Virtual MIMO Network
A Physical Wireless Analysis

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***Abstract*— By analogy with an approach widely used in physics, we consider a discrete set of base stations (BS) as a continuum of transmitters. This model allows us to establish a simple closed-form formula of the interferences and the SINR received by a mobile, whatever its location. As an application of this model, we propose an analytical study of a virtual MIMO network. Indeed, we consider that the useful power received by each user of a cell is the sum of useful powers coming from all base stations of the network. The physical wireless network model enables to quantify, in a simple way, the impact of virtual MIMO on the instantaneous throughput. We moreover show that the outage probability increases.**

I. Introduction

The estimation of wireless networks performances mainly depends on the determination of SINR (Signal to Interference plus Noise Ratio) and more generally on the characterization of interferences. The interference factor f is an important parameter for this characterization in CDMA and OFDMA systems (as established in Section II). The precise knowledge of this parameter allows the derivation of performances such that outage probabilities and capacity evaluation. We focus on downlink direction throughout this paper. However, the framework we propose can easily be extended to the uplink.

The downlink interference factor analysis was developed by using different ways : numerical integration in hexagonal networks [3] [4], Monte Carlo simulations [7] [6]. approximation of the distribution fo f [8].

In contrast to previous works in the field, the modelling key of our approach is to consider the discrete BS entities of a cellular network as a continuum. That approach is used in many fields of physic sciences. More generally, in many papers, wireless networks analysis use methods developed in physics. Guo and Verdu [1] use the replica method developed in statistical physics to analyze CDMA systems. Cover and Thomas [2] propose an analogy between the entropy of thermodynamics and the entropy of information theory. Tanaka [17] develops an approach based on statistical mechanics to analyze physical layer in CDMA systems.

Continuum approach in radio network analysis: Recently, the authors of [26] described a network in terms of macroscopic quantities such as the node density. Authors of this paper show that, considering some given conditions, the optimal distribution of traffic in a wireless sensor network resembles an electrostatic field. Other papers use an approach based on electrostatics [14]. Authors investigate the spatial distribution of wireless nodes that can transport a given volume of traffic in a sensor network. They assume a *massively dense* network: there are so many nodes, that is 'does not make sense to specify their placement in terms of the positions of individual nodes' (as said by the authors).

Assuming massively dense wireless networks, Toumpis [18] presents a few examples on the optimal design of these ones. He shows that these networks contain such a large number of nodes that a macroscopic view of them emerges. Though not detailed, that one preserves sufficient information to allow a meaningful network optimization.

The same idea is used in [27], to analyze the routing of the information in the case of massively dense ad-hoc networks. In his approach, the author assumes a very high density of nodes and unlimited networks.

Physical model of wireless network: We express a wireless network as a continuum of transmitting BS. This approach was used in [9] [11]. This paper shows that an analogy can be done with *Gauss theorem* used in physics. This theorem allows to consider a ponctual mass as a continuum uniformly distributed. We denote this model a *Physical model of wireless network*. This paper also extends the framework proposed in [9] and [11] by providing a simple closed-form formula of the SINR received by a mobile, on the downlink, as a function of the distance to its serving BS.

We also provide a simple formula of the SINR received by a mobile in a Virtual MIMO Network.

The physical approach we propose is however quite different than the existing ones in radio analysis. Indeed, these last ones assume a large set of entities (massively dense radio nodes). In contrast, we express a network as a continuum of transmitting BS, *whatever the density* of base stations, even when this one *is very low* and *whatever the network size*, even when this last one is *limited*.

As an application of this *physical model of wireless network*, we propose a *Virtual MIMO Network* analysis (downlink macrodiversity generalized to all base stations), considering that the total useful power of users is the sum of useful powers transmitted by all base stations of the network. The studies related to macrodiversity were mainly done for the uplink [24] [25]. Hanly [24] described macrodiversity as a scheme in which the cellular structure of a wireless communication network is removed and a user is jointly decoded by all receivers in the network. Bordenave [22] proposed a general model of macrodiversity in CDMA networks in uplink and downlink. Hiltunen and de Bernardi [21] developed a downlink analysis of the macrodiversity to estimate a CDMA network's capacity. Their analysis considers the macrodiversity use to maintain the SIR target of mobiles in soft/softer handover with two base stations. In this paper, we propose an analysis of virtual MIMO networks. The physical network model allows to analytically express the outage probability, in a simple way. Moreover, this model allows to quantify the impact of virtual MIMO. We establish that the cell capacity decreases, and we quantify that decrease.

For clarity of presentation, this paper is focused on CDMA networks. However, the analysis we develop can be used for other technologies such as OFDMA.

