\}$ of Primary Operators (POs), each operating on a separate frequency spectrum, F_n, and having its own primary users, and a set $U = \{1, \ldots, I\}$ of secondary users (SUs), willing to share the frequency spectrums $\{F_1, \ldots, F_N\}$ with the primary users. The basic notation used in this paper is summarized in Table I.

Each SU can transmit simultaneously over multiple spectrum bands, splitting his traffic over the set of available channels, thus choosing which primary operators will transport his traffic. Each SU $i \in U$ has a fixed amount of flow (r^i) to transmit, and aims at minimizing his objective function OF^i, which represents the total congestion cost perceived on all the used channels.

Let f_n^i denote the amount of flow that SU i sends on wireless channel n, and let f_n^{PU} be the total flow sent by primary users on such channel. The secondary user flow configuration $f^i = \{f_1^i, \ldots, f_N^i\}$ is called a spectrum access strategy of SU i, and the set of strategies $S^i = \{f^i \in R^N : f_n^i \geq 0, n \in V\}$ is called the spectrum access strategy space of SU i. The system flow configuration $f = \{f^1, \ldots, f^I\}$ is called a spectrum access *strategy profile* and takes values in the product strategy space S. Furthermore, let f^{-i} represent the flow configuration of all users except SU i.

We denote by A_n (which needs not be symmetric) the interference matrix associated with channel n, and by $a_{i,k}^n$, element of A_n, the interference parameter between secondary users i and k on wireless channel n. More specifically, $a_{i,k}^n$, $i, k \in U, n \in V$ is defined as follows:

$$a_{i,k}^n = \begin{cases} 1 & \text{if SU } i \text{ interferes with SU } k \text{ on channel } n \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 illustrates an example scenario with one primary operator (PO_n) and 3 secondary users (SU_1, SU_2 and SU_3): SU_1 and SU_2 interfere with each other on channel n, while SU_3 does not interfere with any other user. Therefore, in this scenario the interference matrix A_n has the following form:

$$A_n = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that A_n can be also represented using an interference graph, which is still depicted in Figure 1.

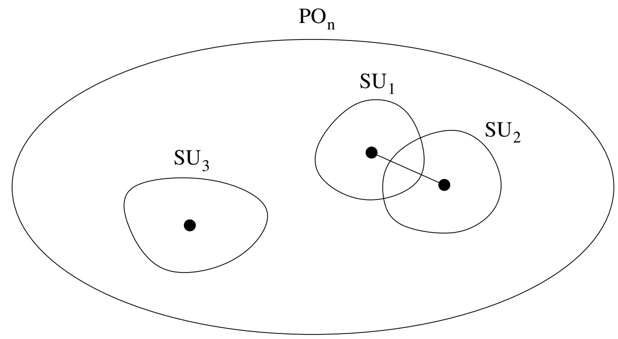


Fig. 1. Example CRN scenario with one primary operator (PO_n) and 3 secondary users (SU_1, SU_2 and SU_3). SU_1 and SU_2 interfere with each other on channel n, while SU_3 does not interfere with any other user.

In the following we present the objective functions of secondary users, as well as the cost functions we propose to adopt for wireless channels.

1) Secondary User Objective Function: We associate to SU $i \in U$ the objective function OF^i, which is a function of the flow transmitted over the wireless channels:

$$OF^i(f^i, f^{-i}) = \sum_{n \in V} f_n^i \cdot J_n^i(f^i, f^{-i}). \tag{1}$$

The term $J_n^i(f^i, f^{-i})$ represents the cost on channel n per unit of flow, and OF^i is the total cost perceived by SU i over all available channels.

