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Optimizing ARQ Strategies in Relay Networks

Aimal Khan, Volker Kuehn
 University of Rostock
 Institute of Communications Engineering
 {aimal.khan,volker.kuehn}@uni-rostock.de

Abstract—In this paper the novel issue of retransmissions allocation in ARQ relay networks is introduced. In ARQ relay networks, the source and relays repeat a signal in response to a request from the destination. The source and relays can repeat once or multiple times with a constant power. Contrary to the protocols where a node is constrained to transmit once, we allow each node to retransmit multiple times. The goal is to allocate the optimal (minimum) number of retransmissions resulting in successful decoding at the destination. We devise an optimization method which is optimal. Moreover, the performance of the protocol with multiple retransmissions is compared to that of protocols with one retransmission per node or all the retransmissions from a single node. The results reveal that multiple retransmission protocols deliver superior performance over the counterparts.

I. INTRODUCTION

In relay networks different forwarding techniques at the relay such as amplify-and-forward (AF), decode-and-forward (DF) [1], [2], estimate-and-forward (EF) [3], and decode-and-forward with soft information relaying [4] have been investigated. AF and EF suffer from the problem of noise amplification, on the other hand DF suffers from error propagation. In order to mitigate these problems the adaptive relaying idea was presented in [5]. In this work, the relay switched from AF to DF and from DF to AF, depending on the decoding status at the relay. This relaying method takes the advantages of both AF and DF and minimizes their disadvantages.

It is obvious that a relay should cooperate only if the destination needs its cooperation, otherwise resources will be used in vain. In [1], a protocol named incremental relaying is introduced in which the relay cooperates if the destination requests its cooperation, the concept of incremental relaying can be viewed as an extension of Automatic Repeat reQuest (ARQ) to the relay context. Efficient resource allocation in conjunction with ARQ is vital for relay networks. In most of the contributions related to ARQ in relay networks with constant transmit power, the source and relays are constrained to repeat only once [6], [7], [8]. However, if this constraint is relaxed, the performance of the relay network can be meliorated. One reason which justify the imposition of this constraint is the fact that all nodes can have equal battery life time. Nevertheless, this is not a problem for the nodes with consistent power supply. Therefore, it is worthwhile to consider ARQ relay networks where the nodes are allowed to retransmit multiple times and to allocate the optimal number of retransmissions to each node.

In this paper the number of retransmission allocation has been treated. It can be also viewed as relay selection, because the relays with zero number of retransmissions do not participate. In Opportunistic relay selection [9], a relay with the “best” end-to-end performance (w.r.t. the first retransmission) among the candidates relays is selected for forwarding. However, a relay which may have the best E2E performance during the first retransmission, may not remain the “best” for the second retransmission even if the channel coefficients remain the same. Therefore, opportunistic relay selection is not always the optimal one when multiple retransmissions are allowed. We devise an optimal solution, which is more computationally efficient as compared to the exhaustive search solution and it delivers identical performance. It should be noted that we believe that the retransmissions selection has been treated in this paper for the first time in ARQ relay network generally and adaptive ARQ relay networks [10] especially.

II. SYSTEM MODEL

The system model in Fig. 1 is restricted to a dual-hop network. The source S encodes its K bits data sequence \mathbf{u} using a code \mathcal{C}_s with rate R_c . BPSK modulation results in the sequence $\mathbf{x}_s \in \{+1, -1\}^{N_c}$ of length N_c . The channel from the source S to the l th relay R_l is parameterized by $h'_{r_l s}$, $a_{r_l s}$, and σ_n^2 , where $1 \leq l \leq L$. Here, $h'_{r_l s}$ is assumed to be i.i.d. Rayleigh distributed block-fading coefficient with $E\{(h'_{r_l s})^2\} = 1$. For each link, the block-fading coefficient is assumed to remain constant during the whole ARQ process. Moreover, $a_{r_l s} = \sqrt{(d_{r_l s})^{-\alpha}}$ is the path loss factor for source-relay distance $d_{r_l s}$ with path loss exponent α . Each element of the noise vector $\mathbf{n}_{r_l}[i]$ is from an AWGN process with zero mean and the variance σ_n^2 per dimension. For N_s retransmissions from the source, the received real baseband signal at the l th relay after maximum ratio combining is given by

$$\begin{aligned} \mathbf{y}_{r_l} &= \sum_{i=1}^{N_s} h_{r_l s} \cdot \mathbf{x}_s + \mathbf{n}_{r_l}[i] \\ &= N_s \cdot h_{r_l s} \cdot \mathbf{x}_s + \mathbf{n}_{r_l}, \end{aligned} \quad (1)$$

with $h_{r_l s} = h'_{r_l s} \cdot a_{r_l s}$. Moreover, each element of the noise vector \mathbf{n}_{r_l} is AWGN with zero mean and variance $\sigma_n^2 = N_s \cdot \sigma_n^2$ per dimension. Similarly, the equivalent signal at the destination after maximum ratio combining becomes

$$\mathbf{y}_{ds} = N_s \cdot h_{ds} \cdot \mathbf{x}_s + \mathbf{n}_{ds}, \quad (3)$$

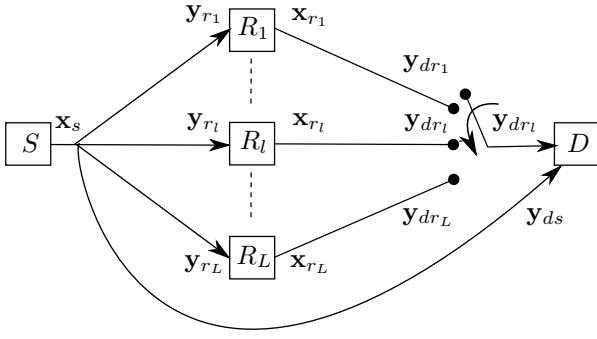


Fig. 1. System model showing one source S , multiple relays R_l and one destination D with respective channels and signals.

with $h_{ds} = h'_{ds} \cdot \sqrt{(d_{ds})^{-\alpha}}$. Furthermore, h'_{ds} and \mathbf{n}_{ds} have the same statistics as h'_{r_1s} and \mathbf{n}_{r_1} respectively, d_{ds} represents the distance between the source and the destination.

