On the Simultaneous Relay Channel with Collocated Relay and Destination Nodes

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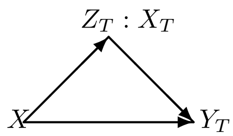
***Abstract*—The simultaneous relay channel with collocated relay and destination nodes is investigated. This models the scenario in which the source user is unaware of the channel controlling the communication but it knows the set of all possible relay channels. Our primary focus is the case where the relay node is physically near to the destination so that Compress-and-Forward (CF) coding is the adequate cooperative strategy. A broadcasting scheme for flexible user cooperation is developed. This enables the encoder to send part of the information regardless of which channel is present and additional information intended for each of the different channels. It can be seen that this problem is equivalent to that of sending common and private information to several destinations in presence of helper relays. This scenarios is identified as the broadcast relay channel (BRC). A general achievable rate region is derived and specific results are obtained for the case of Gaussian BRCs. The advantage of the proposed coding scheme is that the source can adapt its coding rate to the different channels that might be present during the communication.**

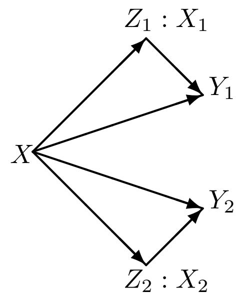
I. INTRODUCTION

Cooperative networks have been a topic of huge interest during recent years between researchers. Using the multiplicity of information in nodes, these networks can increase the capacity and reliability provided by the appropriate coding strategy. The simplest of these networks is the relay channel. The relay channel, first introduced in [1], consists of a sender-receiver pair whose communication is aided by a relay node, as it is shown in Fig.1(a). Significant contributions were made by Cover and El Gamal [2], where the main strategies of Decode-and-Forward (DF) and Compress-and-Forward (CF), and an upper bound were developed for this channel. The performances of DF and CF strategies are strongly dependent on the physical position of the relay and destination nodes. Roughly speaking, one can say that when the relay is near to the source node, i.e., the source-relay channel is not good enough, it is generally better to use DF coding. In contrast with this, if the relay is near to the destination node it is better to use CF coding. Moreover the capacity of the degraded relay channel was established in [2] and a general result that combines DF and CF coding was also provided. This result was improved in [3] where the best known lower bound on the capacity of general relay channels is derived. Based on these strategies, further work has been done on cooperative networks from different aspects, including deterministic channels [4], multiple access relay, broadcast relay and multiple relays, fading relay channels [5]. Particularly the relay broadcast channel was studied in [6]–[8] where capacity theorems, upper and inner bound were developed for the channel.

The simultaneous channel with two nodes was first introduced in [9], [10]. This scenario consists of a source that sends information to a destination through several different channels. A relation between the broadcast channel (BC) and the simultaneous channel was first mentioned in [11]. Broadcast coding can be used to adapt information-rates to the different channel users and thereby improving the general rate by sending additional (private) information layers. Indeed, each channel in the set of possible channels serves as a different branch of the BC. The idea was exploited later in similar cases [12]–[14]. Similarly, extensive research has been done on BCs due to their importance as a main part of scenarios like multi cast, flat fading, multi-hop, ad-hoc networks, and others. This channel consists of a source transmitting different messages to several destinations. The main coding strategies (e.g. superposition coding, Marton coding) were developed in [11], [15]–[17], and shown to be capacity archiving for various classes of channels (e.g. degraded, degraded message sets, less



(a) The simultaneous relay channel



(b) BRC with two relays

Fig. 1. The relay channel

noisy, more capable, deterministic).

In this paper, we focus on the information-theoretic limits of simultaneous relay channels, where the source user is unaware of the channel controlling the communication but it knows the set of possible relay channels. We focus on the case of two possible channels and the relay node is assumed to be physically near to the destination. An achievable rate region is derived which is based on broadcast coding together with CF coding. This paper is organized as follows. In Section II, definition and main results are presented. Next section presents the sketch of proof, and Gaussian examples are relegated to Section IV. Finally, numerical results and discussions are presented in the Section V.