The solution concept adopted is the Nash Equilibrium, i.e., we seek a feasible multi-policy $f^* = f^{i*}, i \in U$ such that $OF^i(f^*) = OF^i(f^{i*}, f^{-i*}) = \min_{f_n^i} OF^i(f^i, f^{-i*})$, $i \in U$, where the minimum is taken over all policies f^i that lead to a feasible multi-policy together with f^{-i*}, which are the optimal flows of all secondary users $j \in U$ with $j \neq i$. Hence, each SU i aims to minimize his cost function OF^i:

$$\min_{f_n^i} \sum_{n \in V} f_n^i \cdot J_n^i(f^i, f^{-i*}) \tag{2}$$

$$\text{s.t.} \sum_{n \in V} f_n^i = r^i \qquad \forall i \in U \tag{3}$$

$$f_n^i \geq 0 \qquad \forall i \in U, n \in V \tag{4}$$

2) Cost Function: In this work, we assume that the cost function is related to the total amount of flow that is transmitted on channel n. More specifically, the cost (or disutility) per flow unit perceived by SU i on channel n has the following form:

$$J_n^i(f) = a_n (F_{t,n}^i)^{\beta(n)} + b_n, \forall n \in V, \tag{5}$$

where $F_{t,n}^i = \sum_{k \in U} a_{k,i}^n f_n^k + f_n^{PU}$ is the total amount of flow

TABLE I
BASIC NOTATION

V	Set of Primary Operators (POs) (or frequency spectrum bands)
U	Set of Secondary Users (SUs)
I	Number of SUs ($I = \|U\|$)
N	Number of available wireless channels ($N = \|V\|$)
r^i	Traffic demand of SU i
f^i	Spectrum access strategy of SU i (i.e., flow vector of SU i)
f^{i*}	Optimal flow vector of SU i
f^{-i*}	Optimal flows of all SUs, except SU i
S^i	Spectrum access strategy space of SU i
f_n^i	Flow transmitted by SU i on wireless channel n
$F_{t,n}^i$	Total amount of flow observed by SU i on wireless channel n
A_n	Interference parameter matrix on wireless channel n
$a_{i,k}^n$	Interference parameter between SU i and k on wireless channel n
f_n^{PU}	Total flow sent by primary users on channel n
a_n, b_n, β_n	Channel-specific pricing parameters
λ_n^i, δ_i	Lagrangian multipliers

observed by SU i over wireless channel n, taking into account the interference produced by all other secondary users as well as f_n^{PU}, which is the total amount of flow sent by primary users on wireless channel n. Parameters a_n, b_n and $\beta(n)$ are positive, and $\beta(n) \geq 1$ (so that the cost function is convex).

In this way, the cost perceived by SU i on channel n, $a_n(F_{t,n}^i)^{\beta(n)} + b_n$, is polynomial (and hence convex) in the users' transmitted flows. We will demonstrate that such cost function has appealing properties and ensures good Nash equilibria, which are, even in the worst cases, close to socially optimal solutions. Finally, we observe that, for $\beta(n) = 1$, $J_n^i(f)$ assumes an *affine* form. In this case, a possible interpretation for this type of secondary user cost in the context of telecommunication networks is that it is the expected delay of a packet in a light traffic regime [9].

3) Comments: We observe that our spectrum access game extends classical *routing games*, like those considered in [9], [10], which represent a particular case of our game when full interference exists between all users. In fact, in our proposed game, the congestion cost perceived by each user depends both on the set of users that are transmitting on a given channel (link) and on the interference matrix. This feature captures the essence of spatial reuse in wireless systems in general (and in CRNs, in particular), and complicates consistently the analysis with respect to routing games. In fact, it has been demonstrated that the routing games studied in [9], [10] are characterized by a unique Nash equilibrium when polynomial cost functions (like those we consider in our work) are used. On the other hand, our spectrum access game is characterized by an infinite number of Nash equilibria, as we will show in Section IV.

III. Existence and Computation of Nash Equilibria

Having defined our proposed interference-aware spectrum access game, in this section we first demonstrate that such game indeed admits at least a Nash equilibrium, and then we illustrate a procedure for computing its Nash Equilibrium Points (NEPs).

To this aim, we consider the cost function (5) introduced before. The objective function of SU $i \in U$ assumes therefore the following expression:

$$OF^i(f^i, f^{-i}) = \sum_{n \in V} f_n^i [a_n (F_{t,n}^i)^{\beta(n)} + b_n]. \qquad (6)$$

SUs' objective functions (6) are continuous in $f = \{f^1, \ldots, f^I\}$ and convex in f_n^i. These properties ensure the existence of the Nash equilibrium according to the Kakutani fixed point theorem [11].

