Evolutionary forwarding games in Delay Tolerant Networks

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***Abstract*—In this paper, we apply evolutionary games to non-cooperative forwarding control of Delay Tolerant Networks (DTN). We focus our study on the probability to deliver a message from source to destination in a DTN using two-hop routing. We derive the success probability as a function of the competition within a large population of mobiles. In particular, for each message generated by a source, each mobile may take a decision that concerns the strategy by which the mobile participates to the relaying. A mobile that participates receives a unit of reward if it is the first to deliver a copy of the packet to the destination. A utility function is introduced as the difference between a reward unit and the energy cost. We show how the evolution dynamics and the equilibrium behavior (called Evolutionary Stable Strategy - ESS) are influenced by the characteristics of inter contact time, energy expenditure and pricing characteristics. We specialize our analysis also to mechanisms that the source may introduce in order to have the message delivered to the destination with high probability within a given deadline.**

I. Introduction

Delay tolerant mobile ad-hoc networks have gained attention in recent networking research. Throughout the paper, Delay Tolerant Networks (DTNs) are sparse and/or highly mobile wireless ad hoc networks where no continuous connectivity guarantee can be assumed [3]. In the context of DTNs, network operations are ensured in spite of intermittent connectivity using the so called *carry-store-and-forward* strategy: messages can arrive at their destination thanks to the mobility of some subset of nodes that carry copies of the message stored in their local memory. In literature, there are also several results of real experiments on DTNs [4], [16].

In such networks a clear tradeoff exists between the number of released message copies and the probability that a message reaches the destination. In literature, several mechanisms for message forwarding in DTNs have been proposed. The basic approach to overcome lack of persistent connectivity is epidemic routing in which any mobile that has the message keeps on relaying it to any other mobile that enters its transmission range and which does not have the message yet [14]. It is clear that doing so the epidemic routing algorithm maximizes the delivery probability, but it is very expensive in terms of network resources. Since energy and memory used for transmission grow with the number of released copies of the message, the need for a more efficient use of network resources has motivated the use of smarter forwarding strategies such as the two-hop routing protocol. In two-hop routing the source transmits copies of its message to all mobiles it encounters; relays in turn are forced to relay the message only to the destination, in case it is encountered. The performance of the two-hop forwarding protocols has been evaluated in [1]. In the rest of the paper we will deal with a novel game theoretical approach to the control of the two-hop forwarding in DTNs.

In particular, the control of forwarding has been recently addressed in the ad hoc networks literature. In [5], the authors describe an epidemic forwarding protocol and show that it is possible to increase the message delivery probability by tuning the parameters of the underlying SIR model. In [14], the authors studied optimal static and dynamic control problems using a fluid model that represents the mean field limit as the number of mobiles becomes very large. In [9] and its follow-up [10], the authors optimize network performance by designing the mobility of message relays. Some other works that explicitly address the control of forwarding and related to our work are [8], [11]. The optimality of a threshold type policy, already established in [12] for the fluid limit framework, was shown to hold in [13] for the actual discrete control problem. Another work [15] proposes the optimization of two hops forwarding based on the theory of linear-quadratic regulators. Finally, a game problem between two groups of DTN networks was studied in [13] and [6], showing the existence of a

Nash equilibrium for that game.

Main contribution: Compared to most existing results in literature, we characterize the possible equilibria of DTN forwarding dynamics within a game theoretical framework. The tool we employ, in particular, is evolutionary game theory, by which we can model the competition of relays in DTNs as a distributed control problem where part of the forwarding policy is determined by the strategies played by the relay nodes. An additional degree of freedom is that the strategies played by relays evolve with time. Once determined the possible equilibria for the game, we could also determine feasibility conditions for optimal forwarding control at the source node through a simple rewarding policy.

This approach provides very interesting insight into the feasibility of optimal mechanism design, which appears novel as well compared to known results in literature.

II. Basic notions on evolutionary games

In the next sections we will consider a standard setting for evolutionary games (see for example [7] for a complete reference):

- There is one population of users. The number of users in the population is large.
- We assume that there are finitely many pure strategies or actions. Each member of the population chooses from the same set of strategies $\mathcal{A} = \{1, 2, \ldots, I\}$.
- Let $M := \{(y_1, \ldots, y_I) \mid y_j \geq 0, \ \sum_{j=1}^{I} y_j = 1\}$ be the set of probability distributions over the I pure actions. M can be interpreted as the set of mixed strategies. In fact, assume that users are drawn uniformly at random from the population: a player using a tagged strategy will confront a user playing strategy j with probability y_j. When several rounds are played, this is equivalent for the user playing j to confront a user playing a mixed strategy $(y_1, \ldots, y_I)$. In the rest of the paper any vector $y \in M$ is referred indifferently as the *profile* or *state* of the population.