II. Problem Definitions and Main Results

A. Problem Definition

The simultaneous relay channel [12] with discrete source and relay inputs $x \in \mathscr{X}$, $x_T \in \mathscr{X}_T$, discrete channel and relay outputs $y_T \in \mathscr{Y}_T$, $z_T \in \mathscr{Z}_T$, is characterized by two conditional probability distributions (PDs) $\big\{P_T : \mathscr{X} \times \mathscr{X}_T \longmapsto \mathscr{Y}_T \times \mathscr{Z}_T\big\}_{T=1,2}$, where T is the channel index. It is assumed here that the transmitter (the source) is unaware of the realization of T that governs the communication, but T should not change during the communication. However, T is assumed to be known at the destination and the relay ends.

Definition 1 (Code): A code for the SRC consists of: an encoder mapping $\{\varphi : \mathcal{W}_1 \times \mathcal{W}_2 \longmapsto \mathscr{X}^n\}$, two decoder mappings $\{\psi_T : \mathscr{Y}_T^n \longmapsto \mathcal{W}_T\}$ and a set of relay functions $\{f_{T,i}\}_{i=1}^n$ such that $\{f_{T,i} : \mathscr{Z}_T^{i-1} \longmapsto \mathscr{X}_{T,i}\}_{i=1}^n$, for some finite sets of integers $\mathcal{W}_T = \big\{1, \dots, W_T\big\}$, where i is the time index. The rates of such code are $n^{-1} \log W_T$ and its maximum error probability

$$e_{\max,T}^{(n)} \doteq \max_{(w_0,w_T)\in\mathcal{W}_0\times\mathcal{W}_T} \Pr\Big\{\psi(\mathbf{Y}_T) \neq (w_0, w_T)\Big\}.$$

Definition 2 (Achievable rates and capacity): For every $0 < \epsilon, \gamma < 1$, a triple of non-negative numbers (R_0, R_1, R_2) is achievable for the SRC if for every sufficiently large n there exist n-length block code whose error probability satisfies $e_{\max,T}^{(n)}\big(\varphi, \psi, \{f_{T,i}\}_{i=1}^n\big) \leq \epsilon$ for each $T = \{1, 2\}$ and the rates $n^{-1} \log W_T \geq R_T - \gamma$. The set of all achievable rates is called the capacity region for the SRC. We emphasize that no prior distribution on T is assumed and thus the encoder must exhibit a code that yields small error probability for every $T = \{1, 2\}$, yielding the BRC setting. A similar definition can be offered for the common-message BC with a single message set $\mathcal{W}_0$ and rate $n^{-1} \log W_0$.

Since the relay and the receiver can be assumed to be cognizant of the realization T, the problem of coding for the SRC can be turned into that of the BRC [12]. This consists of two relay branches where each one equals to a relay channel with $T = \{1, 2\}$, as is shown in Fig. 1(b). The encoder sends common and private messages (W_0, W_T) to destination T at rates (R_0, R_T). The BRC is defined by the PD $\big\{P : \mathscr{X} \times \mathscr{X}_1 \times \mathscr{X}_2 \longmapsto \mathscr{Y}_1 \times \mathscr{Z}_1 \times \mathscr{Y}_2 \times \mathscr{Z}_2\big\}$, with channel and relay inputs (x, x_1, x_2) and channel and relay outputs (y_1, z_1, y_2, z_2). Notions of achievability for (R_0, R_1, R_2) and capacity remain the same as for BCs (see [11], [5] and [7]). Indeed R_0 can be considered as rate that can be transmitted through the both channel, i.e. common information, and R_1, R_2 are the rate corresponding to each channel, i.e. private information.