The l th relay if selected processes the received signal \mathbf{y}_{r_l} and transmits the signal \mathbf{x}_{r_l} . We consider different adaptive relaying schemes which process the received signal in different ways according to the reliability of the received signal. We name these adaptive schemes Amplify-or-Decode & Forward (ADF) and Estimate-or-Decode & Forward (EDF). In the case of ADF, the relay forwards the reencoded signal if it is able to successfully decode the received signal. In the case of a decoding failure, it applies amplify-and-forward. Thus, the signal transmitted from the l th relay is

$$\mathbf{x}_{r_l} = \begin{cases} \mathbf{x}_s & \text{for } N_s \cdot \gamma_{r_1s} \geq \gamma_{th} \\ \frac{\mathbf{y}_{r_l}}{\sqrt{N_s \cdot |h_{r_1s}|^2 + \sigma_n^2}} & \text{for } N_s \cdot \gamma_{r_1s} < \gamma_{th} \end{cases}, \quad (4)$$

where, γ_{r_1s} is the received SNR of the source-relay link for a single transmission and γ_{th} is the minimum SNR required to ensure successful decoding. Therefore, the condition $N_s \cdot \gamma_{r_1s} \geq \gamma_{th}$ represents the successful decoding at the l th relay. Strategy EDF only differs from ADF in that the relay forwards the expectation (estimate-and-forward [3]) of a received bit if the decoder fails. Hence, the signal transmitted from the l th relay can be written as,

$$\mathbf{x}_{r_l} = \begin{cases} \mathbf{x}_s & \text{for } N_s \cdot \gamma_{r_1s} \geq \gamma_{th} \\ C \cdot \tanh\left(\frac{N_s \cdot h_{r_1s} \cdot \mathbf{y}_{r_l}}{\sigma_n^2}\right) & \text{for } N_s \cdot \gamma_{r_1s} < \gamma_{th} \end{cases}, \quad (5)$$

where, C is a normalization constant such that the average transmitted power is 1.

At the destination, the equivalent received baseband signal after maximum ratio combining of N_{r_l} retransmissions from the l th relay can be written as

$$\mathbf{y}_{dr_l} = N_{r_l} \cdot h_{dr_l} \cdot \mathbf{x}_{r_l} + \mathbf{n}_{dr_l}, \quad (6)$$

having identical AWGN and fading channel statistics as in (3) but the path loss depends on the distance from the l th relay to the destination. The destination performs maximum ratio combining over all the received signals from the source and the relays to decode the source's transmitted signal. Furthermore, an orthogonal channel allocation is considered for all retransmissions.

III. END-TO-END PERFORMANCE MEASURES FOR DIFFERENT FORWARDING SCHEMES

In this section we derive the end-to-end (E2E) performance for a relay network presented in Sec. II.

A. Performance of Amplify-or-Decode & Forward

In this subsection, we first consider the situation when the source and a single relay only retransmits once ($N_s = 1$ and $N_{r_l} = 1$). Subsequently, the analysis will be extended to multiple retransmissions and multiple relays. From (1) and the lower part (amplify-and-forward) of (4), the received signal at the destination (6) can be written as,

$$\begin{aligned} \mathbf{y}_{dr_l} &= h_{dr_l} \cdot \frac{h_{r_1s} \cdot \mathbf{x}_s + \mathbf{n}_{r_1s}}{\sqrt{|h_{r_1s}|^2 + \sigma_n^2}} + \mathbf{n}_{dr_l} \\ &= \underbrace{h_{dr_l} \cdot h_{r_1s} \cdot \mathbf{x}_s}_{\text{signal}} + \underbrace{\frac{h_{dr_l} \cdot \mathbf{n}_{r_1s}}{\sqrt{|h_{r_1s}|^2 + \sigma_n^2}} + \mathbf{n}_{dr_l}}_{\text{entire noise}} \end{aligned} \quad (7)$$

In (7), the noise part (entire noise) is zero mean AWGN with variance $\frac{|h_{dr_l}|^2 \cdot \sigma_n^2}{|h_{r_1s}|^2 + \sigma_n^2} + \sigma_n^2$. Therefore, the total SNR delivered via the l th relay for amplify-and-forward is given by

$$\begin{aligned} \gamma_l^{AF} &= \frac{\frac{|h_{r_1s}|^2}{\sigma_n^2} \cdot \frac{|h_{dr_l}|^2}{\sigma_n^2}}{\frac{|h_{r_1s}|^2}{\sigma_n^2} + \frac{|h_{dr_l}|^2}{\sigma_n^2} + \frac{\sigma_n^2}{\sigma_n^2}} \\ &= \frac{\gamma_{r_1s} \cdot \gamma_{dr_l}}{\gamma_{r_1s} + \gamma_{dr_l} + 1}. \end{aligned}$$

Now, we extend the above equation to the case when the source retransmits N_s times and the relay retransmits N_{r_l} times. For this case the received SNR at the destination from the l th relay is given by

$$\gamma_l^{AF} = \frac{N_s \cdot \gamma_{r_1s} \cdot N_{r_l} \cdot \gamma_{dr_l}}{N_s \cdot \gamma_{r_1s} + N_{r_l} \cdot \gamma_{dr_l} + 1}. \quad (8)$$

