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(Invited Paper)

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***Abstract*—We consider the two-user multiple-input single-output (MISO) interference channel and the rate region which is achieved when the receivers treat the interference as additive Gaussian noise and the transmitters have perfect channel state information (CSI). We propose a computationally efficient method for calculating the Pareto boundary of the rate region. We show that the problem of finding an arbitrary Pareto-optimal rate pair, along with its enabling beamforming vector pair, can be cast as a sequence of second-order cone programming (SOCP) feasibility problems. The SOCP problems are convex and they are solved very efficiently using standard off-the-shelf (namely, interior-point) algorithms. The number of SOCP problems that must be solved, for the computation of a Pareto-optimal point, grows only logarithmically with the desired accuracy of the solution.**

I. Introduction

We are interested in the scenario where multiple wireless links operate at the same carrier frequency and mutually interfere with one another. Specifically, we consider two transmitter-receiver pairs $\mathrm{TX}_1 \rightarrow \mathrm{RX}_1$ and $\mathrm{TX}_2 \rightarrow \mathrm{RX}_2$ that are located in the proximity of one another, so that RX_1 receives significant interference from TX_2 and vice versa. In information theory, such a setup is called an interference channel (IFC) [1]. We focus on the transmit strategies for TX_1 and TX_2 that are optimum in the Pareto sense. That is, we look for pairs of transmit strategies such that if any of the strategies is changed so that the transmission rate for the link $\mathrm{TX}_1 \rightarrow \mathrm{RX}_1$ increases, then the rate for the other link $\mathrm{TX}_2 \rightarrow \mathrm{RX}_2$ must necessarily decrease, and vice versa. If the interference is sufficiently weak, then it is optimal from a capacity point of view to treat it as additive noise [2]. Hence, Pareto-optimal transmit strategies directly correspond to Pareto-optimal signal-to-interference-and-noise ratio (SINR) pairs.

Our focus here is on setups where the transmitters have multiple antennas that can be used to perform transmit beamforming. The receivers have a single antenna only. This variant of the IFC is called a multiple-input single-output (MISO) IFC in the literature. For the MISO IFC, the problem of finding Pareto-optimal transmit strategies then specializes to finding the pairs of beamforming vectors that achieve Pareto-optimal rate pairs. This problem has received some previous attention, in our own work in particular. In [3], we provided a parameterization of the Pareto-optimal beamforming vectors, under the assumption that TX_1 and TX_2 have perfect channel state information (CSI). Essentially, we showed that for a beamforming vector to be Pareto optimal, it must lie in a subspace spanned by the forward channels to both receivers.

The main merit of the parameterization derived in [3] is that it provides an analytical tool that has helped to develop an intuitive understanding for what Pareto-optimal beamforming vectors look like. In particular, [4] presented game-theoretic interpretations in terms of "altruistic" and "selfish" strategies. Unfortunately, the parameterization derived in [3] is not particularly efficient when it comes to computing Pareto-optimal beamforming vectors in practice. The main reason for this is that it provides only *necessary* conditions that the beamforming vectors should *separately* fulfil to achieve Pareto-optimality. Hence, using the parameterization as it stands to generate *pairs* of beamforming vectors yields many rate points that are far from optimum, in addition to the desired Pareto-optimal points. We have also earlier presented extensions of the parameterization in [3] to the case where the transmitters only have statistical CSI [5], [6]. Whilst important for analytical studies, these parameterizations are further inefficient for the computation of the Pareto boundary, due to the even larger number of involved parameters.

It becomes apparent, that there is a need to find a way to directly compute pairs of Pareto-optimal beamforming vectors. This need motivated us to propose in [7] an optimization problem, that for each point on the Pareto boundary accepts as input the rate of one link and returns the rate of the other, along with the enabling beamforming vectors. Therein, we considered the scenario where the transmitters have statistical CSI, for which the corresponding (average) rate expressions are involved functions of the beamforming vectors. Because of this, it was hard to solve the proposed optimization problem directly. To overcome this difficulty, we proposed a two-step algorithm to derive a solution. In the first step, the optimization problem was solved separately for each beamforming vector

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and in the second step the solutions were combined.