B. Coding Theorem for the Broadcast Relay Channel

Theorem 2.1: An inner bound on the capacity region of the BRC is given by

$$\begin{aligned}
\mathscr{R}_I \doteq \bigcup_{P\in\mathscr{P}} \Big\{&(R_0 \geq 0, R_1 \geq 0, R_2 \geq 0) : \\
R_0 + R_1 &\leq I(U_0U_1; Y_1\hat{Z}_1|X_1) \\
R_0 + R_2 &\leq I(U_0U_2; Y_2\hat{Z}_2|X_2) \\
R_0 + R_1 + R_2 &\leq I_0 + I(U_1; Y_1\hat{Z}_1|X_1U_0) + \\
&\quad I(U_2; Y_2\hat{Z}_2|X_2U_0) - I(U_1; U_2|U_0) \\
2R_0 + R_1 + R_2 &\leq I(U_0U_1; Y_1\hat{Z}_1|X_1) + I(U_0U_2; Y_2\hat{Z}_2|X_2) \\
&\quad -I(U_1; U_2|U_0)\Big\},
\end{aligned}$$

where the quantity (I_0) is given by

$$I_0 \doteq \min\left\{I(U_0;Y_1\hat{Z}_1|X_1), I(U_0;Y_2\hat{Z}_2|X_2)\right\},$$

and the set of all admissible PDs $\mathscr{P}$ is defined as

$$\begin{aligned}&\mathscr{P} \doteq \{P_{U_0U_1U_2X_1X_2XY_1Y_2Z_1Z_2\hat{Z}_1\hat{Z}_2} = P_{X_2}P_{X_1}\\ &P_{U_0}P_{U_2U_1|U_0}P_{X|U_2U_1}P_{Y_1Y_2Z_1Z_2|XX_1X_2}P_{\hat{Z}_1|X_1Z_1}P_{\hat{Z}_2|X_2Z_2},\\ &I(X_2;Y_2) \geq I(Z_2;\hat{Z}_2|X_2Y_2), I(X_1;Y_1) \geq I(Z_1;\hat{Z}_1|X_1Y_1)\\ &(U_0,U_0,U_1) \ominus (X_1,X_2,X) \ominus (Y_1,Z_1,Y_2,Z_2)\}.\end{aligned}$$

Remark 1: The variables $\hat{Z}_1, \hat{Z}_2$ are the compression of Z_1, Z_2 at the relays. It can be seen that from the probability distribution presented in theorem 2.1 that $\hat{Z}_i$, $i \in \{1,2\}$ is independent of the rest of random variables given X_i, Z_i thus holding the Markov chain with the rest via X_i, Z_i.

Remark 2: Similar to the single relay channel, compress-and-forward rate is not in general, before maximization, convex. So the lower bound can be improved using time-sharing. Notice that this region includes Marton's region [16] with $(X_1, X_2) = \emptyset$, $Z_1 = Y_1$ and $Z_2 = Y_2$.

III. Sketch of Proof

Reorganize first private messages w_i, $i \in \{1,2\}$ into (s'_i, s_i) with non-negative rates (S'_i, S_i) where $R_i = S'_i + S_i$. Merge (s'_1, s'_2, w_0) to one message s_0 with rate $S_0 = R_0 + S'_1 + S'_2$.

Code Generation:

(i) Randomly and independently generate 2^{nS_0} sequences $\underline{u}_0$ draw i.i.d. from $P_{U_0}(\underline{u}_0) = \prod_{j=1}^n p_{U_0}(u_{0j})$. Index them as $\underline{u}_0(s_0)$ with $s_0 \in \left[1, 2^{nS_0}\right]$.

(ii) Randomly and independently generate $2^{nR_{x_b}}$ sequences $\underline{x}_b$ draw i.i.d. from $P_{X_b}(\underline{x}_b) = \prod_{j=1}^n p_{X_b}(x_{bj})$ as $\underline{x}_b(r_b)$, where $r_b \in \left[1, 2^{nR_{x_b}}\right]$ for $b \in \{1,2\}$.

(iii) For each $\underline{x}_b(r_b)$ randomly and independently generate $2^{n\hat{R}_b}$ sequences $\underline{\hat{z}}_b$ each with probability $P_{\hat{Z}_b|X_b}(\underline{\hat{z}}_b|\underline{x}_b(r_b)) = \prod_{j=1}^n p_{\hat{Z}_b|X_b}(\hat{z}_{bj}|x_{bj}(r_b))$. Index them as $\underline{\hat{z}}_b(r_b, \hat{s}_b)$, where $\hat{s}_b \in \left[1, 2^{n\hat{R}_b}\right]$ for $b \in \{1,2\}$.