The rest of the paper is organized as follows. In Section II we establish the SINR expression for HSDPA and OFDMA systems (WiMax, LTE...). In Section III, by analogy with gravitation, we express a transmitting BS as a density of BS. In Section IV, generalizing that approach to a network, we establish a closed form formula of the SINR for HSDPA and OFDMA systems. And we analyze the case of a free space. In Section V, we propose an example of application of the physical model to the analysis of a virtual MIMO network. We establish the SINR expression and we analyse outage probability. In Section VI we conclude.

II. Model and notations

We consider a cellular radio system and we focus on the downlink. BS have omni-directional antennas, so that a BS covers a single cell. The following power quantities are considered:

- $P_{b,u}$ is the transmitted power from station b towards mobile u (for user's traffic);
- $P_b = P_{cch} + \Sigma_u P_{b,u}$ is the total power transmitted by station b. In CDMA systems, P_{cch} represents the amount of power used to support broadcast and common control channels.
- $p_{b,u}$ is the power received at mobile u from station b; we can write $p_{b,u} = P_b\, g_{b,u}$; $g_{b,u}$ designates the pathloss between station b and mobile u
- $S_{b,u} = P_{b,u}\, g_{b,u}$ is the useful power received at mobile u from station b (for traffic data);
- γ_u is the SINR evaluated at mobile u
- γ_u^* is the SINR target for the service requested by MS u. We assume perfect power control, so $SINR = \gamma_u^*$ for all users.

The total amount of power experienced by a mobile station u in a cellular system can be split up into several terms: useful signal (S_u), interference and noise (N_0). It is common to split the system power into two terms: $p_u = p_{int,u} + p_{ext,u}$, where $p_{int,u}$ is the *internal* (or own-cell) received power and $p_{ext,u}$ is the *external* (or other-cell) interference. Notice that we made the choice of including the useful signal S_u in $p_{int,u}$, and, as a consequence, it has to be distinguished from the commonly considered own-cell interference.

With the above notations, we define the interference factor in u, as the ratio of total power received from other BS to the total power received from the serving BS:

$$f_u = p_{ext,u}/p_{int,u}. \tag{1}$$

Since they depend on user u location, the quantities f_u, $p_{ext,u}$, and $p_{int,u}$ are location dependent and can thus be defined in any location as long as the serving BS is known. In downlink, a coefficient α, may be introduced to account for the lack of perfect orthogonality in the own cell.

A. HSDPA system

With the introduced notations, the SINR experienced by u can thus be derived (see e.g. [5]):

$$\gamma_u = \frac{S_u}{\alpha(p_{int,u} - S_u) + p_{ext,u} + N_0} \tag{2}$$

From this relation, we can express S_u as:

$$S_u = \frac{\gamma_u}{1+\alpha\gamma_u}\, p_{int,u}\, \left(\alpha + p_{ext,u}/p_{int,u} + N_0/p_{int,u}\right) \tag{3}$$

and the transmitted power for MS u, $P_{b,u} = S_u/g_{b,u}$, using relations $p_{int,u} = P_b g_{b,u}$ and $f_u = p_{ext,u}/p_{int,u}$ as:

$$P_{b,u} = \frac{\gamma_u}{1+\alpha\gamma_u}(\alpha P_b + f_u P_b + N_0/g_{b,u}). \tag{4}$$

We denote $\kappa_u = \frac{P_{b,u}}{P_b}$. Considering that a unique user is scheduled at each time, the whole available power is dedicated to this user, so we can consider $\kappa_u = \kappa$ as constant. Since $\frac{N_0}{P_b g_{b,u}} << f_u$, typically for cell radii less than about 1 km, this term is neglected. So we have:

$$\gamma_u = \frac{\kappa}{\alpha(1-\kappa) + f_u}. \tag{5}$$

Since f_u depends on the location of the user u, this expression establishes the SINR dependance with the user location.

B. UMTS system

With the introduced notations, the target SINR experienced by u can thus be derived (see e.g. [5]):

$$\gamma_u^* = \frac{S_u}{\alpha(p_{int,u} - S_u) + p_{ext,u} + N_0} \tag{6}$$

From this relation, we can express S_u as:

$$S_u = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}\, p_{int,u}\, \left(\alpha + p_{ext,u}/p_{int,u} + N_0/p_{int,u}\right) \tag{7}$$

Considering the target SINR of a unique service, γ_u^* is a constant. We can express the transmitting useful power towards the mobile u:

$$P_{b,u} = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha P_b + f_u P_b + N_0/g_{b,u}). \tag{8}$$

and so, neglecting thermal noise:

$$\kappa_u = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha + f_u). \tag{9}$$

In this case, κ_u depends on the location of the user u, since f_u depends on the location of the user.

