4TH ORDER DIFFUSION TENSOR ESTIMATION AND APPLICATION

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Introduction Historically Diffusion MRI started with Diffusion Tensor Imaging (DTI), which boosted the development of schemes for estimating positive definite tensors [1, 2] but were limited by their inability to detect fiber-crossings. Recent HARDI techniques have overcome that shortcoming with a plethora of new reconstruction schemes such as radial basis functions, Spherical Harmonics (SH), Higher Order Tensors (HOT), etc. It is appropriate, therefore, to explore HOT while leveraging the extensive framework already established for classical DTI. In this work, we propose a review and a comparison of the existing methods and an extension to the Riemannian framework [1, 2] to the space of 4 th order diffusion tensors.

Methods 4th order tensor modeling: To accommodate a multiple fiber distribution in a voxel, the Gaussian expression of the Stesjkal-Tanner equation was extended in [3], to incorporate a HOT of any order in the diffusivity function, generalizing it to $S=S_0\exp(-bD(\boldsymbol{g}))$ where $D(\boldsymbol{g})=\sum_{i_1}^{3}\sum_{i_2}^{3}\cdots\sum_{i_n}^{3} D_{1,2,\ldots n}\, g_1 g_2 \cdots g_n$.

Restricting n to 4 in the above, results in a 4 th order diffusion tensor model.

Estimation from Linear Regression: The simplest approach to estimate the coefficients of the HOT, is to solve the over-determined linear least squares inverse problem, as employed in [3]. This method is fast, is not limited to 4 th order tensors but does not guarantee positive diffusion.

Estimation from SH with regularization: As proposed in [4], it is possible to compute the HOT coefficients from the SH coefficients of the same order while using regularization. This method is also extensible to any order, but still does not assure positive diffusion.

Estimation of 4 th order diffusion tensors using a ternary quartic formulation: For n=4, the authors in [5] formulate the diffusivity function as a ternary quartic (TQ). Applying Hilbert's theorem on non-negative TQs, the diffusivity function is expressed as: $D(\boldsymbol{g})=(\boldsymbol{v}^T\boldsymbol{q}_1)^2+(\boldsymbol{v}^T\boldsymbol{q}_2)^2+(\boldsymbol{v}^T\boldsymbol{q}_3)^2=\boldsymbol{v}^T\boldsymbol{Q}\boldsymbol{Q}^T\boldsymbol{v}$, where $\boldsymbol{v}=vec(\boldsymbol{g}\boldsymbol{g}^T)$. $\boldsymbol{Q}$ is estimated from the HARDI acquisitions, and the coefficients of $\boldsymbol{D}$ are extracted from $\boldsymbol{QQ}^T$. Since $\boldsymbol{Q}$, however, can only determine $\boldsymbol{D}$ uniquely up to a rotation matrix, it is parameterized as $\boldsymbol{Q}=[\boldsymbol{B},\boldsymbol{A}]^T=[\boldsymbol{TR},\boldsymbol{A}]^T$ where $\boldsymbol{TR}$ is the qr-decomposition of $\boldsymbol{B}$ and $\boldsymbol{T}$ is taken to be $\boldsymbol{I}$ to reduce this indeterminacy. However, this method to reduce the number of parameters is arbitrary. Also, this approach can only estimate positive semi-definite tensors due to its non-negative TQ formulation.

Estimation of 4 th order diffusion tensors using a Riemannian framework: We now propose to extend the Riemannian (Rm) framework for 2 nd order diffusion tensors [1, 2] to the space of 4 th order tensors. First, as proposed in [6, 7], we rewrite the 4 th order tensor $\boldsymbol{D}$ as a 6x6 matrix $\boldsymbol{A}$, and $D(\boldsymbol{g})=\boldsymbol{g}\boldsymbol{g}^T{:}\boldsymbol{D}{:}\boldsymbol{g}\boldsymbol{g}^T=\boldsymbol{v}^T\boldsymbol{A}\boldsymbol{v}=tr(\boldsymbol{AG})$, where $\boldsymbol{v}=vec(\boldsymbol{g}\boldsymbol{g}^T)$, and $\boldsymbol{G}=\boldsymbol{g}\otimes\boldsymbol{g}\otimes\boldsymbol{g}\otimes\boldsymbol{g}$. We then proceed exactly as in [1] and estimate $\boldsymbol{A}$ in S+(6) by using the Riemannian metric defined in that space, with an M-estimator Ψ, minimizing the error energy function $E(\boldsymbol{A})=\sum_{i=1}^{N}\psi\left(\frac{1}{b_i}\ln(S_i/S_0)+\boldsymbol{v}_i^T\boldsymbol{A}\boldsymbol{v}_i\right)$ as a non-linear gradient descent problem. Since $\boldsymbol{A}$, is estimated in S+(6), the diffusivity function is guaranteed to be positive definite. However we also face an indeterminacy, as in S+(6), $\boldsymbol{A}$ has 21 free parameters while $\boldsymbol{D}$ has only 15 coefficients. To overcome this, we notice that $D(\boldsymbol{g})=tr((\boldsymbol{A}^s+\boldsymbol{A}^a)\boldsymbol{G})=tr(\boldsymbol{A}^s\boldsymbol{G})$, where $\boldsymbol{A}^s$ contains the coefficients of the tensor $\boldsymbol{D}$ and $\boldsymbol{A}^a$, the residue contains the excess parameters [7]. We can therefore logically apply the constraint $\left\|\boldsymbol{A}^a\boldsymbol{G}\right\|=0$, by projecting $\boldsymbol{A}$ to its symmetric part $\boldsymbol{A}^s$, to uniquely estimate $\boldsymbol{D}$.