As (8) is only valid for amplify-and-forward, for ADF according to (4) the received SNR at the destination can be written as

$$\gamma_{l, N_{r_l}}^{ADF} = \begin{cases} N_{r_l} \cdot \gamma_{dr_l} & \text{for } N_s \cdot \gamma_{r_1s} \geq \gamma_{th} \\ \gamma_l^{AF} & \text{for } N_s \cdot \gamma_{r_1s} < \gamma_{th} \end{cases}. \quad (9)$$

Finally, the entire SNR from the source and the relays after maximum ratio combining can be given as

$$\gamma^{ADF} = N_s \cdot \gamma_{ds} + \sum_{l=1}^L \gamma_{l, N_{r_l}}^{ADF}, \quad (10)$$

where, γ_{ds} is the signal to noise ratio of the signal at the destination (\mathbf{y}_{ds}) via the direct link with the first transmission.

B. Performance of Estimate-or-Decode & Forward

One method to find the E2E performance of EDF is to calculate the generalized SNR [3] at the destination. Another possible method is to calculate the end-to-end (E2E) mutual information per symbol $I(X_s; Y_{dr_l})$ for each relay. In [11], $I(X_s; Y_{dr_l})$ has already been calculated using serial information concatenation for soft information forwarding with an

ideal code [12]. For convenience, the procedure of serial information concatenation is applied to calculate $I(X_s; Y_{dr_l})$ for estimate-or-decode & forward with multiple retransmissions from each node. In this subsection the procedure of serial information concatenation is explained.

The conditional probability density function for Y_{r_l} at the input of the relay when X_s is transmitted is given by

$$p_r(Y_{r_l}|X_s) = p_N(Y_{r_l} - N_s \cdot h_{r_{s_l}} \cdot X_s, \sigma_n^2), \quad (11)$$

where,

$$p_N(\xi, \sigma^2) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \exp(-\xi^2/2\sigma^2).$$

First we consider the case in (5) when $N_s \cdot \gamma_{r_{l_s}} < \gamma_{th}$ and $\mathbf{x}_{r_l} = C \cdot \tanh\left(\frac{N_s \cdot h_{r_{l_s}} \cdot \mathbf{y}_{r_l}}{\sigma_n^2}\right)$ holds. For this scenario, $p_x(X_{r_l}|X_s)$ is obtained from $p_r(Y_{r_l}|X_s)$ simply by random variable transformation [13].

Similarly, the conditional probability density function for Y_{dr_l} at the output of hop 2 when X_{r_l} is transmitted is given by $p_d(Y_{dr_l}|X_{r_l}) = p_N(Y_{dr_l} - N_{r_l} \cdot h_{dr_l} \cdot X_{r_l}, \sigma_n^2)$.

For this case, the PDF $p(Y_{dr_l}|X_s)$ is calculated by integrating over all the possible values of X_{r_l} which yields Y_{dr_l} for given X_s , i.e.

$$p(Y_{dr_l}|X_s) = \int p_x(X_{r_l}|X_s) \cdot p_d(Y_{dr_l}|X_{r_l}) dX_{r_l}. \quad (12)$$

Consequently, the mutual information for the serially concatenated link for this case can be calculated using the standard mutual information formula,

$$\begin{aligned} I^{ser}(X_s; Y_{dr_l}) \\ = \sum_{X_s} \int_0^\infty p(Y_{dr_l}|X_s) \cdot \log_2 \left(\frac{p(Y_{dr_l}|X_s)}{\frac{1}{2}(\sum_{X_s} p(Y_{dr_l}|X_s))} \right) dY_{dr_l}. \end{aligned} \quad (13)$$

Finally, the entire end-to-end mutual information for estimate-or-decode & forward (EDF) is

$$I_{l, N_{r_l}}^{EDF}(X_s, Y_{dr_l}) = \begin{cases} C_A \left(\frac{N_s \cdot h_{dr_l}^2}{\sigma_n^2} \right) & \text{for } N_s \cdot \gamma_{r_{l_s}} \geq \gamma_{th} \\ I^{ser}(X_s; Y_{dr_l}) & \text{for } N_s \cdot \gamma_{r_{l_s}} < \gamma_{th} \end{cases}. \quad (14)$$

In (14), $C_A(\gamma)$ is the capacity of a Gaussian channel with binary input and signal-to-noise ratio equal to γ . For $N_s \cdot \gamma_{r_{l_s}} \geq \gamma_{th}$, the relay successfully decodes and acts similar to the source. Thus, $I_{l, N_{r_l}}^{EDF}(X_s, Y_{dr_l})$ is equal to the capacity of hop 2.

Finally, the mutual information from the source and via all the relays to the destination is lower bounded by

$$\begin{aligned} I(X_s; Y_{ds}, Y_{dr_1}, Y_{dr_2}, \dots, Y_{dr_L}) &\geq I_{low}(X_s, \mathbf{Y}_{ds}) \\ &= I_{low}^{par} \left(I(X_s, Y_{ds}), I_{1, N_{r_1}}^{EDF}(X_s, Y_{dr_1}), \dots, I_{L, N_{r_L}}^{EDF}(X_s, Y_{dr_L}) \right), \end{aligned} \quad (15)$$

where, $I_{low}^{par}(\dots)$ is the parallel information combining function [12].