In what follows, there are also a few assumptions that are not standard in evolutionary games; they follow from the fact that, in the type of games we deal with, a message is possibly delivered leveraging several relays, i.e., players, at once. To this respect, in our framework, a fundamental assumption is that the incentive to relay a message is a certain reward that each relay may receive upon delivering the message. However, competing relays perceive each other as *interfering* to their final goal to attain the reward.

- The number of users interfering with a given randomly selected user is a random variable K in the set $\{0, 1, \ldots\}$.
- A player does not know how many players would interact with it.
- The payoff function of all players depends of the player's own behavior and the behavior of the other players. The expected payoff of a user playing strategy j when the state of the population is y, is given by
$$U_{av}(j, y) = \sum_{k \geq 0} \mathbb{P}(K = k) U(j, k, y),$$
where $U(j, k, y)$ is the payoff of a user playing strategy j when the state of the population is y and given that the number of users interfering with a given randomly selected user is k. Hence the average payoff of a population in state y is given by
$$F(y, y) = \sum_{i=1}^{I} y_j U_{av}(j, y).$$
- The game is played many times; we call each round of the game a *local interaction* and there are many local interactions at the same time;

A. Evolutionary Stable Strategy

Suppose that, initially, the population profile is $y \in M$. Now suppose that a small group of mutants enters the population playing according to a different profile $mut \in M$. If we call $\epsilon \in (0, 1)$ the size of the subpopulation of mutants after normalization, then the population profile after mutation will be $\epsilon \cdot mut + (1-\epsilon)y$. After mutation, the average payoff of non-mutants will be given by
$$F(y, \epsilon \cdot mut + (1 - \epsilon)y)$$
where $F(x, y) := \sum_{j=1}^{N} x_j U_{av}(j, y)$. Note that U_{av} need not to be linear in the second variable. Analogously, the average payoff of a mutant is
$$F(mut, \epsilon \cdot mut + (1 - \epsilon)x).$$

Definition 1: A strategy $y^* \in M$ is an ESS if for any $mut \neq y^*$, there exists some $\epsilon_{mut} \in (0, 1)$, which may depend on mut, such that for all $\epsilon \in (0, \epsilon_{mut})$ one has
$$F(y^*, \epsilon\, mut + (1 - \epsilon)y^*) > F(mut, \epsilon \cdot mut + (1 - \epsilon)y^*)$$

which can be rewritten as

$$\sum_{j=1}^{N}(y_j^* - mut_j)U_{av}(j, \epsilon \cdot mut + (1-\epsilon)y^*) > 0.$$

That is, y^* is an ESS if, after mutation, non-mutants are more successful than mutants. Borrowing the expression from population dynamics [7], under an ESS profile, mutants cannot invade the population and will eventually get extinct.

III. Activation Control

Consider a DTN network with several message sources s_i, destinations d_i and a large number of mobiles in the system. Each mobile is equipped with some form of proximity wireless communication. Messages are continuously generated at source nodes and need to be delivered to destination nodes. We assume that each such message that is transmitted by a source is relevant for a time interval of length at most τ.

The network is assumed to be sparse, so that, at any time instant, nodes are isolated with high probability. Due to mobility patterns, communication opportunities arise whenever two nodes get within mutual communication range. We refer to such events as "contacts". The time between subsequent contacts of a node with the source s_i or the destination d_i is assumed to follow an exponential distribution with parameter $\lambda > 0$. The validity of this model for synthetic mobility models has been discussed in [1].

Each message contains a time stamp reporting its age, so that it can be deleted at all nodes when it becomes irrelevant. In sight of the lack of connectivity, we exclude the use of feedback that allows the source or other mobiles to know whether the message has been successfully delivered to the destination during time slot τ. Recall that we consider the two hop routing scheme [1] in which a mobile that receives a copy of the packet from the source can only forward it to the destination.

Evolutionary games serve us to study the competition among individual mobile relays in a DTN two-hop routing game. The relays represent the players of the game; such players need to take some action with respect to messages that are generated at a source s_i with destination d_i.

When a message is generated by a source node, a local interaction takes place with duration τ. During a certain local interaction, each mobile has two possible strategies with respect to the forwarding of a message to the destination: either to participate, i.e., pure strategy *transmit* (T), or not to participate, i.e., pure strategy *silent* (S). Each such strategy corresponds to a certain utility for the relay. Clearly, when a relay receives a packet from source s_i, the utility of the relay depends by the actions performed by some N random mobiles that participate to the local interaction.