We now turn to the computation of the equilibrium solutions of our spectrum access game. Each SU i aims at minimizing his objective function OF^i. By definition, a Nash equilibrium is the solution to the individual utility optimization problem for each user given all other users' actions. In our formulation, each individual optimization problem is a nonlinear convex problem with the linear constraints (3) and (4). Hence, the Lagrangian function for user i can be written as:

$$\begin{aligned} \mathcal{L}^i(f^i, f^{-i}) &= \sum_{n \in V} f_n^i [a_n (F_{t,n}^i)^{\beta(n)} + b_n] - \sum_{n \in V} \lambda_n^i f_n^i + \\ &+ \delta^i (\sum_{n \in V} f_n^i - r^i) \qquad (7) \end{aligned}$$

where λ_n^i and δ^i are the Lagrangian multipliers (non negative real numbers). Based on nonlinear convex programming theory [12], the following Karush-Kuhn-Tucker (K.K.T.) conditions are necessary and sufficient for a solution $f = \{f_n^i\}$ to be a Nash equilibrium:

$$a_n(F^i_{t,n})^{\beta(n)} + a_n\beta(n)f^i_n(F^i_{t,n})^{\beta(n)-1} + b_n = \lambda^i_n - \delta_i \quad \text{if } f^i_n > 0, \forall i \in U, n \in V \tag{8}$$

$$a_n(F^i_{t,n})^{\beta(n)} + b_n \geq \lambda^i_n - \delta_i \quad \text{if } f^i_n = 0, \forall i \in U, n \in V \tag{9}$$

$$\sum_{n \in V} f^i_n = r^i \quad \forall i \in U \tag{10}$$

$$f^i_n \geq 0, \lambda^i_n \geq 0, \delta^i \geq 0 \quad \forall i \in U, n \in V \tag{11}$$

As we will show in the next section, our game can admit infinite Nash equilibria (i.e., the system (8)-(11) can have infinite solutions). Therefore, to determine the highest-cost Nash equilibrium necessary to compute the Price of Anarchy, we further look for the feasible solution of (8)-(11) that maximizes the sum of all users' costs, $\sum_{i \in U} OF^i$. The SNOPT 7.2 solver [13] has been used for this end.

IV. Numerical Results

We now measure the sensitivity of the proposed spectrum access game to different parameters like the number of secondary users and wireless channels, the interference between SUs as well as the traffic demands. Furthermore, we study the efficiency of the Nash equilibria by comparing them to the socially optimal solutions, through the determination of bounds to the Price of Anarchy (PoA). Socially optimal solutions minimize the sum of all users' costs, i.e., they minimize $\sum_{i \in U} OF^i$, subject to constraints (3)-(4).

Several CRN scenarios have been considered. Some, very simple, have been studied to discuss preliminarily the main features of our proposed game. Then, more realistic random topologies with a large number of users and wireless channels are used to investigate the system performance.

All the results reported hereafter are the Nash equilibria and optimal solutions of the considered scenarios obtained, respectively, by formalizing the spectrum access models in AMPL, a modeling language for mathematical programming [14], and solving them with SNOPT 7.2 [13].

For the sake of brevity, in the following we discuss the numerical results obtained with an interference matrix A_n that is both symmetric and identical for all frequencies ($A_n = A_m, \forall n, m \in V$). We consider cognitive radio scenarios with affine cost functions ($\beta(n) = 1, \forall n \in V$), and, if not specified differently, we set the cost parameters as follows: $a_n = 1, f^{PU}_n = b_n = 0, \forall n \in V$. Obviously, our proposed model is general and can be applied also to asymmetric instances and with any parameters setting.

A. Simple CRN Scenarios

1) 4-Users CRN Scenarios: We first consider a cognitive radio network with two primary operators (2 wireless channels) and 4 secondary users (SU_1, SU_2, SU_3 and SU_4), all having $r^i = 1$. To evaluate the impact of the interference matrix on the efficiency of our spectrum access game, we study 2 different scenarios: full interference (see Figure 1(a)) and partial (cyclic-like) interference (Figure 1(b)) between the SUs.