IV. ARQ PROTOCOLS & RETRANSMISSIONS ALLOCATION

When the source transmits a signal and a decoding failure occurs at the destination then the source as well as the relays can repeat the signal in response to the ARQ. In this section multiple ARQ protocols are presented. First, a protocol ‘‘Source Multiple (transmissions) and Each Relay with Multiple transmissions (SMERM)’’ is presented. In this protocol source as well as each relay are allowed to retransmit the signal (repetition coding) multiple times. Moreover, ‘‘Source Once and Each Relay with Multiple transmissions (SOERM)’’, differs from SMERM only in that the source transmits only once. Next, in ‘‘Source Once and Each Relay with One transmission (SOERO)’’, the source as well as each relay are allowed to retransmits only once. Finally, in ‘‘Source Once and a Selected Relay with Multiple transmissions (SOSRM)’’, the source transmits once while only a single selected relay can retransmit multiple times. SOERO corresponds to the protocol provided in [6] and [8], while, SOSRM corresponds to the opportunistic relay selection of [9].

First we consider the retransmission allocation for amplify-or-decode & forward. We assume a central node with perfect channel knowledge of all the links. This assumption is too optimistic in practice, however, the goal is to determine an upper bound for the performance of each strategy. In future work, imperfect and partial channel knowledge will be considered. This central node selects the optimal (minimum possible) number of (re)transmissions N_s^* , and N_r^* for source and all the relays, respectively, which results in a successful decoding at the destination. Mathematically, the retransmissions allocation problem for all the protocols with ADF can be written as,

$$\{N_s^*, N_r^*\} = \underset{\{N_s, N_{r_l}\}}{\operatorname{argmin}} \left[(N_s) + \left(\sum_{l=1}^L N_{r_l} \right) \right] \quad (16)$$

such that,

$$\begin{aligned} N_s \cdot \gamma_{ds} + \sum_{l=1}^L \gamma_{l, N_{r_l}}^{ADF} &\geq \gamma_{th}, \\ N_s + \sum_{l=1}^L N_{r_l} &\leq L + 1, \end{aligned} \quad (17)$$

for SMERM: $1 \leq N_s \leq L + 1, 0 \leq N_{r_l} \leq L$. (18)

for SOERM: $N_s = 1, 0 \leq N_{r_l} \leq L$.

for SOERO: $N_s = 1, N_{r_l} \in \{0, 1\}$.

for SOSRM: $N_s = 1, 0 \leq N_{r_{sel}} \leq L$.

For SOSRM, let R_{sel} be the selected relay i.e. $N_{r_l}^* = N_{r_l} = 0 \forall l$ except $0 \leq N_{r_{sel}} \leq L$ holds. In the constraints we have imposed an upper bound on the number of retransmissions i.e. (17). This is due to the reason that SOERO (Source Once Each Relay Once) can have maximum of $L + 1$ (re)transmissions. In order to make the performance comparison fair between SOERO and the rest of the strategies, we impose this bound.

Similarly, for the case of estimate-or-decode the number of

retransmission optimization can be stated as

$$\{N_s^*, \mathbf{N}_r^*\} = \underset{\{N_s, N_{r_l}\}}{\operatorname{argmin}} \left[(N_s) + \left(\sum_{l=1}^L N_{r_l} \right) \right] \quad (19)$$

such that,

$$I_{low}(X_s, \mathbf{Y}_{ds}) \geq R_c, \quad (20)$$

$$N_s + \sum_{l=1}^L N_{r_l} \leq L + 1, \quad (21)$$

$$\text{for SMERM: } 1 \leq N_s \leq L + 1, \quad 0 \leq N_{r_l} \leq L. \quad (22)$$

$$\text{for SOERM: } N_s = 1, \quad 0 \leq N_{r_l} \leq L. \quad (23)$$

$$\text{for SOERO: } N_s = 1, \quad N_{r_l} \in \{0, 1\}.$$

$$\text{for SOSRM: } N_s = 1, \quad 0 \leq N_{r_{sel}} \leq L.$$

Where, $I_{low}(X_s, \mathbf{Y}_{ds})$ the parallel information combining function defined by (15). The first constraint is the condition for successful decoding at the destination for an ideal code with code rate R_c .

V. SELECTING THE OPTIMAL NUMBER OF RETRANSMISSIONS

This section explains the solutions for the optimization problems of the previous section. It is *important to note* that for the algorithms presented in this section the relay set \mathcal{R} is ordered in descending order according to $\gamma_{l, N_{r_l}}^{ADF}$ for ADF and $I_{l, N_{r_l}}^{EDF}(X_s, Y_{dr_l})$ for EDF with $N_{r_l} = 1$ and $N_s = 1$. Thus, after reordering $\gamma_{1,1}^{ADF} \geq \gamma_{L,1}^{ADF}$ holds.

A. Solutions for SMERM

In this subsection two solutions for the optimization of SMERM are described: The Exhaustive Search (SMERM-ES) and a Marginal Returns based allocation (SMERM-MR) [14].

1) *Exhaustive Search*: To show that the algorithm presented in the next subsection (Marginal Returns Based Allocation) provides optimal solution, we need to compare its performance with that of the exhaustive search (ES). ES tests all possible realizations for the sequence $[N_s, N_{r_1}, N_{r_2}, \dots, N_{r_L}]$. It picks the realization (as the optimum one) which minimizes $N_s + \sum_{l=1}^L N_{r_l}$ and fulfils the respective constraints for ADF (16) and EDF (19).

2) *Marginal Returns Based Allocation*: For convenience, for each $1 \leq N_s \leq L + 1$, the optimal number of retransmissions represented by $\mathbf{N}^*[N_s]$ is determined. The l th element of the vector $\mathbf{N}^*[N_s]$ represents the optimal number of retransmissions ($N_{r_l}^*$) for relay l and the given N_s . The optimization procedure given below should be repeated for each N_s . Once $\mathbf{N}^*[N_s]$ is found for each N_s , the overall optimal N_s in (16) is the one which minimizes $N_s + \sum_{l=1}^L N_{r_l}^*$ and the overall optimal \mathbf{N}_r (in (16)) is equal to $\mathbf{N}^*[N_s^*]$.