Let $M := \{(y_T, y_S) \mid y_T + y_S = 1\}$ be the set of probability distributions over the 2 pure actions T and S. As recalled before, M can be interpreted as the set of mixed strategies. It is also interpreted as the set of distributions of strategies among the population, where y_T (resp. y_S) represents the proportion of mobiles choosing the strategy T (resp. S).

Local Interaction: Without loss of generality, we assume that at the i-th local interaction, there is a source-destination pair in which the source has packet generated at some time $t_i = i\tau$, which is then relevant up to time $t_i + \tau = t_{i+q}$, when another message is released and local interaction $(i+1)$-th begins.

In the rest of the paper, for the sake of notation, we will consider interval $[0, \tau]$ as the reference time slot interval for a local interaction.

Remark 1: We observe that in our system, relays do not need to be synchronized to the source clock. In fact, it is sufficient that they decide their strategy at the time when they meet the source. At that time, they can be made aware of the deadline τ using a time-to-live counter that is decreased over time at the source node.

Let N be the number of mobiles (possibly random) in an area which is assumed fixed during time slot $[0, \tau]$. We denote by y_T (resp. $1 - y_T$) the fraction of mobiles using strategy T (resp. S). Consider an active mobile in a local interaction with source s_i, destination d_i and N opponents.

From [2], the probability that the tagged mobile relays the copy of the packet to the destination within time τ is given by $1 - Q_\tau$ where Q_τ is given by

$$Q_\tau = (1 + \lambda\tau)e^{-\lambda\tau} \tag{1}$$

Hence the probability that the tagged mobile receive the unit reward, if it decides to participate is (see [2])

$$\begin{aligned} P_{succ}(N, y_T) &= \beta \sum_{k=1}^{N-1} C_{k-1}^{N-1} \frac{\left(y_T\beta\right)^{k-1}(1 - y_T\beta)^{N-k}}{k} \\ &= \frac{(1 - (1 - y_T\beta)^N)}{N y_T} \end{aligned} \tag{2}$$

where $\beta = 1 - Q_\tau$. The utility for an active mobile is given by

$$U_{av}(T, y_T) = \sum_{N=1}^{\infty} P(K = N) P_{succ}(N, y_T) - g\tau,$$

where $g\tau$ is the energy cost. The linear energy expenditure, in particular, may express the cost of activating the RF interface and the cost of periodic beaconing.

The utility $U_{av}(S, y_T)$ for not participating is zero.

Now suppose that, initially, the population profile is $y_T \in M$, representing the fraction of relays adopting strategy T. The average payoff for relays playing T is

$$F(y_T, y_T) = y_T U_{av}(T, y_T) + (1 - y_T) U_{av}(S, y_T).$$

Let H be the function defined as

$$H : y_T \in (0, 1) \to U_{av}(T, y_T) - U_{av}(S, y_T)$$

It is easy to see that H is decreasing in y_T.

A. Existence and Uniqueness of ESS

In this section, we are now looking at the existence and uniqueness of the ESS

Theorem 1: (1) The Pure strategy T dominates the strategy S if and only if $\sum_{N=1}^{\infty} P(K = N)\frac{(1-Q_\tau^N)}{N} - g\tau \geq 0$.

(2 The pure strategy S dominates the strategy T if and only if $(1 - Q_\tau) - g\tau \leq 0$

(3) If $(1 - Q_\tau) - g\tau > 0$ and $\sum_{N=1}^{\infty} P(K = N)\frac{(1-Q_\tau^N)}{N} - g\tau < 0$, then there exists an unique ESS $y_T^* = H^{-1}(g\tau)$.

Proof:

(1) The strategy T dominates the strategy S if and only if $U(T, y_T) \geq U(S, y_T) = 0$ for all $y_T \in [0, 1]$. Since H is decreasing function and $H(1) = \sum_{N=1}^{\infty} P(K = N)\frac{(1-Q_\tau^N)}{N} - g\tau \geq 0$, thus $H(y_T) = U(T, y_T) > 0$ for all $y_T \in [0, 1]$. This completes the proof for (1)

(2) The strategy S dominates the strategy T if and only if $U(S, y_T) = 0 \geq J(T, y_T)$ for all $y_T \in [0, 1]$. Since the function H is a decreasing function and $H(0) = (1 - Q_\tau) - g\tau \leq 0$, thus $H(y_T) = U(T, y_T) \leq 0 = U(S, y_T)$ for all $\beta \in [0, (1 - Q_\tau]$. This completes the proof for (2)

(3) A strictly mixed equilibrium y_T^* is characterized $U(T, y_T^*) = U(S, y_T^*)$ i.e $H(y_T^*) = 0$. The function H is continuous and strictly decreasing monotone on $(0, 1)$ with $H(0) > 0$ and $H(1) < 0$. Then the equation $H(y_T) = 0$ has a unique solution in the interval $(0, 1)$. This completes the proof. $\diamond$