Synthetic Test Data: We generate synthetic HARDI data using the multi-tensor model [4]. We use a tensor profile with eigenvalues [300,300,1700]x10^{-6} mm^2/s (Fractional Anisotropy = 0.8) and use complex Gaussian noise with Signal to Noise Ratio (SNR) of 35. The data is generated with 1, 2, or 3 fibers chosen randomly with equal volume fractions and crossing perpendicularly, with b-values of 1000 s/mm^2 and 3000 s/mm^2. 1000 such HARDI data are generated separately. To evaluate the above algorithms, we use them to estimate 4 th order diffusion tensors from the noisy synthetic data. We compute the ADCs from these tensors and compare them against the ground truth ADC. Table-1 presents the mean and the standard deviation of the squared error between the ground truth ADC and the estimated ADCs. We clearly see the value added by our Riemannian approach, which provides the best estimates.

Human Brain Data: We use high angular and spatial resolution diffusion-weighted data (60 diffusion encoding gradients with a b-value of 1000 s/mm², twice-refocused spin-echo EPI sequence, TE = 100 ms, GRAPPA/2, 1.72×1.72×1.7 mm³ voxel resolution, three repetitions, corrected for subject motion) acquired on a whole-body 3 tesla Trio scanner (Siemens, Erlangen) [8]. The time taken to estimate *159345* tensors is presente estimate positive definite tensors at every voxel and be

Results

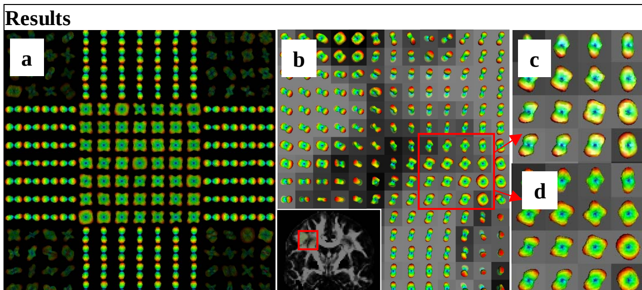

Figure-1: In (a) we have the synthetic data of two fibers crossing perpendicularly, with the ADC estimated from our Riemannian approach. We see that it is capable of discerning the crossings.
(b) A coronal slice, with the region of interest (ROI) with complex fiber structures in the white matter. In the ROI we see crossing fibers between the cortical spinal tract, superior longitudinal fibers (traversing the plane) and the corpus callosum (in the plane). We zoom in where the ADC detects crossings and compare in (c) and (d) the true SH ADC (without HOT) [4], with our Riemannian approximation ADC. In all cases the background is coloured by the GFA and the ADCs are coloured by their values on the sphere – red indicating high and blue low value.

	LS		SH to HOT		TQ		(Rm)	
b=1000 s/mm^2	29.3	9.1	29.4	9.1	16.4	6.2	13.8	4.3
b=3000 s/mm^2	28.8	8.2	28.8	8.2	15.2	4.4	12.8	3.4

Table-1: The ***mean*** and the ***standard deviation*** of the squared error between the ground truth ADC and the ADCs from the four methods are compared. Clearly our Riemannian approach provides the best estimates.

159345 Tensors	LS	SH to HOT	TQ	Rm 4-th	Rm 2-nd
Time (sec)	3	14	25411 ≈ 7 h	63	3572 ≈ 1 h

Table-2: The estimation time of the four methods and the classical 2nd order DTI are tested on the human dataset to estimate *159345* tensors. The two Riemannian methods are the only algorithms to guarantee positive definite diffusion. The TQ scheme estimates all the tensors as positive semi-definite. Our Riemannian extension is the best tradeoff between speed and positive definite diffusion.

computationally viable. The TQ scheme estimates all the tensors as positive semi-definite.

Discussion 4 th order tensor estimation schemes are now maturing to take into consideration diffusion constraints. This enables us to detect fiber-crossings, while ensuring positive diffusion. The linear algorithms like the least squares and the SH to HOT still remain the fastest and are extensible, but cannot guarantee positive diffusion. The ternary quartic estimation is a novel approach, but it guarantees only positive semi-definite tensors. As per our implementation it is also slow. On the other hand our proposed Riemannian extension ensures positive-definite diffusion and is practicable since it is fast. From our experiments, the results also speak strongly in its favour. Therefore it is naturally worthwhile to explore HOTs to generalize this framework to tensors of any order. Equally importantly it also opens the door for now doing accurate tractography.

References [1] Lenglet et al, JMIV 2006 & PhD thesis 2006, [2] Arsigny et al, MRM 2006, [3] Ozarslan et al, MRM 2003. [4] Descoteaux et al, MRM 2006. [5] Barmpoutis et al, IPMI 2007. [6] Basser and Pajevic Sig Pro 2007, [7] Moakher, *Perspectives Workshop: Visualization and Image Processing of Tensor Fields*, number 04172 in Dagstuhl Seminar Proceedings, [8] A. Anwander, M. Tittgemeyer, D. Y. von Cramon, A. D. Friederici, and T. R. Knosche, Cerebral Cortex , 17(4):816–825, 2007.

d in table-2. Our Riemannian extension is the only algorithm to