After fixing N_s , we also fix the total number of retransmissions $\sum_{l=1}^L N_{r_l} = \bar{N}$. Consequently, an equivalent problem to (16) with a fixed N_s is to maximize the sum SNR $\sum_{l=1}^L \gamma_{l, N_{r_l}}$ by distributing \bar{N} retransmissions among all the

relays. The minimum \bar{N} , which results in successful decoding ($N_s \cdot \gamma_{ds} + \sum_{l=1}^L \gamma_{l, N_{r_l}} \geq \gamma_{th}$) leads to the optimal solution for the specific N_s . From the problem description it is clear that for a specific N_s , search should be performed for the minimum \bar{N} which ensures successful decoding at the destination. Thus, mathematically, this equivalent problem to (16) with a given N_s can be written as

$$\mathbf{N}^*[N_s] = \underset{N_{r_l}}{\operatorname{argmax}} \left[\sum_{l=1}^L \gamma_{l, N_{r_l}} \right] \quad (24)$$

such that,

$$\sum_{l=1}^L N_{r_l} = \bar{N}, \quad (25)$$

$$N_s + \bar{N} \leq L + 1, \quad (26)$$

where, \bar{N} may take the values $0, 1, 2, 3, \dots, L + 1 - N_s$. First, \bar{N} is initialized with 0, i.e. $N_{r_l} = 0 \forall l$. If the SNR $N_s \cdot \gamma_{ds}$ is greater than γ_{th} then $\mathbf{N}^*[N_s] = \mathbf{0}$ is the solution. Otherwise, \bar{N} is incremented by 1 i.e. $\bar{N} = 1$. The optimization in (24) is performed and the condition for successful decoding $N_s \cdot \gamma_{ds} + \sum_{l=1}^L \gamma_{l, N_{r_l}} \geq \gamma_{th}$ is checked. This process (increment, optimization and check) continues until the condition is satisfied or $N_s + \bar{N}$ exceeds $L + 1$. In the case of satisfaction $\mathbf{N}^*[N_s]$ is the solution, while, in the case of excess, outage is declared for that specific N_s . Next, solution to (24) is explained.

A similar problem as (24) with an efficient optimization procedure is provided in [14]. However, we invented this optimization procedure independently, before discovering [14]. Moreover, to the best of our knowledge, we are the first one to apply the optimization method of [14] to resource allocation in relay networks.

The SNR marginal return is defined as $\Delta_{l,j} = \gamma_{l,j}^{ADF} - \gamma_{l,j-1}^{ADF}$, where, $j = N_{r_l}$. We show that similar to [14], these marginal returns are diminishing with respect to the increase in N_{r_l} , i.e. $\Delta_{l,j} \geq \Delta_{l,j+1} \geq \Delta_{l,j+2} \dots \geq \Delta_{l,L}$. For the upper part of (9), $\Delta_{l,j} = \Delta_{l,j+1} = \gamma_{dr_l}$ holds, and for the lower part we have

$$\begin{aligned} \Delta_{l,j} &= \gamma_{l,j}^{ADF} - \gamma_{l,j-1}^{ADF} \\ &= \frac{N_s \cdot \gamma_{r_{1s}} \cdot j \cdot \gamma_{dr_l}}{N_s \cdot \gamma_{r_{1s}} + j \cdot \gamma_{dr_l} + 1} - \frac{N_s \cdot \gamma_{r_{1s}} \cdot (j-1) \cdot \gamma_{dr_l}}{N_s \cdot \gamma_{r_{1s}} + (j-1) \cdot \gamma_{dr_l} + 1} \\ &= \frac{N_s \cdot \gamma_{r_{1s}} \cdot \gamma_{dr_l} (N_s \cdot \gamma_{r_{1s}} + 1)}{(N_s \cdot \gamma_{r_{1s}} + j \cdot \gamma_{dr_l} + 1)(N_s \cdot \gamma_{r_{1s}} + (j-1) \cdot \gamma_{dr_l} + 1)}. \end{aligned} \quad (27)$$

Here, (27) shows that $\Delta_{l,j}$ is diminishing with respect to $j \geq 1$.

Table I shows the SNR marginal returns for each relay R_l and each number of retransmissions N_{r_l} . To search for the optimal solution for (24), we examine the entries ($\Delta_{l,j}$) in Table I, and tick mark (select) the \bar{N} largest numbers among them in such a way that $\Delta_{l,j+1}$ should never be tick

N_{r_l}	1	2	3	4	...	L
R_1	$\Delta_{1,1}$ ✓	$\Delta_{1,2}$ ✓	$\Delta_{1,3}$ ✓	$\Delta_{1,4}$...	$\Delta_{1,L}$
R_2	$\Delta_{2,1}$	$\Delta_{2,2}$	$\Delta_{2,3}$	$\Delta_{2,4}$...	$\Delta_{2,L}$
R_3	$\Delta_{3,1}$	$\Delta_{3,2}$	$\Delta_{3,3}$	$\Delta_{3,4}$...	$\Delta_{3,L}$
R_4	$\Delta_{4,1}$ ✓	$\Delta_{4,2}$ ✓	$\Delta_{4,3}$	$\Delta_{4,4}$...	$\Delta_{4,L}$
...
R_L	$\Delta_{L,1}$ ✓	$\Delta_{L,2}$	$\Delta_{L,3}$	$\Delta_{L,4}$...	$\Delta_{L,L}$

TABLE I
THE SNR MARGINAL RETURNS FOR ALL THE RELAYS.