B. Poisson distribution

We consider that nodes are distributed over a plan following a Poisson distribution with density γ. The probability that there exist N nodes during a local interaction is given by the following distribution: $\mathbb{P}(K = k) = \frac{\gamma^{k-1}}{(k-1)!}e^{-\gamma}$, $k \geq 1$. Considering those node distributions and from previous theorems, the unique ESS $y_T^* = \frac{\beta}{1-Q_\tau}$ where $\bar{\beta}$ is unique solution of the following equation: $\frac{(1-Q_\tau)(1-e^{-\gamma\bar{\beta}})}{\gamma y_T \beta} - g\tau = 0$. Thus, the equilibrium is given by

$$y_H^* = \frac{LambertW(-\frac{(1-Q)}{g\tau}e^{\frac{1-Q}{\tau g}})}{(1 - Q_\tau)\gamma} + \frac{1}{g\tau\gamma}.$$

where $LambertW$ is the inverse function of $f(u) = ue^u$.

IV. Strategic 'Live time' control

In this section, we apply evolutionary games to non-cooperative 'live time' selection in delay tolerant networks. The live time of a message copy denotes the maximum time before the message is discarded by a relay that carries it. The message live time in turn is dictated by the strategy adopted by relays: a relay node can decide the duration of the interval of time during which it participates to the the forwarding process. For simplicity, let us assume that each relay node chooses two different live times for the message: τ, i.e., full activation, and τ', i.e., partial activation. Without loss of generality let $\tau' < \tau$.

Notice that switching off the radio interface at $\tau' < \tau$ can be seen as a power saving technique since the relay reduces battery depletion due to idle listening and beaconing.

Also, the activation control problem considered earlier is a special case of this problem where $\tau' = 0$.

Local Interaction: During a local interaction, a tagged mobile confronts N opponents, which as before may in general be a random variable. However, this number is assumed constant during the time slot of a local interaction. We denote by y (resp. $1 - y$) the fraction of mobiles that adopt the strategy (live time) τ (resp. τ'). The probability that the tagged mobile relays the copy of the packet to the destination within live time τ is then given by $1 - Q_\tau$ where Q_τ is given by

$$Q_\tau = (1 + \lambda\tau)e^{-\lambda\tau}$$

and the probability that it relays the copy of the message if it chooses live time τ' is given by is given by $1 - Q_{\tau'}$ where $Q_{\tau'} = (1 + \lambda\tau')e^{-\lambda\tau'}$.

Let $P_{succ}(\tau, N, y)$ (resp. $P_{succ}(\tau', N, y)$) be the probability that the tagged mobile receives the unit reward, if it chooses live time τ (resp. τ'). Now

$$P_{succ}(\tau', N, y) = = (1-Q_{\tau'})\sum_{k=1}^{N-1} C_{k-1}^{N-1}\frac{(1-Q_{\tau'})^{k-1}(1-(1-Q_{\tau'}))^{N-k}}{k} = \frac{1-Q_{\tau'}^N}{N} \quad (3)$$

where $\beta' = (1-Q_{\tau'})$. The utility for a mobile using live time τ' is

$$U_{av}(\tau', y) = \sum_{N=1}^{\infty} P(K=N)P_{succ}(\tau', N, y) - g\tau'$$

where g is energy cost. Now the probability that a mobile receives the unit award, if it chooses live time τ, is given by

$$P_{succ}(\tau, N, y) = P_{succ}(\tau', N, y) + (Q_{\tau'})^N \beta \sum_{k=1}^{N-1} C_{k-1}^{N-1}\frac{\beta^{k-1}y^{k-1}(1-y\beta)^{N-k}}{k} = P_{succ}(\tau', N, y) + (Q_{\tau'})^N \frac{1-(1-\beta y)^N}{Ny} \quad (4)$$

where $\beta = 1 - \frac{1+\lambda\tau}{1+\lambda\tau'}e^{-\lambda(\tau-\tau')}$.