(iv) Partition the set $\left\{1, \ldots, 2^{n\hat{R}_b}\right\}$ into $2^{nR_{x_b}}$ cells and label them as S_{r_2}. In each cell there are $2^{n(\hat{R}_b - R_{x_b})}$ elements.

(v) For each pair $\underline{u}_0(s_0)$, randomly and independently generate 2^{nT_b} sequences $\underline{u}_b$ draw i.i.d. from $P_{U_b|U_0}(\underline{u}_b|\underline{u}_0(s_0)) = \prod_{j=1}^n p_{U_b|U_0}(u_{bj}|u_{0j}(s_0))$. Index them as $\underline{u}_b(s_0, t_b)$, where $t_b \in \left[1, 2^{nT_b}\right]$ for $b \in \{1,2\}$.

(vi) For $b \in \{1,2\}$, partition the set $\left\{1, \ldots, 2^{nT_b}\right\}$ into 2^{nS_b} subsets and label them as S_{s_b}. In each subset, there are $2^{n(T_b - S_b)}$ elements for $b \in \{1,2\}$.

(vii) Then the encoder looks for $t_1 \in S_{s_1}$ and $t_2 \in S_{s_2}$ such that $\left(\underline{u}_1(s_0,t_1), \underline{u}_2(s_0,t_2)\right)$ are jointly typical given the RV $\underline{u}_0(s_0)$. The constraints for the coding step is:

$$T_1 + T_2 - S_1 - S_2 \geq I(U_2;U_1|U_0). \tag{1}$$

At the end, it finds a pair (t_1, t_2).

(viii) Finally, use a deterministic function for generating $\underline{x}$ as $f\left(\underline{u}_1, \underline{u}_2\right)$ indexed by $\underline{x}(s_0, t_1, t_2)$.

Encoding Part: In block i, the source wants to send (w_{0i}, w_{1i}, w_{2i}) by reorganizing them into (s_{0i}, s_{1i}, s_{2i}). Encoding steps are as follows:

(i) Relay b knows from the previous block that $\hat{s}_{b(i-1)} \in S_{r_{bi}}$ and it sends $\underline{x}_b(r_{bi})$ for $b \in \{1,2\}$.

(ii) From (s_{0i}, s_{1i}, s_{2i}), the source finds (t_{1i}, t_{2i}) and sends $\underline{x}(s_{0i}, t_{1i}, t_{2i})$.

Decoding Part: After the transmission of the block $i+1$, the relays start to find $\hat{Z}_b$. The destinations decode $\hat{Z}_b$ and then use it with Y_b to decode the messages.

(i) Relay b searches for $\hat{s}_{bi}$ after receiving $\underline{z}_b(i)$ such that $\left(\underline{x}_b(r_{bi}), \underline{z}_b(i), \underline{\hat{z}}_b(\hat{s}_{bi}, r_{bi})\right)$ are jointly typical subject to

$$\hat{R}_b \geq I(Z_b;\hat{Z}_b|X_b). \tag{2}$$

(ii) Destination b searches for $r_{b(i+1)}$ such that $\left(\underline{y}_b(i+1), \underline{x}_b(r_{b(i+1)})\right)$ is jointly typical. Then in finds $\hat{s}_{bi}$ such that $\hat{s}_{bi} \in S_{r_{b(i+1)}}$ and $\left(\underline{\hat{z}}_b(\hat{s}_{bi}, r_{bi}), \underline{y}_b(i), \underline{x}_b(r_{bi})\right)$ are jointly typical. Conditions for reliable decoding are:

$$R_{x_b} \leq I(X_b;Y_b),\ \hat{R}_b \leq R_{x_b} + I(\hat{Z}_b;Y_b|X_b). \tag{3}$$

(iii) Decoding in block i is done such that $(\underline{u_0}(s_{0i}), \underline{u_b}(s_{0i}, t_{bi}), \underline{y}_b(i), \underline{\hat{z}}_b(\hat{s}_{bi}, r_{bi}), \underline{x}_b(r_{bi}))$ are all jointly typical. This leads to the next constraints

$$S_0 + T_b \leq I(U_0U_b;Y_b\hat{Z}_b|X_b), \tag{4}$$

$$T_b \leq I(U_b;Y_b\hat{Z}_b|U_0X_b). \tag{5}$$

After decoding of (s_{0i}, s_{1i}, s_{2i}) at destinations, the original messages (w_{0i}, w_{1i}, w_{2i}) can be extracted. One can see that the rate region of theorem 2.1 follows form equations (1)-(5), the equalities between the original rates and reorganized rates, the fact that all the rates are positive and by using Fourier-Motzkin elimination. Similarly to [2], the necessary condition