In this paper, we show that the optimization problem, proposed in [7] to find pairs of Pareto-optimal beamforming vectors, can be solved jointly when the transmitters have perfect CSI. This is now possible because the corresponding (instantaneous) rates are simple functions of the beamforming vectors. Specifically, we exploit the fact that the rates are monotonously increasing with the SINR's and recast the original rate optimization problem as a sequence of SINR feasibility problems. These feasibility problems have nonconvex quadratic constraints, but nevertheless admit an equivalent reformulation as second-order cone programming (SOCP) problems [8]. The SOCP problems are convex [9] and they are solved very efficiently using standard off-the-shelf (namely, interior-point) algorithms [10]. The number of SOCP problems that needs to be solved grows only logarithmically with the desired accuracy of the solution. Taken together, we obtain a computationally very efficient method of calculating the Pareto boundary of the rate region for the MISO IFC, that it is achieved when the transmitters have perfect CSI.

II. Preliminaries

A. System Model

We assume that both transmitters employ n antennas and that their transmissions consist of scalar coding followed by beamforming. For the setup under consideration, where the transmitters know the forward channels perfectly, beamforming is optimal [11]. We further assume that all the propagation channels are frequency-flat. Hence, the matched-filtered symbol-sampled complex baseband data received by RX_1[1] is modeled as

$$y_1 = \boldsymbol{h}_{11}^H \boldsymbol{w}_1 s_1 + \boldsymbol{h}_{21}^H \boldsymbol{w}_2 s_2 + e_1. \tag{1}$$

In (1), $\boldsymbol{h}_{11} \in \mathbb{C}^n$ denotes the (conjugated) channel vector of the direct link $\mathrm{TX}_1 \rightarrow \ \mathrm{RX}_1$, whereas $\boldsymbol{w}_1 \in \mathbb{C}^n$ and $s_1 \sim \mathcal{CN}(0,1)$ are the employed beamforming vector and the transmitted symbol of TX_1. Respectively, $\boldsymbol{h}_{21} \in \mathbb{C}^n$ is the (conjugated) channel vector of the crosstalk link $\mathrm{TX}_2 \rightarrow \ \mathrm{RX}_1$, whereas $\boldsymbol{w}_2 \in \mathbb{C}^n$ and $s_2 \sim \mathcal{CN}(0,1)$ are the beamforming vector and the transmitted symbol of TX_2. Finally, $e_1 \sim \mathcal{CN}(0,\sigma^2)$ models the receiver noise.

The transmission power of each TX is bounded due to regulatory and hardware constraints. Without loss of generality we set this bound to 1. Hence, the set of feasible beamforming vectors for each TX is

$$\mathcal{W} \triangleq \{\boldsymbol{w} \in \mathbb{C}^n \mid \|\boldsymbol{w}\|^2 \leq 1\}. \tag{2}$$

Note that the set $\mathcal{W}$ is convex. In what follows, a specific choice for $\boldsymbol{w}_1 \in \mathcal{W}$ is denoted as a *transmit strategy* of TX_1.

B. Instantaneous Rate Region

When TX_1 has perfect CSI and RX_1 treats the interference as noise, the achievable instantaneous rate (in bits/channel use) for the link $\mathrm{TX}_1 \rightarrow \ \mathrm{RX}_1$ is [2]

[1] In this section, the expressions are introduced with respect to link 1. The respective expressions for link 2 can be found by interchanging the indexes.

$$R_1(\boldsymbol{w}_1, \boldsymbol{w}_2) = \log_2\left(1 + \frac{|\boldsymbol{h}_{11}^H \boldsymbol{w}_1|^2}{|\boldsymbol{h}_{21}^H \boldsymbol{w}_2|^2 + \sigma^2}\right). \tag{3}$$