marked unless $\Delta_{l,j}$ has already been tick marked. Thus, we do not need to compute $\Delta_{l,j+1}$ unless $\Delta_{l,j}$ is marked, this improves the computational efficiency. The optimal allocation of retransmissions ($N_{r_l}^*$) for relay R_l with specific \bar{N} can be immediately determined simply by counting the number of tick marks that each relay R_l has received. As an example, in Table I for $L = 8$ and $\bar{N} = 6$, $\mathbf{N}^*[N_s] = [3, 0, 0, 2, 0, 0, 0, 1]$, i.e. $N_{r_1}^* = 3, N_{r_2}^* = 0, N_{r_3}^* = 0, N_{r_4}^* = 2, N_{r_5}^* = 0, N_{r_6}^* = 0, N_{r_7}^* = 0, N_{r_8}^* = 1$ hold. In the simulation results the computational complexities of SMERM-ES and SMERM-MR are compared.

The optimization problem of estimate-or-decode & forward (19) for SMERM can be solved in a similar way as done for amplify-or-decode & forward. However, the following modifications are required to the above solution. In (24), $I_{low}(X_s, \mathbf{Y}_{ds})$ should be maximized. Second, the condition for successful decoding should be $I_{low}(X_s, \mathbf{Y}_{ds}) \geq R_c$. Moreover, for each entry of the marginal returns in Table I, $\Delta_{l,j} = I_{l,j}^{EDF}(X_s, Y_{dr_l}) - I_{l,j-1}^{EDF}(X_s, Y_{dr_l})$ holds.

It is involved to show analytically that for EDF $\Delta_{l,j}$ is diminishing, however, it was found by numerical simulations that similar to ADF it is also decreasing with respect to increase in N_{r_l} .

B. Solution for SOERM

It should be noted that SOERM is a special case of SMERM, therefore the same solution as in subsection V-A should be applied with $N_s^* = N_s = 1$.

C. Solution for SOERO

The solution for SOERO with amplify-or-decode (16) is straight forward. As the relay set \mathcal{R} is ordered in descending order, first, R_1 with $N_{r_1} = 1$ is selected. If it does not ensure successful decoding at the destination ($\gamma_s + \gamma_{1,N_{r_1}}^{ADF} < \gamma_{th}$) then the next relay R_2 is activated. Again the successful decoding status is checked. If unsuccessful decoding occurs the next relay R_3 is activated with $N_{r_3} = 1$. This process continues until successful decoding occurs or all the relays have been activated.

For SOERO, estimate-or-decode (19) can be solved in a similar manner, however for successful decoding the summation of SNR should be replaced with the respective information combining function $I_{low}(\dots) \geq R_c$.

D. Solution for SOSRM

After ordering the relay set \mathcal{R} , R_1 is the “best” relay with one retransmission. Thus, for the solution of SOSRM

we have to find the minimum number of retransmissions N_{r_1} , which lead to a successfully decoding at the destination i.e. for ADF ($\gamma_s + \gamma_{1,N_{r_1}}^{ADF} \geq \gamma_{th}$) while for EDF $I_{low}^{\text{par}}(I_s, I_{l,N_{r_1}}^{EDF}(X_s, Y_{dr_1})) \geq R_c$ holds. However, according to the constraint in SOSRM, N_{r_1} should not exceed L .

VI. RESULTS

Four relays ($L = 4$) were placed at different positions to observe the effect of distances between the nodes. Let $Pos \in \{NS, MD, ND\}$ is a variable defining the position of the set of the L relays in Cartesian coordinate, where, $NS = (0.2 + 0.2i)$, $MD = (0.5 + 0.2i)$ and $ND = (0.8 + 0.2i)$. For each value of Pos , R_1 and R_L are placed at positions Pos and Pos^* respectively¹, the remaining $L - 2$ relays are equally distributed between the relays R_1 and R_L such that all the relays positions have the same real part values. Thus, for $Pos = MD$ the relays are placed in the middle and for ND the relays are placed near the destination. Furthermore, source and destination are placed at position $(0 + 0i)$ and $(1 + 0i)$ respectively. The path loss coefficient α is set to 2 and $N = 10000$ Rayleigh channel realization were simulated.

An ideal code with code rate $R_c = 0.5$ is considered, which ensures successful decoding when the symbol-wise mutual information at the decoder is greater than or equal to the code rate R_c [11]. Therefore, γ_{th} is chosen such that $C_A(\gamma_{th}) = R_c$ holds.

A. Simulation parameters

This section explains how the parameters outage probability (P_{outage}), throughput (η), and the energy per bit to noise ratio ($\bar{\gamma}_{E_b}$) are calculated. The outage probability is defined as

$$P_{outage} = \frac{V_{out}}{N}, \quad (28)$$

where V_{out} represents the number of events when the destination could not decode successfully at the end of the second phase.

The throughput is defined as

$$\eta = \frac{V_{succ} \cdot R_c}{S_s + S_R}, \quad (29)$$

where, V_{succ} denotes the number of events which resulted in successful decoding at the destination. Moreover, S_s and S_R represent the total number of transmissions from the source and the relays respectively over all the channel realizations. The maximum throughput that can be achieved for the adaptive schemes with ARQ is $\eta_{max} = R_c$. This is achieved when $S_R = 0$ and $V_{succ} = N = S_s$ hold.

The energy per bit to noise ratio is given by

$$\bar{\gamma}_{E_b} = E_s/N_0 \cdot \frac{1}{R_c} \cdot \frac{S_s + S_R}{N}. \quad (30)$$

It has to be emphasized that E_s/N_0 is the ratio of the transmit energy per BPSK symbol to the one sided noise spectral density (pseudo SNR), identical noise statistics are assumed at all the receivers.