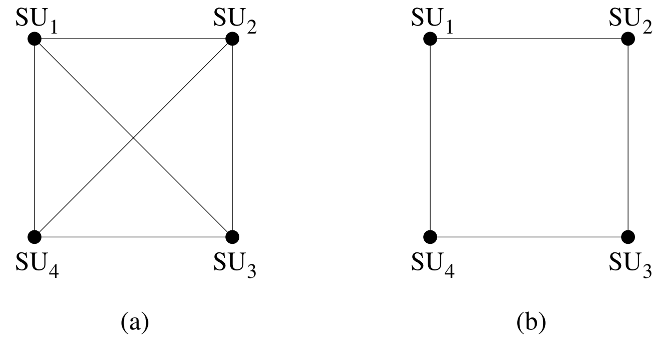


Fig. 2. 4-Users CRN scenarios: interference graphs.

In the full interference case, the PoA is equal to 1, since both in the Nash equilibrium and in the optimal solution all SUs split their traffic equally on the available channels. On the other hand, in the partial interference scenario, there exist infinite Nash equilibrium points, i.e. those where SU_1 and SU_3 transmit p and $1 - p$ traffic units on channels 1 and 2, respectively, while SU_2 and SU_4 transmit $1 - p$ and p traffic units on these channels ($p \in [0, 1]$). The total cost of these equilibria is equal to $-8p^2 + 8p + 4$, which assumes its maximum value, 6, for $p = 1/2$; on the other hand, at the optimum, SU_1-SU_3 send all their traffic on one channel, and SU_2-SU_4 on the other, with a total cost of 4, thus leading to a $PoA = 3/2$.

Hence, it can be observed that the quality of the equilibria reached by cognitive radio users depends significantly on the specific interference scenario, and counter-intuitively, with full interference, we obtain a PoA which is lower than that obtained for the partial interference scenario.

We then consider a variation of this scenario, where SU_1 and SU_3 have a rate r^i equal to 3/2 while the other two users have $r^i = 1/2$. As for the homogeneous traffic case, in the full interference scenario we again have $PoA = 1$, while for the partial interference scenario we have an infinite number of Nash equilibria: SU_1 and SU_3 transmit p and $3/2 - p$ traffic units on channels 1 and 2, respectively, while SU_2 and SU_4 transmit $1 - p$ and $p - 1/2$ traffic units on these channels ($p \in [1/2, 1]$). The total cost at the NEP is in this case equal to $-8p^2 + 12p + 1$, which is maximum for $p = 3/4$, where its value is 5.5. At the optimum, SU_1 and SU_3 send 5/4 and 1/4 traffic units on channels 1 and 2, respectively, while SU_2 and SU_4 send all their traffic on channel 2, with a social cost of 4.75, thus leading to a $PoA = 5.5/4.75 = 1.158$, which is lower than the one determined in the homogeneous traffic case.

This result confirms on the one hand the behavior already observed for the homogeneous traffic case, where the PoA with partial interference is higher than the one with full interference. However, this effect is mitigated by the presence of heterogeneous traffic demands, as we will discuss more in detail for random CRN scenarios.

2) Chain-like Interference CRN Scenario: We now consider a chain-like interference scenario, illustrated in Figure 3, with 2 wireless channels and I secondary users. All users have $r^i = 1$.

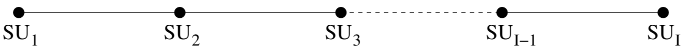


Fig. 3. Chain CRN scenario: interference graph.

The socially optimal solution sees in this case users SU_1, SU_3, SU_5, ... transmit all their traffic on one channel, while users SU_2, SU_4, SU_6, ... transmit exclusively on the other, with a total cost of I. At the Nash equilibrium, instead, all users split their traffic equally on the available channels, thus leading to a cost equal to 3/2 for players SU_2, SU_3, ..., SU_{I-1}, and equal to 1 for external users SU_1 and SU_I. Therefore, the PoA has in this case the following expression:

$$PoA = \frac{3/2(I-2)+2}{I} = \frac{3I-2}{2I} \qquad (12)$$

which increases with I, and is upper bounded by 3/2 for $I \to \infty$. Hence, increasing the number of secondary users leads to more inefficient network behaviors.