$I(X_b;Y_b) \geq I(Z_b;\hat{Z}_b|X_bY_b)$ follows from (2) and (3) for $b \in \{1,2\}$.

IV. Gaussian Examples

In this section the Gaussian example is investigated, where the relay is collocated with the destination in the both channels. No interference is allowed from the relay b to the destination $\overline{b}$, $b \in \{1,2\}$. The relationship between RVs are as follows:

$$\begin{gathered} Y_1 = \frac{X}{\sqrt{d^{\delta}_{y_1}}} + \frac{X_1}{\sqrt{d^{\delta}_{z_1y_1}}} + \mathcal{N}_1 \ , \ Y_2 = \frac{X}{\sqrt{d^{\delta}_{y_2}}} + \frac{X_2}{\sqrt{d^{\delta}_{z_2y_2}}} + \mathcal{N}_2, \\ Z_1 = \frac{X}{\sqrt{d^{\delta}_{z_1}}} + \tilde{\mathcal{N}}_1 \ , \ Z_2 = \frac{X}{\sqrt{d^{\delta}_{z_2}}} + \tilde{\mathcal{N}}_2. \end{gathered} \tag{6}$$

The channel inputs are constrained to satisfy the power constraint P, while the relay inputs must satisfy power constraint P_1 and P_2. The Gaussian noises $\tilde{\mathcal{N}}_1, \tilde{\mathcal{N}}_2$, $\mathcal{N}_1$ and $\mathcal{N}_2$ are zero-mean of variances $\tilde{N}_1, \tilde{N}_2, N_1$ and N_2. We choose also $\hat{Z}_1 = Z_1 + \hat{\mathcal{N}}_1$ and $\hat{Z}_2 = Z_2 + \hat{\mathcal{N}}_2$ where $\hat{\mathcal{N}}_1, \hat{\mathcal{N}}_2$ are zero-mean Gaussian noises of variances $\hat{N}_1, \hat{N}_2$. The results are calculated in two cases, private and common information.

A. Achievable Rates for Private Information

As for the classical broadcast channel, by using superposition coding, we decompose X as a sum of two independent RVs such that $\mathbb{E}\left\{X_A^2\right\} = \alpha P$ and $\mathbb{E}\left\{X_B^2\right\} = \overline{\alpha} P$, where $\overline{\alpha} = 1-\alpha$. The codewords (X_A, X_B) contain the information intended to receivers Y_1 and Y_2. We choose the dirty-paper coding (DPC) schemes to deal with the problem. A DPC scheme is applied to X_B for canceling the interference X_A, while for the relay branch of the channel is similar to [2]. Hence, the auxiliary RVs (U_1, U_2) are set to

$$U_1 = X_A \quad U_2 = X_B + \gamma X_A, \tag{7}$$

Notice that in this case, instead of only Y_2, we have also $\hat{Z}_2$ present in the rate. Thus DPC should be also able to cancel the interference in both, received and compressed signals which have different noise levels. Calculation should be done again with $(Y_2, \hat{Z}_2)$ which are the main message X_B and the interference X_A.We can show that the optimum γ has a similar form to the classical DPC with the noise term replaced by an equivalent noise which is like the harmonic mean of the noise in $(Y_2, \hat{Z}_2)$. The optimum γ^* is given by $\frac{\overline{\alpha}P}{\overline{\alpha}P+N_{t1}}$ where $N_{t1} = \left((d^{\delta}_{z_2}(\tilde{N}_2 + \widehat{N}_2))^{-1} + (d^{\delta}_{y_2}(N_2))^{-1}\right)^{-1}$. As we can see the equivalent noise is twice of the harmonic mean of the other noise terms. For calculating the private rate we use the theorem 2.1 with choosing $U_0 = \phi$. we can see that the current definitions yield the rates: $R_1 = I(U_1;Y_1\hat{Z}_1|X_1)$ and $R_2 = I(U_2;Y_2\hat{Z}_2|X_2) - I(U_1X_1;U_2)$. The rate for optimal γ is as follows:

