A Polynomial Based Approach to Extract Fiber Directions from the ODF and its Experimental Validation

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Introduction: In Diffusion MRI, spherical functions are commonly employed to represent the diffusion information. The ODF is an intuitive spherical function since its maxima are aligned with the dominant fiber directions. Therefore, it is important to correctly determine these maximal directions, as they are the key to tracing fiber tracts. A tractography algorithm will suffer from cumulative error when the maximal directions are incorrectly estimated locally. The goal of this work is to present a polynomial based approach for estimating the maximal directions correctly. The paper will also present a measure of the correctness of the estimation. This approach will be tested on synthetic, phantom, and real data, and will be compared to an existing discrete "mesh-search" approach [2]. It will be shown how this approach naturally overcomes the inherent shortcomings of the discrete search. Finally, although, the approach is demonstrated on the ODF, it can be equally applied to any spherical function.

Materials and Methods: Spherical functions can be naturally represented in the Spherical Harmonic (SH) basis. The ODF was written in the SH basis in [1,2] for an analytical q-Ball Imaging (QBI) algorithm. However, as shown in [3,4] the SH basis can be linearly transformed to the Homogeneous Polynomial (HP) basis by computing the coefficients of a HP constrained to the unit sphere from the SH coefficients, when they both describe the same spherical function *S*. Therefore, an ODF estimated in the SH basis can be rewritten as the polynomial *P* constrained to the unit sphere. In this form, computing the maximal directions of the ODF turns out to be a constrained optimization problem: $\max_X P(X) \; subject \; to \|X\|_2^2 = 1$. This can be solved using Lagrange multipliers: $F(X,\lambda) = P(X) - \lambda(\|X\|_2^2 - 1)$, where the maximal directions of the ODF, *P*, are among the solutions of: $\|\nabla F\| = \|X\|_2^2 - 1 = 0$. This polynomial system can be solved algebraically using state of the art algorithms such as the Subdivision Methods [5] and Generalized Normal Forms [6], which guarantee numerical stability and also compute all the stationary points at once. The stationary points can then be ranked as maxima, minima or saddle points using a generalized Hessian test. Interestingly, if *d* were to be a stationary point of *F* (or a maximal direction of *P*), then $\|\nabla F(d)\|$ provides a natural measure of the quality of the maxima extraction since *F* and *d* need to satisfy $\|\nabla F(d)\| = 0$.

Results: Synthetic Dataset: 1 and 2 fiber voxels were simulated using the multi-tensor model with a diffusion tensor profile of diag(**D**) = [1390,355,355]x10^{-6} mm^2/s, FA = 0.7. Simulations used known ground truth directions $\{d_i\}$ chosen randomly, and perpendicular in the 2-fiber case. An ODF of rank-4 was estimated and then the HP maxima extraction method was applied to extract a set of maxima $\{d_i'\}$. $\|\nabla F(d)\|$ and $\|\nabla F(d')\|$ were computed. $\|\nabla F(d')\|$ provides a measure of the quality of the HP maxima extraction approach. $\|\nabla F(d)\|$ on the other hand provides a measure of the quality of the ODF estimation step, since *F* is formulated from *P* which is the ODF in the HP basis, and *d* are known directions, implying that $\|\nabla F(d)\|$ should be close to zero when the ODF is correctly estimated (Table-1).

Phantom Dataset [7]: Acquired on a GE Healthcare Signa 1.5T ExciteII scanner, 4000 directions, b = 4000 mm^2/s, 32x32 matrix, and an SNR greater than 4. The phantom had a geometry of two fiber bundles crossing perpendicularly close to the X-axis and the Y-axis (Fig-1). ODFs of rank-4 were estimated. To compare the HP approach with a discrete "mesh search" approach [2] a mesh with 81 directions on a hemisphere was used to test the Discrete Search (DS) method. To show the dependency of the DS on the mesh-orientation, the mesh was turned from [-4,4] degrees in 100 steps about the Z-axis and at every step the DS and HP approaches were applied. This was computed only within the ROI where crossings were present (Fig-1). The extracted maximal directions and their corresponding function values as a function of the Z-rotation are presented in Figs-2,3,4,5. Essentially the HP approach is independent of the mesh-orientation and the DS approach is dependent on it.