¹ * denotes the complex conjugate.

B. Outage and Throughput

The exhaustive search (ES) algorithm was only applied to SMERM with ADF. Fig. 2 shows the outage probabilities for the scenario when the relays are near the source. It can be observed that the performance of SMERM-ES (Exhaustive Search) and SMERM-MR (Marginal Returns Based Allocation) is identical. Moreover, SMERM with ADF outperforms all other schemes, e.g. SOERO by 4 dB, SOSRM and SOERM by 2 dB and 1 dB respectively. Here, strategies with ADF perform similar to EDF except for SMERM, where, EDF is 0.5 dB away from ADF.

Fig. 3 illustrates the outage probabilities when the relays are in the middle. It is evident from this figure that the outage performance of the proposed solution (SMERM-MR) is identical to that of the optimum SMERM-ES, which validates the optimality claim made at the start of the paper. A gain of more than 4 dB is possible for SMERM over SOSRM (Source Once and Selected Relay transmit Multiple times) and SOERO (Source Once and Each Relay transmit Once). Furthermore, the amplify-or-decode & forward (ADF) and estimate-or-decode & forward (EDF) performs the same for all the cases. Moreover, SMERM-MR delivers nearly 2 dB gain over the SOERM-MR (Source Once and Each Relay transmit Multiple times).

Fig. 4 shows the outage probabilities for the scenario when the relays are near the destination. Again, the performance of the devised solution (SMERM-MR) is identical to the optimal (SMERM-ES). Moreover, as in the previous figure, the superior asymptotic performance of the devised solution over the rest of the solutions is eminent.

It can be observed from the above outage probability figures that as the distance between the source and relays grows, multiple source transmissions becomes more important. This is the reason as the relays move toward the destination, the gap between the outage performance of SMERM and that of the other strategies increases. In Fig. 5, the relays are placed near the source. Here, SMERM-MR and SMERM-ES have identical throughput. Moreover, the performance of ADF with all the strategies is similar to that of EDF. SMERM yields a gain of 2 dB over SOERO. The difference between SMERM and SOERM is small, the reason for this is the fact that at this position (near the source) all the relays can decode successfully most of the time and can act similar to the source. Fig. 6 depicts the throughput for the relays positioned in the middle. In this figure, SMERM-MR and SMERM-ES surpass all other strategies. Both of the solutions deliver 1 dB gain over SOERM-MR with ADF and EDF. Again, as in the above results, for all the strategies ADF and EDF behave the same. SOERO is more than 2 dB away from the devised solution at low SNR.

In Fig. 7, the relays are placed near the destination. Here, SMERM-MR and SMERM-ES outperform SOERM-MR by more than 1 dB. The same solutions yield 2.5 dB gain over SOSRM and SOERO. For each protocol ADF and EDF have very small difference in performance.

$\frac{1}{2 \cdot \sigma_n^2}$	-8	-7	-6	-5	-4	-3	-2	-1
ES	100	92	85.3	81.1	66.7	60	51.4	43
MR	5.3	3.7	2.9	2.3	1.8	1.6	1.3	1
Q	18.8	24.8	29	35.2	37	37.5	39.5	43

TABLE II
AVERAGE NUMBER OF COMPUTATIONAL STEPS FOR SMERM-ES AND SMERM-MR AT DIFFERENT SNR ($\frac{1}{2 \cdot \sigma_n^2}$) VALUES IN DECIBEL

Similar to the outage figures, it can be seen in the throughput figures that as the distance between source and relays grows, multiple source transmissions becomes more important. This is the reason as the relays move toward the destination, the gain deliver by SMERM over the rest of the strategies increases.

C. Complexity Comparison

For the comparison in this subsection, the relays were placed in the middle. To compare the computational complexity of SMERM-ES (exhaustive search) with that of SMERM-MR (marginal returns based allocation), we consider the average number of comparisons made for γ^{ADF} for ADF ($I_{low}(X_s, Y_{ds})$ for EDF) as a measure of computational complexity. The comparison of each γ^{ADF} or $I_{low}(X_s, Y_{ds})$ is termed as a computational step. Table II shows the average number of computational steps required by SMERM-ES (2nd row) and SMERM-MR (3rd row) to solve the optimization problem (16) at different SNRs ($\frac{1}{2 \cdot \sigma_n^2}$ in dB). It should be noted that the computational complexity for ADF and EDF only differ in the third digit after the decimal point, however, after rounding they have the same values for SMERM-ES and SMERM-MR as shown in 2nd and 3rd rows. Here, Q (last row) represents the ratio of the number of computational steps of SMERM-ES to that of SMERM-MR. From this table it is evident that SMERM-MR is more efficient than SMERM-ES. At SNR equal to -8 dB, SMERM-ES requires 18.8 times more computational steps as compared to SMERM-MR. Similarly, at -1 dB, SMERM-MR is 43 times more efficient than SMERM-ES.

VII. CONCLUSION

This paper has presented the novel issue of retransmission allocation in ARQ relay networks. An optimal SNR marginal returns based solution is devised for the allocation. The results further reveal that the allowance of multiple retransmissions from each node can significantly improve the network performance, especially if the source transmits multiple times. Moreover, the multiple retransmissions protocol also outperforms the opportunistic relaying technique where a single node with the best end-to-end SNR (per single retransmission) is selected for all the transmissions.

The complexity comparison shows that the devised procedure SMERM-MR is significantly computationally efficient as compared to the exhaustive search alternative.

Sofar, the improvement of a relay's received signal by retransmissions of other relays has not been considered.