$$\begin{aligned} R_1^* &= C\left(\frac{\alpha P}{d^{\delta}_{y_1}N_1 + \overline{\alpha}P} + \frac{\alpha P}{d^{\delta}_{z_1}(\widehat{N}_1 + \tilde{N}_1) + \overline{\alpha P}}\right), \\ R_2^* &= C\left(\frac{\overline{\alpha}P}{d^{\delta}_{y_2}N_2} + \frac{\overline{\alpha}P}{d^{\delta}_{z_2}(\widehat{N}_2 + \tilde{N}_2)}\right), \end{aligned} \tag{8}$$

where $C(x) = \frac{1}{2}\log(1+x)$. Note that since (X_A, X_B) are chosen independent, destination 1 sees X_B as an additional channel noise. The compression noise is chosen as follows:

$$\begin{aligned} \hat{N}_1 &= \tilde{N}_1 \left(P\left(\frac{1}{d^{\delta}_{y_1}N_1} + \frac{1}{d^{\delta}_{z_1}\tilde{N}_1}\right) + 1\right) \Big/ \frac{P_1}{d^{\delta}_{z_1y_1}N_1}, \\ \hat{N}_2 &= \tilde{N}_2 \left(P\left(\frac{1}{d^{\delta}_{y_2}N_2} + \frac{1}{d^{\delta}_{z_2}\tilde{N}_2}\right) + 1\right) \Big/ \frac{P_2}{d^{\delta}_{z_2y_2}N_2}. \end{aligned} \tag{9}$$

One can see the similarity between this case and the case of classical dirty paper coding.

B. Inner and Upper Bounds on the Common-Rate

The definition of the channels remain the same. We define $X = U_0$ and evaluate the theorem 2.1 for $U_1 = U_2 = \phi$. The goal is to send common-information at rate R_0. It is easy to verify the following results based on the theorem 2.1:

$$\begin{aligned} R_0 &\leq C\left(\frac{P}{d^{\delta}_{y_1}N_1} + \frac{P}{d^{\delta}_{z_1}(\widehat{N}_1 + \tilde{N}_1)}\right), \\ R_0 &\leq C\left(\frac{P}{d^{\delta}_{y_2}N_2} + \frac{P}{d^{\delta}_{z_2}(\widehat{N}_2 + \tilde{N}_2)}\right). \end{aligned} \tag{10}$$

The constraint for the compression noise remains unchanged, exactly like the previous section.

Here we also present an upper bound for the common information without proof. The bound is indeed a combination of two cut-set bounds for the relay channel. The final bound is the minimum of these two upper bounds. It means that if a common rate is to be transmitted, it cannot exceed the upper bound of both channels, otherwise the respective receiver cannot decode the message. The upper bound can be written as $R_1^* = \max_{0\leq\beta_1,\beta_2\leq 1} \min$

$$\left\{ C\left(\beta_1 P\left[\frac{1}{d^{\delta}_{z_1}\tilde{N}_1} + \frac{1}{d^{\delta}_{y_1}N_1}\right]\right), C\left(\frac{\frac{P}{d^{\delta}_{y_1}} + \frac{P_1}{d^{\delta}_{z_1y_1}} + 2\sqrt{\frac{\overline{\beta}_1 PP_1}{d^{\delta}_{y_1}d^{\delta}_{z_1y_1}}}}{N_1}\right), \right.$$
$$\left. C\left(\beta_2 P\left[\frac{1}{d^{\delta}_{z_2}\tilde{N}_2} + \frac{1}{d^{\delta}_{y_2}N_2}\right]\right), C\left(\frac{\frac{P}{d^{\delta}_{y_2}} + \frac{P_2}{d^{\delta}_{z_2y_2}} + 2\sqrt{\frac{\overline{\beta}_2 PP_2}{d^{\delta}_{y_2}d^{\delta}_{z_2y_2}}}}{N_2}\right) \right\}.$$