The utility for a mobile using live time τ

$$U_{av}(\tau, y_H) = \sum_{N=1}^{\infty} P(K=N)P_{succ}(\tau, N, y) - g\tau$$

. The average payoff is

$$\begin{aligned} F(y,y) &= yU_{av}(\tau,y) + (1-y)U_{av}(\tau',y) \\ &= \sum_{N=1}^{\infty} P(K=N)\Big((1-y)P_{succ}(\tau',N,y) \\ &\quad +yP_{succ}(\tau,N,y)\Big) - y(g\tau - g\tau') - g\tau' \end{aligned}$$

Let $\tilde{H}$ be the function defined as

$$\tilde{H} : y \in (0,1) \to U_{av}(\tau, y) - U_{av}(\tau', y) \quad (5)$$
$$= \sum_{N=1}^{\infty} P(K=N)(Q_{\tau'})^N \frac{1-(1-\beta y)^N}{Ny} - g(\tau-\tau')$$

A. Existence and Uniqueness of ESS

In this section, we are now looking at the existence and uniqueness of the ESS in the case of live time strategies. Even in this case, a very compact result exists that ties together the main parameters of the system:

Theorem 2: (1) The strategy τ dominates the strategy τ' if and only if

$$\sum_{N=1}^{\infty} P(K=N)\frac{Q_{\tau'}^N(1-(1-\beta)^N)}{N} - g(\tau-\tau') \geq 0.$$

(2 The strategy τ' dominates the strategy τ if and only if

$$\sum_{N=1}^{\infty} P(K=N)Q_{\tau'}^N\beta - g(\tau-\tau') \leq 0$$

(3) If $\sum_{N=1}^{\infty} P(K=N)Q_{\tau'}^N\beta - g(\tau-\tau') > 0$ and $\sum_{N=1}^{\infty} P(K=N)\frac{Q_{\tau'}^N(1-(1-\beta)^N)}{N} - g(\tau-\tau') < 0$, then there exists an unique ESS y^* which is given by

$$y^* = \tilde{H}^{-1}\Big(g(\tau-\tau')\Big)$$

The proof is similar to that of the Thm. 1.

B. Poisson distribution

We consider that nodes are distributed over a plan following a Poisson distribution with density γ. The probability that there are N nodes in local interaction is given by the following distribution: $\mathbb{P}(K=k) = \frac{\gamma^{k-1}}{(k-1)!}e^{-\gamma},\ k \geq 1$. Considering those node distributions and from previous theorems, the unique ESS y^* for all cases, is solution of the following equation: $\frac{e^{-\gamma(1-Q_{\tau'})}}{\gamma y}(1-e^{-\gamma\beta y Q_{\tau'}}) = g(\tau-\tau')$. Thus, the equilibrium is given by

$$y^* = \frac{Z}{\gamma\beta Q_{\tau'}}$$

where Z is given by $Z = \frac{LambertW(-\frac{1}{X}e^{\frac{1}{X}})}{\gamma} + \frac{1}{X\gamma}$, and $X = \frac{g(\tau-\tau')e^{\gamma(1-Q_{\tau'})}}{\beta Q_{\tau'}}$.

V. Convergence to ESS through Delayed Evolutionary Dynamics

Next, we shall see how evolutionary dynamics, particularly the Replicator Dynamics, can be used to achieve dynamic convergence to equilibrium. The replicator dynamics describes the evolution in a population of various strategies in which each member follows the following imitation protocol: after every interaction it switches to any strategy which achieved more payoff than its own with probability proportionate to the payoff difference. In the resulting dynamics, the share of a strategy j in the population grows at a rate that is proportional to the difference between the payoff of that strategy and the average payoff of the population. As it turns out the game that we have considered is one with an interior ESS under the assumptions in statement 3 of Thm. 1 and 2 and in such games the convergence of replicator dynamics (or other major evolutionary dynamics) to the ESS is guaranteed (see [7]).

Delayed evolutionary dynamics describes the evolution of a population in which the payoffs from each local

$$\frac{d\,y^1(t)}{dt} = \mu\, y^1(t)(U_{av}(\tau, y^1(t-T_1)) - [y^1(t)U_{av}(\tau, y^1(t-T_1)) + y^2(t)U_{av}(\tau', y^1(t-T_2))]) \tag{6}$$
$$\frac{d\,y^2(t)}{dt} = \mu\, y^2(t)(U_{av}(\tau', y^1(t-T_2)) - [y^1(t)U_{av}(\tau, y^1(t-T_1)) + y^2(t)U_{av}(\tau', y^1(t-T_2))])$$

interaction are received with time delays. More precisely, if a relay chooses the strategy j at time t when the population profile is y, then it will receive the payoff only T_j time later. In the replicator dynamics with time delays, the share of a strategy j in the population grows at a rate that is proportional to the difference between the payoff of that strategy delayed by an average time delay T_j and the average delayed payoff of the population. Convergence of Replicator Dynamics in a game does not imply the convergence of delayed Replicator dynamics and its convergence generally strongly depends on the time delays T_j and on an additional parameter μ. In particular, consider the Live Time control game described in the previous sections. The delayed evolutionary dynamics model the evolution of the population according to equation (6) where $y^1(t), y^2(t) \geq 0$ and $y^1(t)+y^2(t) = 1$, μ is a parameter which can be used to tune the rate of convergence and it can be interpreted as the rate at which a member of the population participates in a local interaction game. We take up an example to study the convergence to ESS under these dynamics; we use the following parameters: $\mu = 1$, $\tau = 1$, $\tau' = 0.5$, $\lambda = 3$, $\gamma = 3$, $g = 0.15$.