B. Random CRN Scenarios

Random cognitive radio network scenarios are obtained using a custom generator which considers a square area with edge equal to 1000, and randomly extracts the position of I nodes, each corresponding to a SU. A SU i interferes with SU k only if this latter is at a distance not greater than R, the interference range of i. We assume for simplicity that such range is the same for all secondary users.

For each random CRN scenario, the results are obtained averaging each point on 3000 instances, thus obtaining very small 95% confidence intervals, which are not shown in the figures for the sake of clarity.

1) Effect of the number of SUs (I): We first evaluate the effect of the number of SUs on the Price of Anarchy in random CRN scenarios with $N = 2$ wireless channels; all SUs are characterized by the same traffic demand $r^i = 1, \forall i \in U$. We consider several R values, in the 0 to 1500 range, thus increasing the interference between SUs.

Figure 4 shows the average PoA in function of the interference range for different I values ($I \in [2, 20]$). It can be observed that for both small and high interference ranges, the PoA is very small (i.e., close to 1). These two scenarios correspond, respectively, to complete absence of interference and full interference between SUs. In the first case, obviously, the Nash equilibrium coincides with the social optimum; the same happens for the full interference case, as we have discussed before for the square interference pattern of Figure 1(a). Partial interference (i.e., intermediate R values) leads to larger gaps between Nash equilibria and optimal solutions, as it was already observed for the corresponding scenario of Figure 1(b), even though the average PoA is in all cases quite small (always less than 1.16).

Furthermore, the PoA increases when the number of secondary users becomes larger, as in the chain-like interference scenario. We can therefore argue that it is more difficult to

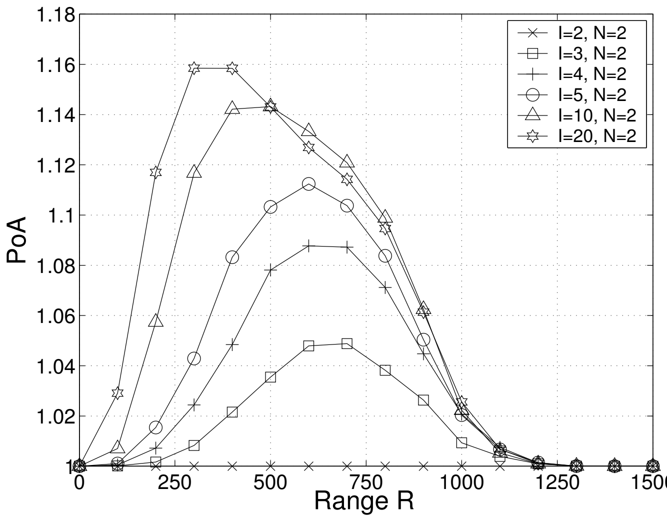


Fig. 4. Average PoA measured in CRN scenarios with $N = 2$ available wireless channels and different numbers of SUs ($I \in [2, 20]$).

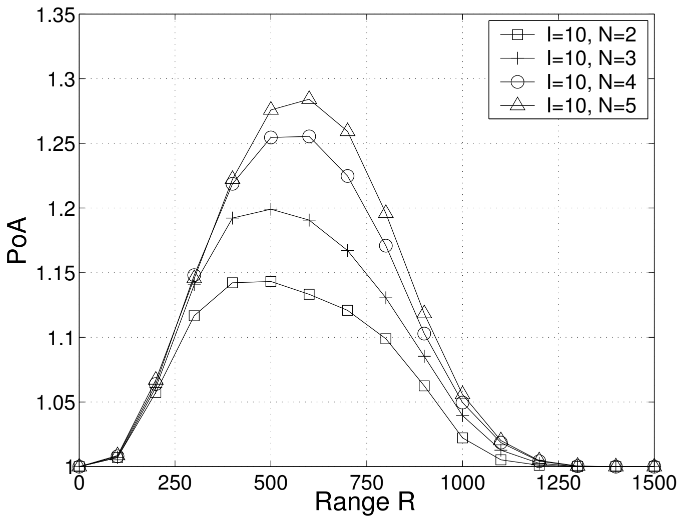
coordinate several secondary users, thus leading to more inefficient network equilibria. However, we must further observe that the worst PoA we could measure in all the considered instances, and for all I values, was equal to 1.606, which is still an acceptable loss of efficiency with respect to the achievable network optimum.