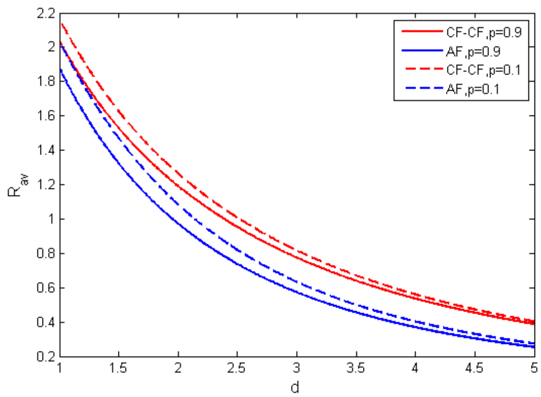


Fig. 2. The average rates for CF-CF and AF scenarios.

V. Numerical Results and Examples

In this section, we compare the numerical results for Amplify-and-Forward (AF) and Compress-and-Forward (CF). The AF rates are calculated with the assumption of (6) and also, the assumption that $X_{1i} = a_1 Z_{1(i-1)}, X_{2i} = a_2 Z_{2(i-1)}$ similar to [5] where a_1, a_2 are the amplification factor in the relays and should satisfy the following conditions:

$$a_1^2 \leq \frac{P_1}{\frac{P}{d_{z_1}^{\delta}} + \tilde{N}_1}$$

$$a_2^2 \leq \frac{P_2}{\frac{P}{d_{z_2}^{\delta}} + \tilde{N}_2}$$

Here we assume that all channel noises are set to the unit variance and $P = P_1 = P_2 = 10$. The distance between X and (Y_1, Y_2) is d, while $d_{z_1} = d - 0.1, d_{z_1 y_1} = 0.1, d_{z_2} = d - 0.4, d_{z_2 y_2} = 0.4$. In other words, the source, the relay and the destination in each channel are on the same line. Now we focus on the scenario 2 where the two possible channels occur randomly with probability $p, 1-p$. In this case, the average rate achievable by the AF strategy is equal to $R_{AF,av} = pR_{AF1} + (1-p)R_{AF2}$. While in our strategy the average rate will be $R_{CF-CF,av} = \max_{0 \leq \alpha \leq 1} \{pR_1 + (1-p)R_2\}$. Based in the definitions, relays are collocated with the destinations, i.e. the CF is the proper cooperative strategy. And the source is moving which is obvious from the definition. We plotted the average rates in 2 based on these assumptions and for $p = 0.9, 0.1$. As one can see the CF-CF strategy outperforms the AF in this case. The current scenario is based on a fast fading assumption. It is clear that the CF-CF strategy proposed here outperforms the AF in the case of quasi-static fading channel. Because it guarantees the transmission of a private rate adapted to each channel using the broadcasting idea. While AF and the time sharing use the fix coding which is basically limited regarding the channel change, in the case quasi-static fading channel. But the results show that the current coding can make a significant improvement compared to AF not only in the quasi-static fading channel but also a fast fading channel which was presented here. This characteristic provides the flexibility over the channel variations.

VI. Summary and Discussion

The two-user simultaneous relay channel with collocated relay and destination nodes was studied. It is assumed that the source node is unaware of the specific channel controlling the communication but knows the set of all possible relay channels. Since the relay and destination nodes are collocated, Compress-and-Forward (CF) strategy turns to be the best known cooperative strategy to transmit information to the destinations. We explained that this problem is equivalent to that of the two-user broadcast relay channel. By using this connection, common information can be transmitted to the destination via any possible channel and simultaneous, private information can be decoded on each of the specific channels. The proposed coding strategy exploits broadcast coding (superposition and Marton coding) and CF coding. The resulting region includes Marton's region for the general broadcast channel and the CF rate for the relay channel. Afterward Gaussian examples was studied for the case of common and private information. Finally, it was shown via numerical results that significant improvement can be made by using our broadcasting coding compared to standard techniques as Amplify-and-Forward (AF) coding.

Acknowledgment

This research is supported by the *Institut Carnot C3S*.

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