C. OFDMA system

In OFDMA, the data are multiplexed over a large number of subcarriers. Since $p_{ext,u} = \sum_{j \neq b} P_j g_{j,u}$, the SINR received by a mobile u can be written, for a subcarrier

$$\gamma_u = \frac{P_{b,u} g_{b,u}}{\sum_{j \neq b} P_j g_{j,u} + N_0} \tag{10}$$

so we have

$$\gamma_u = \frac{1}{f_u + \frac{N_0}{P_{b,u} g_{b,u}}} \tag{11}$$

Neglecting the thermal noise, we write

$$\gamma_u = \frac{1}{f_u} \tag{12}$$

For each subcarrier of an OFDMA system (WiMax, LTE...), the parameter f represents the inverse of the SIR (Signal to Interference Ratio).

D. Throughput

Since the SINR ($\approx$ SIR for cell radii < 1 km) received by a mobile allows to calculate the throughput D_u by using Shannon relation

$$D_u = \log_2(1 + \gamma_u) \tag{13}$$

in bps/Hz, expressions (5) and (12) allow to evaluate performances of HSDPA and OFDMA systems. We showed that the interference factor is a key parameter to analyze wireless networks. In the aim to propose a closed form formula of f, we first develop a physical model of the network.

III. Physical Analysis

The physical network model proposed in this paper considers that a discrete set of BS of a cellular network can be replaced by a continuum of BS. This approach can be compared to other approaches in physics, where discrete entities are considered as a continuum. For example, to analyze certain types of problems concerning the gravitation, a discrete mass M can be considered as a continuum. Hereafter, we remind this approach based on the Gauss theorem. Afterwards we propose an analogy with a wireless network, which allows to consider a ponctual BS as a density of BS.

A. Gauss theorem

In a mechanical system, let's consider a mass m inducing a gravitational force at a given point of the space, located at the distance r from the mass. The gravitation is characterized by a function form $\frac{1}{r^2}$. The Gauss theorem shows that the gravitational effect of a non punctual object with a mass density ρ_m, a volume V and a mass m can be replaced by an equivalent one with the same mass which is located at its gravity mass centre. For example the Earth can be replaced by a point in its centre where all the Earth mass would be concentrated. The gravitational effects of these two approaches are equivalent. In fact the Gauss theorem expresses that a field created by a *non ponctual* object of mass m at a given point of a system can be calculated by considering only the distance to the mass centre, and *not* the distances to each elementary mass. The Gauss theorem allows *replacing* a mass density uniformly distributed by one unique ponctual mass.

1) Global expression: The gravitation force $\vec{F}$ flow through a closed surface is equal to the sum of the interior masses m_i multiplied by $-4\pi G$ (G is the gravitation constant).

$$\int\int_S \vec{F}.d\vec{S} = -4\pi G \sum m_i \tag{14}$$

where $\sum m_i = \int\int\int_V \rho_m dV$ and ρ_m is the density of mass in the volume V.

Remark

We notice that (14) is true whatever the density of mass ρ_m.

2) Local expression: From the local expression of the Gauss theorem written as the *Poisson* equation, the gravitation field can interpreted as the *Response of the Isotropic Space* to a mass density solicitation [13].

The force $\vec{F}(\vec{r}, t)$ created at each point of a system where there is a density of mass $\rho(r', t)$ is obtained [13] as the sum of all the infinitesimal contributions of a continuum distributed matter:

$$\vec{F}(\vec{r}, t) = \int G\rho(r', t) \frac{\vec{r} - \vec{r'}}{\|\vec{r'} - \vec{r'}\|^3} d^3r'. \tag{15}$$

Introducing a gravitational potential $\psi(r, t)$ and write $\vec{F}(\vec{r}, t) = -grad(\psi(r, t))$. Finally the potential can be expressed as a convolution:

$$\psi(r, t) = -G\rho(r, t) * \frac{1}{\|r\|}. \tag{16}$$

And so $grad(\psi(r, t)) = -Ggrad(\rho(r, t) * \frac{1}{\|r\|})$. The function $\frac{1}{\|r\|}$ is the Green function of the Laplacien Δ. So we have

$$\Delta(\psi(r, t)) = 4\pi G\rho(r, t) * \delta(r) \tag{17}$$

where $\delta(r)$ is the Dirac distribution. Since the Dirac is the neutral element of the convolution algebra, we finally obtain the Poisson equation:

$$\Delta(\psi(r, t)) = 4\pi G\rho(r, t) \tag{18}$$

These equations mean that Gravitation can be interpreted as the **Response of the isotropic Space to a density mass solicitation**.

Remark Expression (17) can be generalized to obtain the classical definition of the gravitational potential in a D-dimensionnal space [13]. So ψ can be written as proportional to $\frac{1}{r^{D-2}}$ in a multidimensional space where D represents the dimension of the space. That property can be interpreted as follows: the response of the system to a solicitation depends on the dimension of the space considered for the system. For example the gravitational force could be written as proportional to $\frac{1}{r^{D-1}}$. Generally, D=3 for the "normal" space.