It is evident that the instantaneous rate is monotonously increasing with the SINR, which is defined by the ratio within the right-hand-side term of (3). Furthermore, the rate depends on the choice of both beamforming vectors. We denote as $p_{11}(\boldsymbol{w}_1)$ the useful signal power received in the direct link $\mathrm{TX}_1 \rightarrow \ \mathrm{RX}_1$ and as $p_{21}(\boldsymbol{w}_2)$ the interference power received in the crosstalk link $\mathrm{TX}_2 \rightarrow \ \mathrm{RX}_1$, i.e.,

$$p_{11}(\boldsymbol{w}_1) \triangleq |\boldsymbol{h}_{11}^H \boldsymbol{w}_1|^2 \quad \text{and} \quad p_{21}(\boldsymbol{w}_2) \triangleq |\boldsymbol{h}_{21}^H \boldsymbol{w}_2|^2. \tag{4}$$

It can be easily seen and understood that $R_1(\boldsymbol{w}_1, \boldsymbol{w}_2)$ is monotonously increasing with $p_{11}(\boldsymbol{w}_1)$ for fixed $p_{21}(\boldsymbol{w}_2)$ and monotonously decreasing with $p_{21}(\boldsymbol{w}_2)$ for fixed $p_{11}(\boldsymbol{w}_1)$.

A conflict situation is associated with the choice of beamvectors. A beamvector $\boldsymbol{w}_1$ which increases the useful signal power $p_{11}(\boldsymbol{w}_1)$ received by RX_1, might also increase the interference power $p_{12}(\boldsymbol{w}_1)$ experienced at RX_2. The question that naturally arises is which rate pairs $(R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1))$ are achievable for given channel vectors. The union of the rate pairs that can be obtained by all feasible beamvector pairs defines the achievable rate region

$$\mathcal{R} = \bigcup_{(\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2} (R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1)). \tag{5}$$

Note, that for fixed channel vectors the region $\mathcal{R}$ is compact, since $\mathcal{W}$ is compact and the mapping of $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ to $(R_1(\boldsymbol{w}_1, \boldsymbol{w}_2), R_2(\boldsymbol{w}_2, \boldsymbol{w}_1))$ is continuous. However, the region $\mathcal{R}$ is not necessarily convex.

C. Pareto Boundary

The rate points which are optimal in the strong Pareto sense are the ones for which it is not possible to improve the rate of one link without decreasing the rate of the other link. More precisely:

Definition 1. A rate pair $(R_1^\star, R_2^\star) \in \mathcal{R}$ is Pareto-optimal if there is no other pair $(R_1, R_2) \in \mathcal{R}$ with $(R_1, R_2) \geq (R_1^\star, R_2^\star)$ and $(R_1, R_2) \neq (R_1^\star, R_2^\star)$. (The inequality is componentwise.)

The union of all Pareto-optimal points defines the so-called Pareto boundary of the rate region $\mathcal{R}$. We denote as $\underline{R}_1$ and $\overline{R}_1$ the minimum and maximum, respectively, Pareto-optimal rates of link $\mathrm{TX}_1 \rightarrow \ \mathrm{RX}_1$. The Pareto boundary consists of all Pareto-optimal points on the outer (upper-right) boundary of $\mathcal{R}$, between $(\underline{R}_1, \overline{R}_2)$ and $(\overline{R}_1, \underline{R}_2)$; see Fig. 1.

As evidenced by (3), the maximum rate $\overline{R}_1$ is achieved when TX_1 operates "selfishly" to maximize the useful signal power $p_{11}(\boldsymbol{w}_1)$ and TX_2 operates "altruistically" to null the interference power $p_{21}(\boldsymbol{w}_2)$. The selfish operation of TX_1 corresponds to the maximum-ratio (MR) transmit strategy [4]

$$\boldsymbol{w}_1^{\mathrm{MR}} = \arg\max_{w_1 \in \mathcal{W}} \ p_{11}(\boldsymbol{w}_1) = \frac{\boldsymbol{h}_{11}}{\|\boldsymbol{h}_{11}\|}. \tag{6}$$

Note that the computation of the MR beamforming vector in (6) requires knowledge only of the direct channel vector

