Resource Allocation for Cross-Layer Utility Maximization in Multi-Hop Wireless Networks in the Presence of Self Interference

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***Abstract*—The cross-layer utility maximization problem subject to stability constraints is considered for a multi-hop wireless network. A time slotted network, where the channel gains are changing randomly from slot to slot is assumed. The optimal cross-layer network control policy can be decomposed into three subproblems: 1) *flow control*, 2) *next-hop routing and in-node scheduling*, and 3) *power and rate control*, also known as *resource allocation* (RA).**

In the case of multi-hop networks, RA subproblem is particularly difficult to solve due to the *self interference* problem which arises when a node simultaneously transmits and receives in the same channel. According to relative distances between networks nodes, the self interference coefficients can be several order of magnitude larger than the power gains between distinct nodes. Thus, standard RA methods for bipartite networks can not be applied directly.

The main contribution of this paper is to derive a novel RA algorithm for multi-hop wireless networks which is capable to deal with the self interference problem and does not rely on combinatorial constraints for finding the set of links which can be simultaneously activated. The numerical results show that the proposed RA algorithm can provide significant gains at network layer in terms of end-to-end rates and network congestion, even though the solution is local.

Index Terms: Multi-hop wireless networks, self interference, cross-layer optimization, network utility maximization, backpressure, resource allocation, signomial programming.

I. Introduction

The concept of network utility maximization (NUM) was introduced by Kelly et. al. in [1], [2] for fairness control in wireline networks. In [3]–[7] Kelly's NUM framework was extended to cover certain aspects of wireless networks. It has been shown that an optimal cross-layer control policy, which achieves data rates arbitrarily close to the optimal operating point, can be decomposed into three subproblems that are normally associated with different network layers. Specifically, *flow control* resides at the transport layer, *routing and in-node scheduling* resides at the network layer, and *resource allocation* (RA) is usually associated with the medium access control (MAC) and physical (PHY) layers [4].

The flow control and routing/scheduling subproblems are convex optimization problems and can be solved relatively easily. Under reasonably mild assumptions, the RA subproblem can be cast as a general weighted sum-rate maximization over the instantaneous achievable rate region [4], [8]–[10]. The weights of the links are given by the differential backlogs and the policy resembles the well known *backpressure algorithm* introduced by Tassiulas and Ephremides in [11], [12] and further extended in [13], [14] to dynamic networks with power control capabilities. Tutorials [8]–[10] provide detailed surveys on the wireless network control approaches mentioned above.

In the case of wireless networks, the achievable rates on the links are interdependent due to interference, i.e., the achievable rate of a particular link depends on the powers allocated to all other links. This coupling makes the RA subproblem a difficult nonconvex optimization problem [8], [9]. Another challenge, particularly specific to multi-hop networks, is that only certain subsets of links can be simultaneously activated in order to avoid the large *self interference* encountered when a node transmits and receives in the same time [8], [15]–[21]. This typically leads to a *node-exclusive interference model* which often requires combinatorial optimization approaches. Therefore, the problem of finding the global optimum for the RA subproblem is essentially non-tractable, even for off line optimization of moderate size networks.

Several approximations have been proposed for the case when all links in the network operate in certain signal-to-interference-and-noise-ratio (SINR) regions. For instance, the assumption that the achievable rate is a linear function of the SINR (i.e., low SINR region) is widely used in the ultra-wide-band systems [22]–[24]. In addition [3], [25], [26] provide solutions for the power and rate control in low SINR region. The high SINR region is treated in [27]–[29]. The above mentioned methods ignored the *self interference* problem and they fail to solve the general problem, where at the optimal operating point different links correspond to different SINR regions, which is usually the case for the multi-hop networks. The RA subproblem can be cast into a signomial programming (SP) formulation [30, Sec. 9] or into a complementary geometric program (CGP) [31] and locally optimal solutions can be obtained efficiently [32]–[35]. So far, no globally optimal solutions are known for the general case of multi-hop networks operating at arbitrary SINR values [36]. Furthermore, in the case of multi-hop networks, novel techniques specially designed to handle the self interference problem are still required.

In this paper we develop an iterative RA algorithm for multi-hop wireless networks which is capable to deal with the self interference problem and does not rely on combinatorial constraints for finding the set of links which can be simultaneously activated. The numerical results show that the proposed RA algorithm can provide significant gains at network layer in terms of end-to-end rates and network congestion, even though the solution is local.

The rest of the paper is organized as follows. The system model and the problem formulation are presented in Section II. The proposed power and rate control algorithm is presented in Section III. The numerical results are presented in Section IV and Section V concludes our paper.

II. System Model and Problem Formulation

The wireless network consists of a collection of nodes which can send, receive and relay data across wireless links. The set of all nodes is denoted by $\mathcal{N}$ and we label the nodes with the integer values $n = 1, \ldots, N$. A wireless link is represented as an ordered pair (i, j) of distinct nodes. The set of links is denoted by $\mathcal{L}$ and we label the links with the integer values $l = 1, \ldots, L$. We define $tran(l)$ as the transmitter node of link l, and $rec(l)$ as the receiver node of link l. The existence of a link $l \in \mathcal{L}$ implies that a direct transmission is possible from node $tran(l)$ to node $rec(l)$. Furthermore, we define $\mathcal{O}(n)$ as the set of links that are outgoing from node n, and $\mathcal{I}(n)$ as the set of links that are incoming to node n.