Concluding remark

It is equivalent to say that the gravitation is the response of the isotropic space to a density mass solicitation and to write the expression (17) which expresses that it exists a gravitational potential ψ whose Laplacien is proportional to a mass density. That expression represents the local expression of Gauss theorem.

B. Analogy for a Base Station

At a distance r from a transmitting BS, we can measure a power proportional to $\frac{1}{r^\eta}$, where η =2 in a free space and $\eta > 2$ in wireless networks. This measure can be interpreted as the **Response of the environment to the BS solicitation**. By analogy with expression (17) and using the property of the Dirac function, a localized transmitting BS can be expressed as a density of transmitting BS.

Remark

In a real environment the parameter η is generally comprised between 3 and 4. This is due to the reflections, refractions of radio propagation ... all phenomena which transform an isotropic and free environment to a non one. The observed increase of η can be interpreted as follows. The non isotropy and the environment constraints of the wireless system play an analog role as a multidimensional system where the dimension $D > 3$.

IV. A Physical Wireless Network Model

In this section, we first present the model, derive the closed-form formulas for f and SIR, and analyze the limit cases of free space ($\eta = 2$) and large network.

A. Model

We expressed a ponctual BS as a density of BS. We generalize that approach to a network constituted by a set of base stations. We propose to consider a given fixed finite number of interfering BS as an equivalent density of transmitters which are spatially distributed in the network. The interfering base stations of a network are characterised by a base station density ρ_{BS} [9] [10]. We assume that BS are uniformly distributed in the network, so that ρ_{BS} is constant. Considering a homogeneous network, all base stations have the same output power P_b.

1) Interference factor formula: We focus on a given cell and consider a round shaped network around this centre cell with radius R_{nw} [9] [10]. The half distance between two base stations is R_c (see Figure 1).

We use a propagation model where the path gain, $g_{b,u}$, only depends on the distance r between the BS b and the MS u. The power, $p_{b,u}$, received by a mobile at distance r_u can thus be written $p_{b,u} = P_b K r_u^{-\eta}$, where K is a constant and $\eta > 2$ is the path-loss exponent.

Let's consider a mobile u at a distance r_u from its serving BS. Each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{BS} zdzd\theta$ base stations which contribute to $p_{ext,u}$. Their contribution to the external interference is $\rho_{BS} zdzd\theta P_b K z^{-\eta}$.

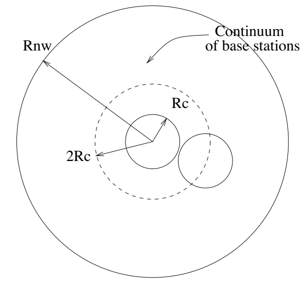

Fig. 1. Network and cell of interest in the physical model; the distance between two BS is $2R_c$ and the network is made of a continuum of base stations.

We approximate the integration surface by a ring with centre u, inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$.

$$p_{ext,u} = \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} P_b K z^{-\eta} zdzd\theta$$

$$= \frac{2\pi \rho_{BS} P_b K}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \quad (19)$$

Moreover, MS u receives internal power from b, which is at distance r_u: $p_{int,u} = P_b K r_u^{-\eta}$. So, the interference factor $f_u = p_{ext,u}/p_{int,u}$ can be expressed by:

$$f_u = \frac{2\pi \rho_{BS} r_u^\eta}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \quad (20)$$

Since we assumed an homogeneous network, all base stations emit the same power. As a consequence, f_u does not depend on the BS output power. In our model, f only depends on the distance r to its serving BS. It can be defined in each location. We can write f as a function of r, $f(r)$. Note that for a large network i.e. $R_{nw} >> R_c$, f_u can be approximated by:

$$f(r) = \frac{2\pi \rho_{BS} r^\eta}{\eta - 2} (2R_c - r)^{2-\eta}. \quad (21)$$

We notice that formula can be applied to *any wireless system* (CDMA, OFDMA, TDMA...) as long as ρ_{BS} represents the density of *interfering transmitters*.

Remark We used the physical approach developed in Section III to establish formula (20) used in [9] [10] [11] for CDMA networks.

2) SIR formula : HSDPA system: For a mobile located at a distance r from its serving BS, SIR formula (denoted γ) is deduced from (12) and (20):

$$\gamma(r) = \frac{\kappa}{\alpha(1 - \kappa) + f(r)}. \quad (22)$$

and so

$$\gamma(r) = \frac{\kappa}{\alpha(1 - \kappa) + \frac{2\pi \rho_{BS} r^\eta}{\eta - 2} \left[(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta} \right]}. \quad (23)$$

3) SIR formula : OFDMA system: For a mobile located at a distance r from its serving BS, SIR formula (denoted γ) is deduced from (12) and (20):

$$\gamma(r) = \frac{\eta - 2}{2\pi\rho_{BS}r^{\eta}} \frac{1}{[(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta}]}. \quad (24)$$

4) Useful power : UMTS system: For a mobile located at a distance r from its serving BS, using (20), κ_u (9) can be expressed :

$$\kappa(r) = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha + \frac{2\pi\rho_{BS}r^{\eta}}{\eta - 2}\left[(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta}\right]) \quad (25)$$

The physical wireless network model is developed whatever the density of base station. As a consequence, the expressions of f (20) and SIR (23) (24) are established *whatever the base station's density*, even when this one is low.