Figure 1 shows the convergence to ESS under delayed replicator dynamics without delay. The dynamic finally converges to the ESS. For very high values of delays the dynamics may not converge as shown in figure 2 (where $T_1 = T_2 = 200$). It is observed that the amplitude of the oscillations increase as the time delay further increases.

Remark 2: The results showed Fig. 1 demonstrate that the design of forwarding mechanisms that leverage rewarding in order to forward messages in DTNs cannot overlook the delays in realizing the payoffs; in fact, if there exist other delays than the delay caused by the duration of the local interaction, the convergence to an ESS is not guaranteed and oscillatory dynamics may in turn appear.

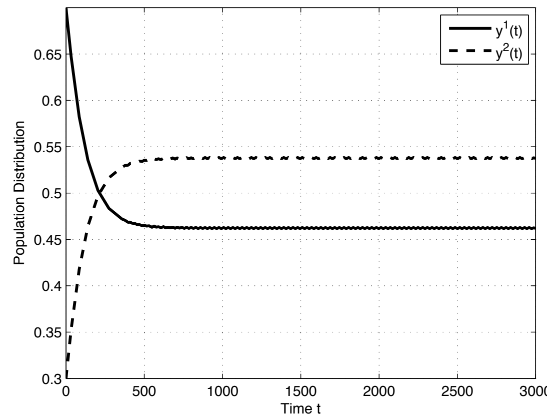

Fig. 1. Evolution of the replicator dynamic with delay equal to τ

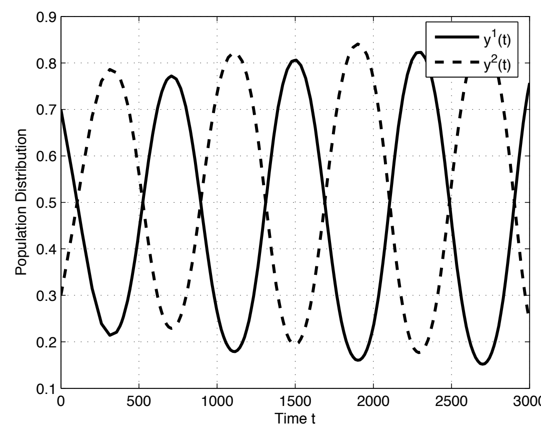

Fig. 2. Impact of the time delay on the stability of the replicator dynamics

VI. Mechanism design

It sight of the characterization of the ESS for the system described before, we are interested in controlling the system in order to optimize for the energy consumption and the delivery probability. Let us assume that the source node controls the forwarding of message copies: during a local interaction a copy of the message is relayed with constant probability u upon meeting a node without a message copy, i.e., using a static forwarding policy [12]. The main quantity of interest is denoted P_s and it is the success probability of a message at a local interaction; under the same assumptions of linearity in [12], the average energy expenditure at the source node is $\mathcal{E} = \varepsilon\Psi$, where $\varepsilon > 0$ is the source energy expenditure per relayed message copy and Ψ is the corresponding expected number of copies released.

A. Activation Control

Let us consider first the activation control. Due to the forwarding control, the expression for (1) is slightly modified: in particular, a tagged mobile relay playing T may deliver a copy of the message to the destination within time τ with probability $1-Q_\tau^u$ where Q_τ^u is given by $Q_\tau^u = \frac{e^{-\lambda u \tau} - u e^{-\lambda \tau}}{1-u}$ so that the success probability in a local interaction with N mobiles

$$P_s(u|N=k) = 1 - \left(Q_\tau^u\right)^{k y_T(u,k)}$$
$$\Rightarrow P_s(u) = 1 - \sum_{k=0}^{\infty} P(N=k)\left(Q_\tau^u\right)^{k y_T(u,k)} \quad (7)$$

where $y_T(u,k)$ is the fraction of mobiles playing T when there are k nodes in the local interaction. It is then possible to define all relevant quantities as a function of u, i.e., $U^u(T, y_T, k)$, $U^u(S, y_T, k)$ and $H^u(\cdot, k)$. Notice that, in order to have a positive delivery probability, it should hold indeed $y_T(u) > 0$, which in turn requires $(1-Q_\tau^u) - g\tau > 0$ as stated in Thm. 1. We obtain the following conditions

Proposition 1:
1. If $g > \frac{1}{\tau}(1-(1+\lambda\tau)e^{-\lambda\tau})$, then $P_s(u) = 0$;
2. If $g < \frac{1}{\tau}(1-(1+\lambda\tau)e^{-\lambda\tau})$, then $P_s(u) > 0$ iff $u \in [u_{\min}, 1]$ where $u_{\min}$ is the unique root of $Q_\tau^u - (1-y)$.