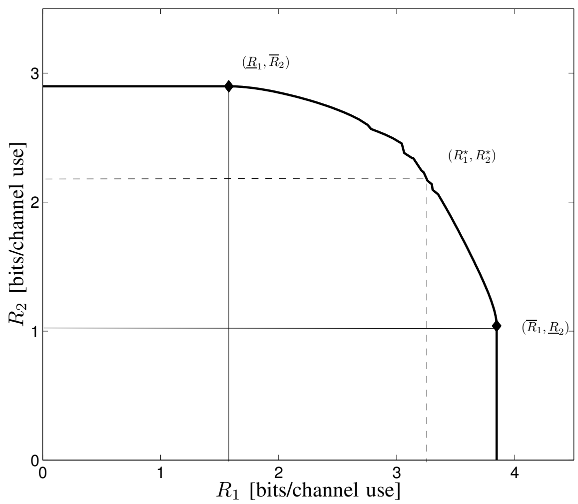

Fig. 1. Example of an achievable rate region for the two-user MISO IFC with perfect CSI

$\boldsymbol{h}_{11}$. That is, the MR strategy does not take into account the interference that it causes to the other communication link. Contrarily, the altruistic operation maximizes the useful signal power under the hard constraint of causing no interference. The so-called zero-forcing (ZF) strategy of TX_1 is [4]

$$\boldsymbol{w}_1^{\mathrm{ZF}} = \arg \max_{\substack{w_1 \in \mathcal{W} \\ p_{12}(\boldsymbol{w}_1)=0}} p_{11}(\boldsymbol{w}_1) = \frac{\boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{12}}{\left\| \boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \boldsymbol{h}_{11} \right\|}, \tag{7}$$

where $\boldsymbol{\Pi}_{\boldsymbol{h}_{12}}^{\perp} \triangleq \boldsymbol{I} - \boldsymbol{h}_{12}(\boldsymbol{h}_{12}^H \boldsymbol{h}_{12})^{-1} \boldsymbol{h}_{12}^H$ is the orthogonal projection onto the orthogonal complement of $\boldsymbol{h}_{12}$ and $\boldsymbol{I}$ is the identity matrix. In [3], we showed that all the Pareto-optimal transmit strategies can be represented as linear combinations of the MR and the ZF ones.

The maximum and minimum Pareto-optimal rates on link $\mathrm{TX}_1 \rightarrow \mathrm{RX}_1$ are explicitly written as functions of the MR and ZF strategies of TX_1 and TX_2 as follows

$$\overline{R}_1 \triangleq R_1(\boldsymbol{w}_1^{\mathrm{MR}}, \boldsymbol{w}_2^{\mathrm{ZF}}) \quad \text{and} \quad \underline{R}_1 \triangleq R_1(\boldsymbol{w}_1^{\mathrm{ZF}}, \boldsymbol{w}_2^{\mathrm{MR}}). \tag{8}$$

III. Computation of the Pareto boundary

As discussed in Sec. II-C, we accurately know the endpoints of the Pareto boundary and the transmit strategies that enable them. In this section, we propose an efficient method to calculate the Pareto-optimal pair of beamforming vectors $(\boldsymbol{w}_1^\star, \boldsymbol{w}_2^\star)$ that corresponds to an arbitrary operating point $(R_1^\star, R_2^\star)$ on the Pareto boundary of the achievable rate region. We do so by building on our previous work [7]. Therein, we observed that every Pareto-optimal point is uniquely defined when the rate of one communication link is known. The other rate is then the maximum one that can be simultaneously achieved (see the dashed lines in Fig. 1). This motivated us to propose the following optimization problem, which accepts as input the coordinate $R_1^\star$ of the sought Pareto-optimal rate pair.

$$\max_{(\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2} \quad R_2(\boldsymbol{w}_2, \boldsymbol{w}_1) \tag{9}$$

$$\text{s.t.} \qquad R_1(\boldsymbol{w}_1, \boldsymbol{w}_2) = R_1^\star. \tag{10}$$

The optimal value of the problem (9)–(10) is the other coordinate $R_2^\star$. The optimal solution of (9)–(10) is the pair $(\boldsymbol{w}_1^\star, \boldsymbol{w}_2^\star)$ of transmit strategies that have to be employed in order to achieve $(R_1^\star, R_2^\star)$. It is apparent that the optimization is always feasible and that the entire Pareto boundary can be calculated when $R_1^\star$ is chosen in the range $[\underline{R}_1, \overline{R}_1]$.