B. Case of exponential pathloss η = 2

Though in real networks, the exponential pathloss η is different of 2, it is interesting to analyze what happens in the limit of $\eta \to 2$ since that case characterizes the propagation in a free space. Using the physical model, we can write (19) for the whole network, when $\eta = 2$ as (dropping the index u):

$$\begin{aligned} p_{ext} &= \int_0^{2\pi}\int_{2R_c - r}^{R_{nw}-r} \rho_{BS}P_bKz^{-2}zdzd\theta \\ &= 2\pi\rho_{BS}P_bK \log\frac{R_{nw}-r}{2R_c - r}. \end{aligned} \quad (26)$$

Since $p_{int} = P_bKr^{-2}$, the interference factor (Eq. 1), denoted $f_2(r)$ in this case, is expressed as:

$$f_2(r) = 2\pi\rho_{BS}r^2 \log\frac{R_{nw}-r}{2R_c - r}. \quad (27)$$

Considering the expression (20) of the interference factor $f(r)$ for $\eta > 2$, we notice that

$$\lim_{\eta\to 2} f(r) = 2\pi\rho_{BS}r^2 \log\frac{R_{nw}-r}{2R_c - r} = f_2(r). \quad (28)$$

Since r and R_c have finite values, the interference factor tends towards an infinite value for an infinite network ($R_{nw} \to \infty$). This means that in an infinite network, if no phenomena modifies the free propagation exponential pathloss factor (i.e. η = 2), the SIR (denoted $\gamma_2(r)$) given by (OFDMA case)

$$\gamma_2(r) = \frac{1}{2\pi\rho_{BS}r^2 \log\frac{R_{nw}-r}{2R_c - r}} \quad (29)$$

would be close to zero. As a consequence, the performances of a cell in term of throughput (by using Shannon relation 13) should decrease to zero due to the amount of interferences generated by the whole network. However, in a real network, though R_{nw} can be very large, the ratio $\frac{R_{nw}}{2R_c}$ remains finite. The interference factor remains limited and the capacity is low, but does not decrease to zero.

Remark

Let us notice that non uniform density of base station $\rho_{BS}(r)$ can be considered, in the aim to analyze a non homogeneous network. This analysis was done in [10]. Moreover, for sake of simplicity, we did not consider the shadowing in the model. However, in [12][20], we propose a formula of f taking into account the shadowing.

C. Comparison with hexagonal model network

It appears interesting to compare the physical network model presented above to an hexagonal classical one. In this perspective, we compare the figures obtained with Eq.23 with those obtained numerically by simulations. The simulator assumes an homogeneous hexagonal network made of several rings of BS around a centre cell. Figure 2 shows an example of such a network.

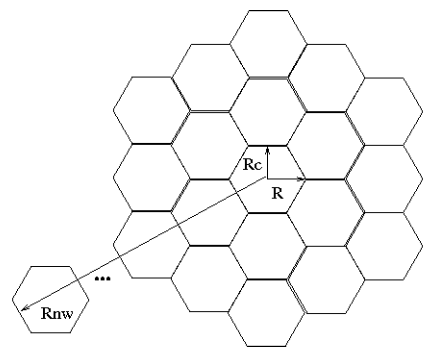

Fig. 2. Hexagonal network and main parameters of the study.

The comparison is done numerically by computing the SIR in each point of the cell and averaging the values at a given distance from the BS. Figure 3 shows the simulated SIR as a function of the distance to the base station. Simulation parameters are the following: R = 1 Km, η between 2 and 4.5, $\rho_{BS} = (3\sqrt{3}R^2/2)^{-1}$, the number of rings is 8 (except for $\eta = 2$). Eq.23 is also plotted for comparison. For $\eta = 2$, the number of rings is 100, to simulate a very large network, and Eq.29 is plotted for comparison (with κ=1). In

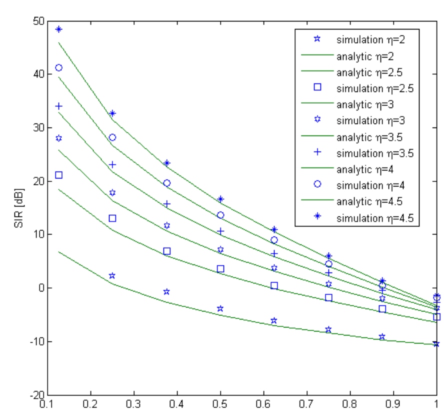

Fig. 3. SIR vs. distance to the BS; comparison of the physical model with simulations on an hexagonal network with $\eta = 2$, 2.5, 3, 3.5, 4, 4.5.

all cases, the physical model matches very well the simulations on an hexagonal network for various figures of the path-loss exponent.