Proof. The first part of the statement is obvious from Thm. 1 and observing that $Q_\tau^u \geq Q_\tau^1 > 1 - g\tau$ for $0 \leq u < 1$. The second statement is again a direct calculation; observe that $Q_\tau^1 < 1 - g\tau \leq 1 = Q_\tau^0$, and Q_τ^u is a continuous decreasing function of u so that there exists a unique solution $u_{\min}$ of the equation $Q_\tau^u = 1 - g\tau$ in $[0,1]$. ◇

We reported in Fig 3 the representation of the plane $(\lambda g, \lambda\tau)$ partitioned into the two regions indicated in Prop. 1. Also, in Fig 4 we depicted the threshold control value that corresponds to the minimal value $u_{\min}$ described in the previous statement. In particular, the effect of the average energy expenditure $g\tau$ and the average number of contacts $\lambda\tau$ is visible there: the larger the energy expenditure of relays during local interactions, the higher the $u_{\min}$ value the source node has to use. In particular, in the left side of that graph, we observe that the source has to forward with higher probability in order to incentive relays to forward the message. For larger values of $g\tau$, however, no control is feasible for low number of contacts.

The constraint on the expected number of copies released at each local interaction writes $\lambda\tau\mathbf{E}\{N y_T(u,N)\}u = \Psi$. Based on this simple relation, it is possible to rephrase the previous results in order to design an optimal forwarding mechanism at the source node, e.g.,

Proposition 2: Assume that $g < \frac{1}{\tau}(1-(1+\lambda\tau)e^{-\lambda\tau})$, and let $u_{\min}$ be as defined in Prop. 1:
1. Let $u^* = \min\{\frac{\Psi}{\lambda\tau\mathbf{E}\{N\}}, 1\}$: if $u^* \geq u_{\min}$ and $\sum_{N=1}^{\infty} P(K=N)\frac{1-(Q_\tau^u)^N}{N} - g\tau \geq 0$, then u^* is the optimal control. If not go to step 2.
2. Let u^* solve for $\frac{\Psi}{u\lambda\tau} = \mathbf{E}\left\{N H_u^{-1}\big(g\tau, N\big)\right\}$ and be $u_{\min} \leq u^* \leq 1$, then u^* is the optimal control. If not go to step 3
3. No control is feasible for the given constraint.

Remark 3: In [12] it was showed that an optimal static policy may not exist when $u \geq \alpha > 0$ for τ large enough. We avoided to include that result in Prop. 2 for the sake of clearness. However, we notice that the feasibility condition showed above is very different. Basically, there exists a threshold in the forwarding probability below which the reward obtained by relays – due to the small probability to obtain a copy from the source – is so small to drive the system to the idle ESS $y_S = 1$.

B. Live time control

Now consider the live time control. Again, we assume that the source wishes to control the system in order to optimize for the energy consumption, i.e., the number of message copies, and the delivery probability by static forwarding policies. In this case, a tagged mobile relay playing $\theta \in \{\tau, \tau'\}$ may deliver a copy of the message to the destination within time θ with probability $1 - Q_\theta^u$ where Q_θ^u is given by $Q_\theta^u = \frac{e^{-\lambda u \theta} - u e^{-\lambda\theta}}{1-u}$, so that the success probability in a local interaction with N mobiles at the equilibrium is

$$P_s(u,N) = 1 - \left[\left(Q_{\tau'}^u\right)^{k(1-y(u))} . \left(Q_\tau^u\right)^{k y(u)}\right] \quad (8)$$
$$\Rightarrow P_s(u) = 1 - \sum_{k=0}^{\infty} P(N=k)\left[\left(Q_{\tau'}^u\right)^{k(1-y(u))} . \left(Q_\tau^u\right)^{k y(u)}\right]$$

where $y(u)$ is the fraction of mobiles playing $\theta = \tau$ when k nodes are present in the local interaction. As done before for the activation control, we can define all relevant quantities as a function of u, i.e., $U^u(\tau, y)$, $U^u(\tau', y)$ and also $H^u(\cdot, k)$. Recall that the source wants to maximize the delivery probability of the message to the destination and meet a give constraint on the energy expenditure, i.e., message copies. In the case of live time