In [7], we considered the case where the transmitters only have statistical CSI, for which the achievable rates are computed by averaging over the channel realizations. The resulting rate expressions are involved functions of the beamforming vectors, comprising exponential integrals with quadratic terms in their limits. For this reason, it was difficult in [7] to solve the problem (9)–(10) directly and we had to break the optimization in two parts, each involving a different beamforming vector. Herein, we assume that the transmitters have perfect CSI. Hence, the corresponding (instantaneous) rate expressions (3) are relatively simple functions of the beamforming vectors. As shown in the sequel, this allows us to perform a number of reformulations to the original problem (9)–(10) in order to eventually cast it in convex form, that enables us to very efficiently yield the optimal pair of beamforming vectors.

Towards this end, the first step is to replace the rate expression (3) in (9)–(10) and yield

$$\max_{(\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2} \quad \log_2 \left(1 + \frac{|\boldsymbol{h}_{22}^H \boldsymbol{w}_2|^2}{|\boldsymbol{h}_{12}^H \boldsymbol{w}_1|^2 + \sigma^2} \right) \tag{11}$$

$$\text{s.t.} \qquad \log_2 \left(1 + \frac{|\boldsymbol{h}_{11}^H \boldsymbol{w}_1|^2}{|\boldsymbol{h}_{21}^H \boldsymbol{w}_2|^2 + \sigma^2} \right) = R_1^\star. \tag{12}$$

Note that the equality constraint (12) can be equivalently relaxed to a lower-bounded inequality. It is rather straightforward to show that due to the concept of Pareto optimality, the bound will be tight at the optimum. Furthermore, since the instantaneous rates are monotonously increasing with the SINR's, the rate optimization (11)–(12) is equivalent to the SINR optimization

$$\max_{(\boldsymbol{w}_1, \boldsymbol{w}_2) \in \mathcal{W}^2} \quad \frac{|\boldsymbol{h}_{22}^H \boldsymbol{w}_2|^2}{|\boldsymbol{h}_{12}^H \boldsymbol{w}_1|^2 + \sigma^2} \tag{13}$$

$$\text{s.t.} \qquad \frac{|\boldsymbol{h}_{11}^H \boldsymbol{w}_1|^2}{|\boldsymbol{h}_{21}^H \boldsymbol{w}_2|^2 + \sigma^2} \geq c_1, \tag{14}$$

where $c_1 \triangleq 2^{R_1^\star} - 1$ is the SINR level which is required to achieve a rate equal to $R_1^\star$.

The objective function (13) is a fraction of quadratic terms; hence, it is neither convex nor concave. Since, we are dealing with a maximization problem, we would like to have a concave or a linear objective function. This is easily achieved by performing the standard trick of introducing a nonnegative real auxiliary variable t [9]. The auxiliary variable can now serve as the new objective function, provided that it will also bound from below the original objective function. Implementing these changes, we equivalently reformulate (13)–(14) as

$$\max_{(\boldsymbol{w}_1,\boldsymbol{w}_2)\in\mathcal{W}^2,\ t\geq 0} \quad t \tag{15}$$

$$\text{s.t.} \quad \frac{|\boldsymbol{h}_{11}^H\boldsymbol{w}_1|^2}{|\boldsymbol{h}_{21}^H\boldsymbol{w}_2|^2+\sigma^2} \geq c_1, \tag{16a}$$

$$\frac{|\boldsymbol{h}_{22}^H\boldsymbol{w}_2|^2}{|\boldsymbol{h}_{12}^H\boldsymbol{w}_1|^2+\sigma^2} \geq t. \tag{16b}$$