We notice that the distance between BSs is 2 Km which corresponds to a low base station density. The physical model and the hexagonal model are two simplifications of the reality, and none is a priori better than the other. We propose hereafter an application of the model to the analysis of virtual MIMO networks.

V. Virtual MIMO Network Analysis

A. Useful Transmitting Power

For the downlink, we express that the useful power received by a mobile u connected to the base station b comes from its serving base station and all the other $B-1$ base stations of the network. The SIR γ_u of user u can be written as:

$$\gamma_u = \frac{P_{b,u}g_{b,u} + \sum_{q\neq b}^{B-1} P_{q,u}g_{q,u}}{\alpha(P_b - P_{b,u})g_{b,u} + \sum_{q\neq b}^{B-1}(P_q - P_{q,u})g_{q,u} + N_0}. \quad (30)$$

The term $\sum_{q\neq b}^{B-1}(P_q - P_{q,u})g_{q,u}$ represents the interferences due to the other B-1 base stations of the network.

Let remind that $P_{q,u}$ represents the useful transmitting power coming from base station q towards the mobile u belonging to the base station b, and $g_{q,u}$ is the pathloss between the base station q and the mobile u, P_q is the total transmitting power of the base station q.

Dropping the index u in γ_u we denote

$$\mu = \frac{1+\gamma_u}{1+\alpha\gamma_u} \quad (31)$$

and

$$\beta = \frac{\gamma_u}{1+\alpha\gamma_u}. \quad (32)$$

So we can write

$$P_{b,u} = \beta(\alpha P_b + f_u P_b + \frac{N_0}{g_{b,u}}) - \frac{\mu}{g_{b,u}}\sum_{q\neq b}^{B} P_{q,u}g_{q,u} \quad (33)$$

It is reasonable to consider a limitation of the available transmitting powers $P_{q,u}$ dedicated to a mobile u connected to the BS b. Indeed, the base stations transmitting powers are limited. Moreover, considering the large distances between the other base stations of the network and the mobile u belonging to base station b, we can assume that the base stations use the maximum power (denoted P) available for the transmitting power $P_{q,u}$.

Using Eq. (1) we can write, when all the base stations transmitting powers are identical : $P_q = P_b$ for q = 1...B,

$$f_u = \frac{1}{P_b g_{b,u}}\sum_{q\neq b}^{B-1} P_q g_{q,u} = \frac{1}{g_{b,u}}\sum_{q\neq b}^{B-1} g_{q,u}, \quad (34)$$

and when all the transmitting powers $P_{q,u}$ equal P, we have

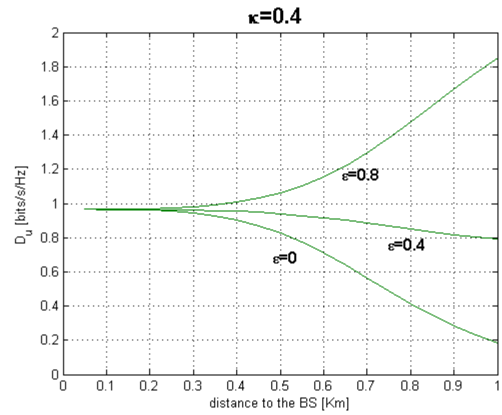

Fig. 4. Instantaneous throughput vs distance from serving BS, for κ = 0.4.

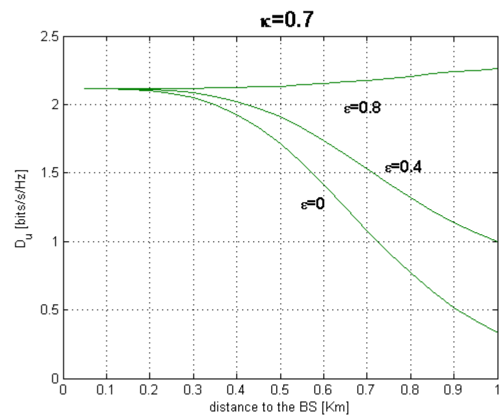

Fig. 5. Instantaneous throughput vs distance from serving BS, for κ = 0.7.

$$\frac{1}{g_{b,u}}\sum_{q\neq b}^{B-1} P_{q,u}g_{q,u} = P\frac{1}{g_{b,u}}\sum_{q\neq b}^{B-1} g_{q,u} = Pf_u. \quad (35)$$

These last assumptions can be verified for example if the network is homogeneous, or when base stations transmit the maximum power.