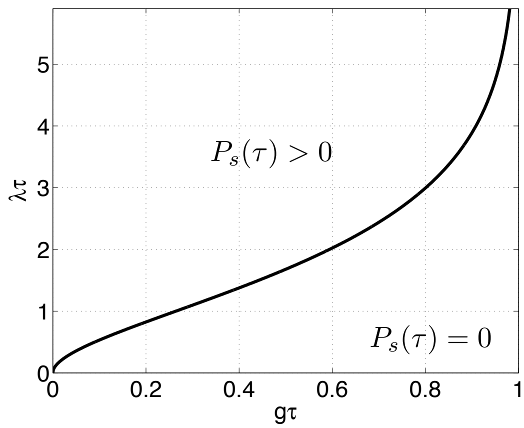

Fig. 3. The partition of the plane into the two regions: zero delivery probability (right), and positive delivery probability (left)

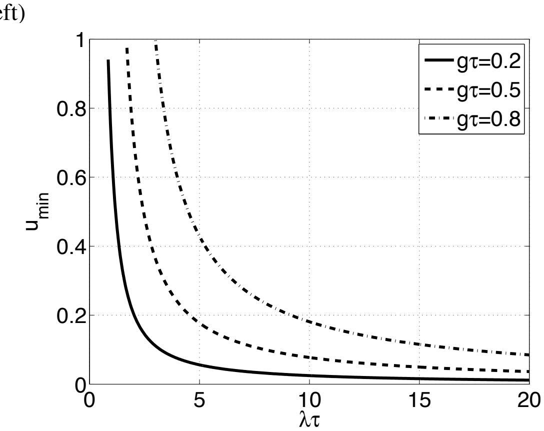

Fig. 4. The minimum forwarding control $u_{\min}$ as a function of the average number of intermeetings $\lambda\tau$ parametrized in the energy expenditure $g\tau$.

control, the number of copies generated given k nodes in local interaction is $u\lambda k(\tau y(u) + \tau'(1 - y(u)))$, from which the constraint writes

$$\lambda u(\mathbf{E}\{Ny(u)\}(\tau - \tau') + \mathbf{E}\{N\}\tau') = \Psi$$

Using the previous relation, it is possible to design an optimal control mechanism at the source node.

Proposition 3: Assume $0 < \tau' \le \tau$:

1. Let $u^* = \min\{\frac{\Psi}{\lambda\tau'\mathbf{E}\{N\}}, 1\}$: if $\sum_{N=1}^{\infty} P(K = N)\frac{(Q_{\tau'})^N\beta}{\tau-\tau'} \le g$, then u^* is the optimal control. If not go to step 2.
2. Let $u^* = \min\{\frac{\Psi}{\lambda\tau\mathbf{E}\{N\}}, 1\}$: if $\sum_{N=1}^{\infty} P(K = N)\frac{Q_{\tau'}^N(1-(1-\beta)^N)}{N} - g(\tau - \tau') \ge 0$, then u^* is the optimal control. If not go to step 3.
3. Let u^* solve for $\frac{\Psi - \mathbf{E}\{N\}\lambda u\tau'}{\lambda\mathbf{E}\{N\}u(\tau-\tau')} = \mathbf{E}\left\{N\tilde{H}_u^{-1}(g(\tau - \tau'), N))\right\}$, then u^* is the optimal control.

Notice that in this case an optimal static forwarding policy always exists: this is a consequence of the fact that all relays are active. Also, should we consider the case $0 = \tau' < \tau$, then the problem would fall back into the case of the activation game described in Prop. 3.

Remark 4: Deterministic case. Observe that if N is known at the source, the results stated in Prop. 2 and Prop. 3 still hold: it is sufficient to drop the expectation operator in all the above equations.

VII. Conclusions

In this paper we presented a general framework for competitive forwarding in DTNs under the two hops routing. Within the context of routing games, part of the forwarding control is demanded to relays, which may accept to spend some energy and participate to the forwarding mechanism and trade energy expenditure for reward. In this work, we addressed competitive games where the activation of a relay during a local interaction depends solely on the reward and the cost obtained by the relay. Compared to existing works in literature, we introduced an evolutionary game theoretical framework and provided necessary and sufficient conditions for the existence of evolutionary stable strategies, depending on energy and delivery probability only. In the case when hybrid ESS exist, we derived the related replicator dynamics of the game, describing the effect of delayed rewarding, finding typical oscillations around the ESS for the system in case of large delays.

We observe that this is a novel context for routing control in DTNs: in fact the decision to forward or not at the source depends on the whole evolutionary dynamics and ultimately on the ESS that is reachable for any given forwarding control at the source.

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