Since the denominators of the ratios in the left-hand-side terms of the inequalities (16) are positive, we can multiply with them and equivalently obtain

$$\max_{(\boldsymbol{w}_1,\boldsymbol{w}_2)\in\mathcal{W}^2,\ t\geq 0} \quad t \tag{17}$$

$$\text{s.t.} \quad |\boldsymbol{h}_{11}^H\boldsymbol{w}_1|^2 \geq c_1|\boldsymbol{h}_{21}^H\boldsymbol{w}_2|^2 + c_1\sigma^2, \tag{18a}$$

$$|\boldsymbol{h}_{22}^H\boldsymbol{w}_2|^2 \geq t|\boldsymbol{h}_{12}^H\boldsymbol{w}_1|^2 + t\sigma^2. \tag{18b}$$

Now that the objective function (17) is linear, we may focus on the feasible set of (17)–(18). If this was convex, then the optimization problem would be convex and its global optimal solution could be efficiently found. We observe that the constraint (18a) defines a quadratic inequality, which is nonconvex due to the left-hand-side term. As we will later show, this poses no problem since it is possible to recast the quadratic inequality in (18a) as a second-order cone (SOC) constraint, which is convex. However, this is not directly possible for the inequality (18b), because of the fact that the quadratic term in its right-hand-side is multiplied with the variable t.

In order to overcome this difficulty, we propose solving (17)–(18) as a sequence of optimization problems for given values of t. We basically need to perform a line search over the values of t to determine which is the maximum feasible value. We can employ the bisection algorithm, because the nonnegative variable t is also bounded from above, since it corresponds to the achievable SINR of the link 2. As initial lower and upper bounds we may set the values that correspond to the endpoints of the Pareto boundary, i.e., $L := 2^{\underline{R}_2} - 1$ and $U := 2^{\overline{R}_2} - 1$, respectively. In each iteration of the bisection algorithm, t takes the value $(U-L)/2$. Since t is not a variable anymore, the formulation (17)–(18) denotes a feasibility problem with respect to the variables $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$. The feasibility problems are optimization problems with constraints but without an objective function [9]. They return a binary solution to answer the question whether the feasible set, defined by the constraints, is empty or not. If a solution to the feasibility problem exists, then the lower bound is updated with the feasible value of t, i.e., $L := t$. Otherwise, the upper bound is updated with the infeasible value of t, i.e., $U := t$.

Let us now see what is the actual form of the feasibility problem that needs to be solved in each iteration of the bisection algorithm. For a given value of t, (17)–(18) reads

$$\text{find} \quad (\boldsymbol{w}_1,\boldsymbol{w}_2)\in\mathcal{W}^2 \tag{19}$$

$$\text{s.t.} \quad |\boldsymbol{h}_{11}^H\boldsymbol{w}_1|^2 \geq c_1|\boldsymbol{h}_{21}^H\boldsymbol{w}_2|^2 + c_1\sigma^2, \tag{20a}$$

$$|\boldsymbol{h}_{22}^H\boldsymbol{w}_2|^2 \geq t|\boldsymbol{h}_{12}^H\boldsymbol{w}_1|^2 + t\sigma^2. \tag{20b}$$

Now, both constraints in (20) are nonconvex quadratic inequalities of the same form. As shown in [8], for the multiuser downlink beamforming problem with SINR constraints, the inequalities of the form (20) admit a reformulation as SOC constraints. This is possible because the beamforming vectors are only involved in homogeneous quadratic terms in problem (19)–(20). Hence, there is inherent ambiguity on the phases of the optimal beamforming vectors. The trick is to constrain the phases of the beamforming vectors to the values that yield real-valued the signals that are received on the direct links. Then, it is possible to take the square root on the inequalities (20) and discard the absolute value operator from the left-hand-side terms. With this change, the feasibility problem (19)–(20) is now rewritten as

$$\text{find} \quad (\boldsymbol{w}_1,\boldsymbol{w}_2) \tag{21}$$

$$\text{s.t.} \quad \boldsymbol{h}_{11}^H\boldsymbol{w}_1 \geq \sqrt{c_1|\boldsymbol{h}_{21}^H\boldsymbol{w}_2|^2 + c_1\sigma^2}, \tag{22a}$$