B. SINR Expression

From (33) and (35), we consider that the power P is a fraction of the total power P_b, so we write $P = \epsilon P_b$. We can express the SINR of a mobile in a virtual MIMO network, assuming a negligible noise N_0:

$$\gamma_u = \frac{\epsilon f_u + \kappa}{\alpha(1-\kappa) + f_u(1-\epsilon)}, \quad (36)$$

where $\kappa = \frac{P_{b,u}}{P_b}$. By using the physical model, we can express f_u as $f(r)$ at any location r. We can write:

$$\gamma(r) = \frac{\epsilon f(r) + \kappa}{\alpha(1-\kappa) + f(r)(1-\epsilon)}. \quad (37)$$

Figures 4 and 5 show the instantaneous throughput variations with the distance to the serving BS, for two values of κ.

We observe that the throughput given by expressions (13) and (37) may increase or decrease with the distance r from the serving BS, according to the values of ϵ and κ. For example, for ϵ=0.8 and κ = 0.4, we observe a maximum increase of about 80% (from r= 0.3 to r = 1). In this case, the instantaneous throughput reaches a maximum value of 1.85 bits/s/Hz. And for ϵ=0.4 and κ = 0.7, we observe a maximum decrease of about 50% (from r= 0.3 to r = 1). In this case, the instantaneous throughput reaches a minimum value of 1 bit/s/Hz.

We notice, from (37), that $\gamma(r)$ is an increasing function of $f(r)$ (and consequently of the distance r) when $\epsilon > \frac{\kappa}{\alpha-\alpha\kappa+\kappa}$.

In the aim to analyse these figures, let first consider a virtual MIMO network where one only mobile is scheduled at each time in the whole network. It means that the whole useful transmitting power of each BS is dedicated to *one unique mobile* at each time. In this case, a reasonnable value of κ is about 0.7 (Fig. 5). For α = 0.7, values of ϵ have thus to be higher than 0.76 to observe an increase of $\gamma(r)$. Another reasonnable value of κ is 0.4, when we assume that two mobiles are scheduled at each time in the whole network (Fig. 4). In this case, for $\gamma(r)$ to increase, values of ϵ have thus to be higher than 0.48. Though we considered mobiles connected to BS b, this analysis can be done whatever the base station q.

These figures also show that the *instantaneous throughput* reached by a mobile is higher for a virtual MIMO network than for a *non virtual MIMO network* (ϵ = 0).

However, let us notice that the *average throughput* reached by a mobile may be lower in the case of a virtual MIMO network than for a non virtual MIMO network, since we considered that in the whole network, a unique mobile is scheduled at each time.

C. Base Station Transmitting Power

We assume now that the bandwidth is used by all mobiles in *the same time*. We consider that each base station of the network contributes to the useful power received by any mobile belonging to any base station. Assuming N mobiles per cell, the total transmitting power of the base station b can thus be written as:

$$P_b = \sum_{u=1}^{N} P_{b,u} + \sum_{q\neq b}^{B-1} \sum_{u=1}^{N} P + P_{cch} \tag{38}$$

Finally, from (33) and (35) and considering that the power P is a fraction of the total power P_b, we can write, $P = \epsilon P_b$, and when $P_{cch} = \varphi P_b$ we can write (38) as:

$$P_b = \frac{\sum_{u=1}^{N} \frac{\beta^* N_0}{g_{b,u}}}{1-\varphi - N\beta^*\alpha - (\beta^* - \mu^*\epsilon)\sum_{u=1}^{N} f_u - N.(B-1)\epsilon} \tag{39}$$

where

$$\mu^* = \frac{1+\gamma_u^*}{1+\alpha\gamma_u^*} \tag{40}$$

and

$$\beta^* = \frac{\gamma_u^*}{1+\alpha\gamma_u^*}. \tag{41}$$

We notice that in this case, typical values of ϵ cannot exceed 10^{-3}. Otherwise, the available power P of a BS, dedicated to mobiles connected to other cells, would be too high. Indeed, we assumed that a BS has to serve at each moment its own mobiles and all mobiles connected to the other base stations.