Real Brain Dataset [8]: Acquired on a 3T Siemens scanner, with 60 directions, b = 1000 s/mm^2, and 1.7x1.7x3 mm^3 voxels. An ROI on a coronal slice was chosen within a region containing crossings between the cortico-spinal tract, superior longitudinal fibers (traversing the plane) and the corpus callosum (in the plane). ODFs of rank-4 were estimated and the HP approach was applied (Fig-6). The DS approach was tested on this ROI to show its dependency on the mesh-resolution. Meshes with 21, 81, 321, 1281, 5121 points on a hemisphere respectively were used to test the DS approach. For the directions extracted in every voxel, the $\|\nabla F(d)\|$ error was computed and from there the mean and the variance. These are plotted as a function of the mesh-resolution and compared to the HP approach in Fig-7.

Discussion: This paper presented a HP approach to extract the maxima of a spherical function, namely the ODF, whose maxima correspond to fiber directions. Solving a polynomial system allowed this approach to compute all the maxima at once. It also provided a natural measure of the quality of the extraction. It was tested on synthetic, phantom and real brain data and compared to an existing discrete "mesh search" approach. Synthetic data tests showed this HP method to be numerically stable. Phantom data tests showed it to be inherently better than the discrete search approach which was dependent on the mesh-orientation it used. The HP method was also tested on real brain data, where it again proved to be better than the discrete search which was shown to be also dependent on the mesh-resolution.

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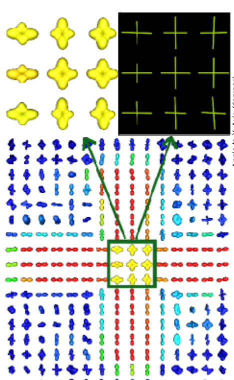

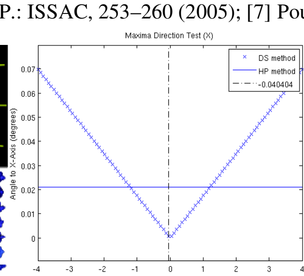

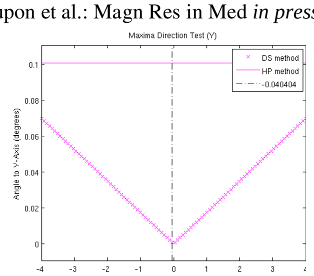

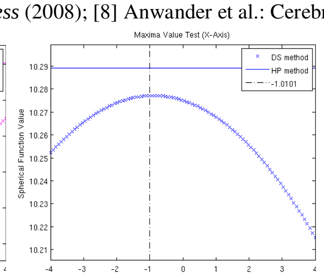

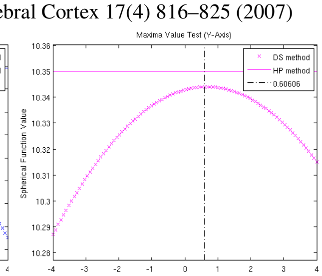

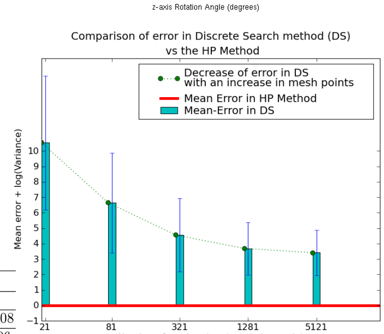

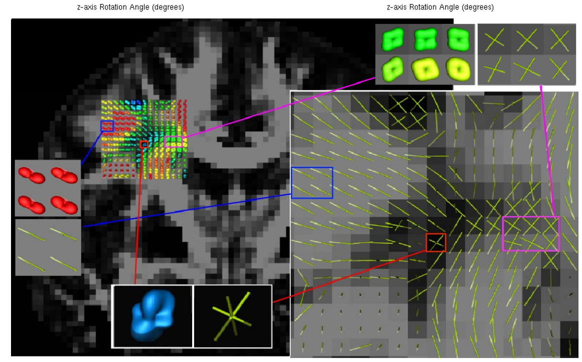

(From Top-Left CW): Fig1: Phantom and ROI with crossings; Fig2,3: Max direction and Z-rot; Fig4,5: Function value and Z-rot; Fig6: HP method on coronal slice; Fig7: DS method and mesh-resolution; Table1: Synthetic data and estimation quality on 1000 test cases.

	$ang(d,d')^o$		$\|\|\nabla\mathcal{F}(d')\|\|$		$\|\|\nabla\mathcal{F}(d)\|\|$	
	m	v	m	v	m	v
1-Fib	0.0104625	4.22152e-05	2.07053e-11	1.49751e-21	0.000490269	9.16956e-08
2-Fib$_{90^o}$	0.0254106	3.85231e-05	5.5777e-11	1.7602e-21	0.0036013	2.4827e-06