2) Effect of the number of wireless channels (N): We then increase the number of wireless channels from 2 to 5, fixing the number of SUs to 10; the interference range R varies from 0 to 1500.

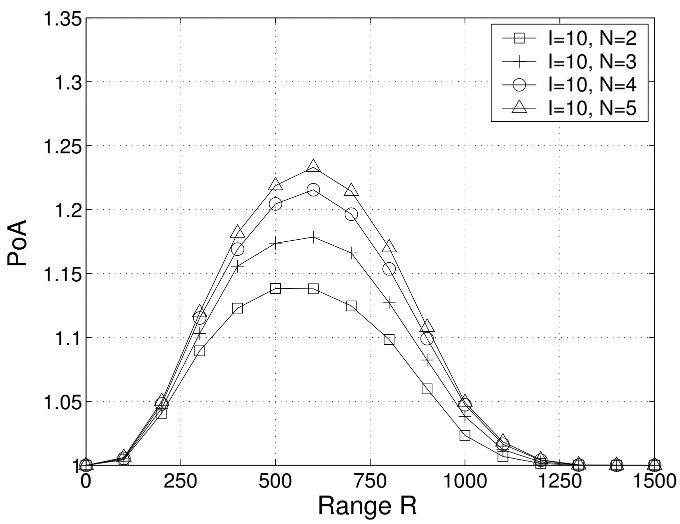
Figure 5(a) illustrates the average PoA obtained in such scenarios. It can be observed that such performance figure increases with the number of wireless channels. Intuitively, this is due to the fact that when N increases, the strategy space of the SUs also increases, thus leading to potentially worse Nash equilibria. Therefore, the availability of a larger number of wireless channels further amplifies the loss of efficiency, which can be observed especially for intermediate R values (i.e., $250 \leq R \leq 750$).

3) Effect of the transmission rate (r^i): We now consider a variation of the previous scenario, assuming that 5 users offer a rate equal to 1/2, while the other 5 have $r^i = 3/2$, thus maintaining the same total offered traffic (equal to 10), for the sake of comparison.

If we compare the results measured in this scenario, reported in Figure 5(b), with those obtained for homogeneous traffic demands (Figure 5(a)), we observe that the PoA is always smaller when traffic demands are different. This is essentially due to the fact that the quality of the equilibria is more influenced by the choices of "elephant" users (those who offer $r^i = 3/2$), and less by those of "mouse" users ($r^i = 1/2$). Hence, the network behavior is closer to that of a cognitive radio network with a smaller number of SUs and, as we observed before, when the number of such users decreases, the PoA also decreases (see Figure 4).



(a) $r^i = 1$



(b) $r^i = \{1/2, 3/2\}$

Fig. 5. Average PoA measured in CRN scenarios with $I = 10$ users and different numbers of available wireless channels ($N \in [2, 5]$); (a) 10 SUs all having $r^i = 1$, and (b) 10 heterogeneous SUs (5 SUs have $r^i = 1/2$ and the others have $r^i = 3/2$).

V. Conclusion

This paper addressed the spectrum access problem in cognitive radio networks from a game theoretical perspective. The problem has been modeled as a non-cooperative game where secondary users access simultaneously multiple spectrum bands left available by primary users, optimizing their objective function which takes into account the congestion-dependent cost functions.

As a key innovative feature with respect to existing works, we modeled accurately the interference between SUs, capturing the effect of spatial reuse, and we also used effective congestion cost functions that ensure good Nash equilibria. Furthermore, we demonstrated the existence of the Nash equilibrium, and we computed equilibrium flow settings. Finally, we performed a thorough numerical analysis of the proposed model, studying the impact of several parameters, like the number of SUs and wireless channels as well as the interference between SUs, on the game efficiency. Our results indicate that the PoA depends significantly on the interference between SUs and increases with both the number of SUs and that of wireless channels. Furthermore, the cost functions adopted in this paper enable good Nash equilibria, thus representing a good starting point for designing pricing mechanisms that foster cooperation in cognitive radio networks.

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