D. Cancellation of the other cell interferences

Expression (39) shows that the term characterizing the other cell interferences $\sum_{u\neq 1}^{N} f_u$ vanishes for $\beta = \mu\epsilon$, as if the interferences felt by a mobile, due to the other base stations of the network, were balanced by the fraction of their transmitting powers dedicated to that mobile. From the expression of β and μ, we have $\epsilon = \frac{\gamma_u^*}{1+\gamma_u^*}$. In this case the cell n capacity can be written:

$$n = \frac{1-\varphi}{\beta^*\alpha + \frac{B-1}{\mu^*}}. \tag{42}$$

E. Outage probabilities

For a given number of users per cell N, outage probability, $P_{out}^{(N)}$, is the proportion of configurations, for which the needed BS output power exceeds the maximum output power: $P_b > P_{max}$. In a CDMA system, if noise (N_0) is neglected and if we assume a single service network ($\gamma_u^* = \gamma$ for all u), we deduce from Eq.39, if $\beta - \mu\epsilon > 0$:

$$P_{out}^{(N)} = Pr\left[\sum_{u=0}^{N-1} f_u > \frac{1-\varphi - N\beta^*\alpha - N(B-1)\epsilon}{\beta^* - \mu^*\epsilon}\right], \tag{43}$$

and if $\beta - \mu\epsilon < 0$:

$$P_{out}^{(N)} = Pr\left[\sum_{u=0}^{N-1} f_u < \frac{1-\varphi - N\beta^*\alpha - N(B-1)\epsilon}{\beta^* - \mu^*\epsilon}\right], \tag{44}$$

where $\varphi = P_{cch}/P_{max}$.

F. Gaussian Approximation

In order to compute these probabilities, we rely on the Central Limit theorem and use a Gaussian approximation. As a consequence, we need to compute the spatial mean and standard deviation of $f_u = f(r)$ given by the physical model (20). The area of a cell is $1/\rho_{BS} = \pi R_e^2$ with $R_e = R_c\sqrt{2\sqrt{3}/\pi}$. So, we integrate $f(r)$ on a disk of radius R_e. As MS are uniformly distributed over the equivalent disk, the probability density function (pdf) of r is: $p_r(t) = \frac{2t}{R_e^2}$. Let ν_f and σ_f be respectively the mean and standard deviation of $f(r)$, when r is uniformly distributed over the disk of radius R_e. As a conclusion of this section, the outage probability can be approximated by

$$P_{out}^{(N)} = Q\left(\frac{\frac{1-\varphi-N\beta^*\alpha-N(B-1)\epsilon}{\beta^*-\mu^*\epsilon} - N\nu_f}{\sqrt{N}\sigma_f}\right), \tag{45}$$

where Q is the error function. We focus on $\beta^* - \mu^*\epsilon > 0$. An analogue expression can be written when $\beta^* - \mu^*\epsilon < 0$.

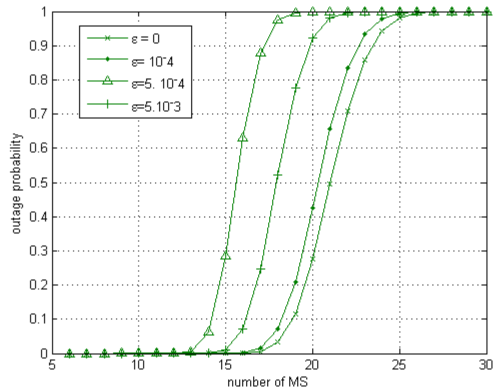

Fig. 6. Outage probability vs percentage of transmitting power ϵ.

G. Results

Figure 6 shows the kind of results we are able to obtain instantaneously thanks to the model we developed and the simple formulas derived in this paper for voice service ($\gamma_u = -16$ dB), B=100, η =3.5 and α =0.7. This figure allows us to easily find the capacity of the network for any given percentage of outage probability. And we are able to quantify the impact of virtual MIMO in term of outage probabilities as a function of the number of users per cell for various values of ϵ. For a given outage probability, the cell capacity decreases when ϵ increases. For example, we establish there are 19 users per cell for an outage probability of 10% when we assume a *non virtual MIMO network* (ϵ=0). For the same outage, there are only 16 users per cell when $\epsilon = 5.10^{-4}$ (virtual MIMO network). And for a given number of users, outage probability increases when ϵ increases. For example, when $\epsilon = 5.10^{-4}$, we observe that the outage probability reaches 80%, when there are 19 users per cell.

VI. Conclusion

Using an analogy with physics, we considered a transmitting BS as a density of BS. Generalizing that approach to a whole network, we developed the *physical wireless network model*. The key idea consists in replacing a discrete set of transmitters by a continuum of transmitters spatially distributed in the network. It allowed us to establish a closed form formula of the SINR received by a user, at any location r.

As an application of this model, we proposed an analysis of a virtual MIMO network. We first established a simple expression of the SINR at any location r, in a virtual MIMO network. We established that the instantaneous throughput reached by a mobile (when scheduled alone) can increase or decrease with the distance to its serving BS. We showed that these variations depend on certain parameters, and quantified these variations according to these parameters. We moreover established that in a virtual MIMO network, the instantaneous throughput of a mobile is higher than in a non virtual MIMO network. Afterwards we analysed the case of N mobiles sharing the bandwidth in the same time. In this case, we established that, compared to a non virtual MIMO network, the outage probability increases.

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