$$\boldsymbol{h}_{22}^H\boldsymbol{w}_2 \geq \sqrt{t|\boldsymbol{h}_{12}^H\boldsymbol{w}_1|^2 + t\sigma^2}, \tag{22b}$$

$$\operatorname{Im}\left\{\boldsymbol{h}_{11}^H\boldsymbol{w}_1\right\} = 0, \tag{22c}$$

$$\operatorname{Im}\left\{\boldsymbol{h}_{22}^H\boldsymbol{w}_2\right\} = 0, \tag{22d}$$

$$\|\boldsymbol{w}_1\|^2 \leq 1, \tag{22e}$$

$$\|\boldsymbol{w}_2\|^2 \leq 1. \tag{22f}$$

The inequalities (22a) and (22b) define SOC constraints. The equalities (22c) and (22d) which constrain the phases of the beamforming vectors are linear. The constraints (22e) and (22f) explicitly define the power constraints, that have so far been implicitly denoted as $(\boldsymbol{w}_1,\boldsymbol{w}_2)\in\mathcal{W}^2$. They are quadratic norm constraints, which can be seen as special cases of SOC constraints. Thus, taken together problem (21)–(22) is a convex (specifically, SOCP) feasibility problem.

A. Computational Complexity

The SOCP problem (21)–(22) is a small-scale problem involving 2 n-dimensional variables, 4 SOC constraints and 2 linear constraints. It is solved very efficiently, requiring at most $O(n^{3.5})$ arithmetic operations, by modern interior-point algorithms [10]. There exist freely-distributed software packages, e.g. [12], that use interior-point algorithms to yield the optimal solution of such problems.

The number of SOCP problems that needs to be solved to determine the Pareto-optimal pair of beamforming vectors that corresponds to a point on the Pareto boundary is very small. This is because the bisection algorithm converges exponentially fast to the desired accuracy, since the search interval is halved in every iteration. Note that the sought value corresponds to an SINR value; hence, the accuracy of the solution need not be very high. Typically, a handful of iterations suffices to find a solution that is good-enough from engineering perspective.

IV. Conclusion

We proposed an efficient method to yield the Pareto boundary of the achievable rate region for the two-user MISO IFC when the transmitters have perfect CSI. The merit of the proposed approach, as opposed to previous ones, is that it directly computes pairs of Pareto-optimal beamforming vectors using optimization techniques.

References

[1] A. B. Carleial, "Interference channels," *IEEE Trans. Inform. Theory*, vol. 24, no. 1, pp. 60-70, Jan. 1978.

[2] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge Univ. Press, 2005.

[3] E. A. Jorswieck, E. G. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5292–5296, Oct. 2008.

[4] E. G. Larsson and E. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Select. Areas Commun.*, vol. 26, no. 7, pp. 1059-1069, Sep. 2008.

[5] J. Lindblom, E. Karipidis, and E. G. Larsson, "Selfishness and altruism on the MISO interference channel: the case of partial transmitter CSI," *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 667–669, Sep. 2009.

[6] J. Lindblom, E. G. Larsson, and E. A. Jorswieck, "Parametrization of the MISO IFC rate region: The case of partial channel state information," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 500–504, Feb. 2010.

[7] E. Karipidis, A. Gründinger, J. Lindblom, and E. G. Larsson, "Pareto-optimal beamforming for the MISO interference channel with partial CSI," in *Proc. 3rd IEEE Int. Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, Aruba, Dec. 2009, pp. 5–8.

[8] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. Boca Raton, FL: CRC, 2001, ch. 18.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, 2004.

[10] Y. Ye, *Interior Point Algorithms: Theory and Analysis*, Wiley, 1997.

[11] X. Shang and B. Chen, "Achievable rate region for downlink beamforming in the presence of interference," in *Proc. IEEE Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2007, pp. 1684–1688.

[12] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," version 1.21, http://cvxr.com/cvx, April 2010.