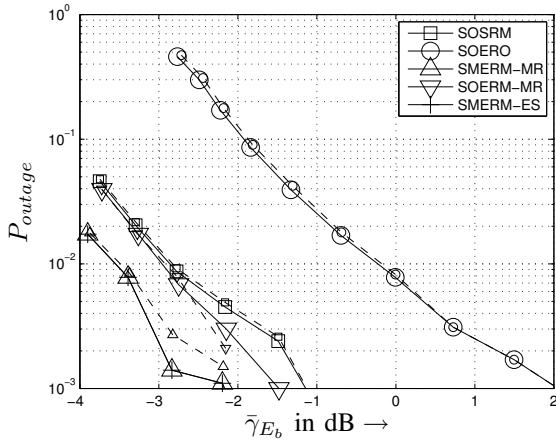


Fig. 2. Outage probability for different relay protocols, relays are positioned near the source, solid lines: ADF protocols, dashed lines (with small marker): EDF protocols

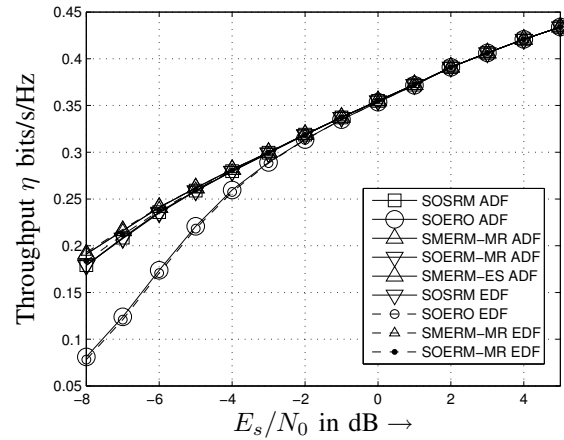


Fig. 5. Throughput for different relay protocols, relays are positioned near the source.

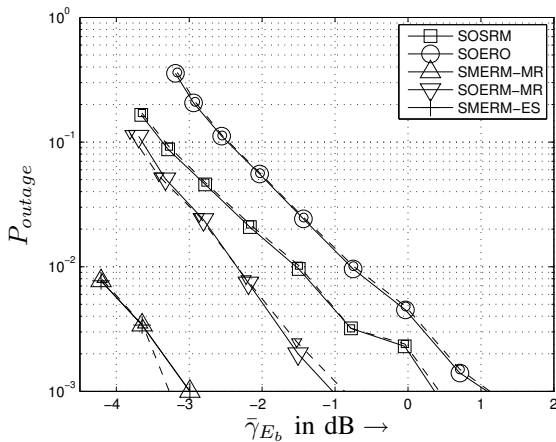


Fig. 3. Outage probability for different relay protocols, relays are positioned in the middle, solid lines: ADF protocols, dashed lines (with small marker): EDF protocols

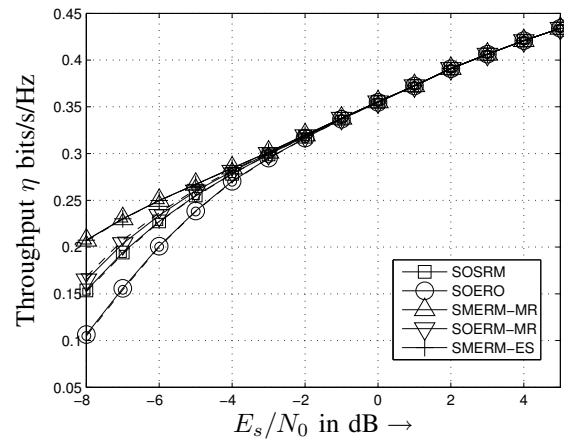


Fig. 6. Throughput for different relay protocols, relays are positioned in the middle, solid lines: ADF protocols, dashed lines (with small marker): EDF protocols

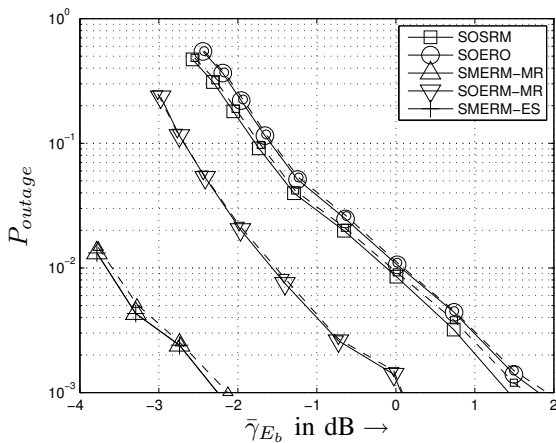


Fig. 4. Outage probability for different relay protocols, relays are positioned near the destination, solid lines: ADF protocols, dashed lines (with small marker): EDF protocols

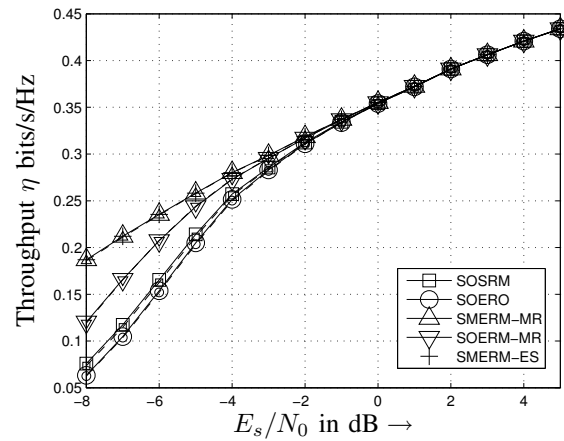


Fig. 7. Throughput for different relay protocols, relays are positioned near the destination, solid lines: ADF protocols, dashed lines (with small marker): EDF protocols

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