The network is assumed to operate in slotted time with slots normalized to integer values $t \in \{1, 2, 3, \ldots\}$. In every time slot, a network controller decides the power and rates allocated to each link. We denote by $p_l(t)$ the power allocated to each link l during time slot t. The power allocation is subject to a maximum power constraint $\sum_{l\in\mathcal{O}(n)} p_l(t) \leq p_n^{\max}$ for each node n. Let $g_{ij}(t)$ denote the power gain from the transmitter of link i to the receiver of link j during time slot t. Note that when $i \in \mathcal{O}(n)$ and $j \in \mathcal{I}(n)$, the term $g_{ij}(t)$ represents the power gain within the same node from its transmitter to its receiver, and is referred to as the self interference gain (see Figure 1). Let us denote the set of all link pairs (i, j) for which the transmitter of link i and the receiver of link j coincide as $\mathcal{A} = \{(i, j)_{i,j\in\mathcal{L}} |\ tran(i) = rec(j)\}$. We let $g_{ij}(t) = 1$ for all link pairs $(i, j) \in \mathcal{A}$ to model the very large self interference that would affect the incoming links of a node if it simultaneously transmitted and received in the same channel. Note that, according to relative distances between network's nodes, these gains can be several order of magnitude larger than the power gains between distinct nodes.

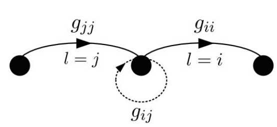

Fig. 1. Self interference for a link pair $(i, j) \in \mathcal{A}$.

In this paper we restrict ourselves to the case where all receivers perform single-user detection (i.e., they decode each of their intended signals by treating all other interfering signals as noise) and assume that the achievable rate of link l during time slot t is given by

$$r_l(t) = \log\left(1 + \frac{g_{ll}(t)p_l(t)}{\sigma^2 + \sum_{j\neq l} g_{jl}(t)p_j(t)}\right), \qquad (1)$$

where σ^2 represents the power of the thermal noise at the receiver. The use of Shannon formula for achievable rate is approximate in the case of finite length packets and is used to avoid the complexity of rate-power dependence in practical modulation and coding schemes. This is a common practice but it must be noted that this is not strictly correct. However, as the packet length increases it is asymptotically correct. Finally, we denote with $\mathbf{p}(t) \in \mathbb{R}_+^L$ the overall power allocation matrix, i.e., $p_l(t) = [\mathbf{p}(t)]_l$.

Exogenous data arrive at the source nodes and they are delivered to the destination nodes over several, possibly multi-hop, paths. We identify the data by their destinations, i.e., all data with the same destination are considered as a single commodity, regardless of its source. We label the commodities with integers $s = 1, \ldots, S$ ($S \leq N$) and the destination node of commodity s is denoted by d_s. For every node, we define $\mathcal{S}_n \subseteq \{1, \ldots, S\}$ as the set of commodities which can arrive exogenously at node n.

We consider a network utility maximization (NUM) framework similar to the ones considered in [4, Sec. III-A] and [9, Sec. 5.1]. Specifically, exogenously arriving data is not directly admitted to the network layer. Instead, the exogenous data is first placed in the transport layer storage reservoirs. At each source node, a set of flow controllers decides the amount of each commodity data admitted every time slot in the network. Let $x_n^s(t)$ denote the amount of data of commodity s admitted in the network at node n during time slot t. At the network layer, each node maintains a set of S internal queues for storing the current backlog (or unfinished work) of each commodity. Let $q_n^s(t)$ denote the current backlog of commodity s data stored at node n. We formally let $q_{d_s}^s(t) = 0$, i.e., it is assumed that data which is successfully delivered to its destination exits the network layer. Let $\bar{x}_n^s$ be the average rate with which the data of commodity s is sent from node n to d_s over the used paths, i.e., $\bar{x}_n^s = \lim_{T\to\infty} 1/T \sum_{t=1}^{T} \mathrm{E}\{x_n^s(t)\}$. Associated with each node-commodity pair $(n, s)_{s\in\mathcal{S}_n}$ we define a concave and non-decreasing *utility function* $g_n^s(\bar{x}_n^s)$, representing the "reward" received by sending data of commodity s from node n to node d_s at an average rate of $\bar{x}_n^s$ [bits/slot]. The NUM problem under stability constraints can be formulated as[1]

$$\begin{array}{ll} \text{maximize} & \sum_{n\in\mathcal{N}} \sum_{s\in\mathcal{S}_n} g_n^s(\bar{x}_n^s) \\ \text{subject to} & \{\bar{x}_n^s | n \in \mathcal{N}, s \in \mathcal{S}_n\} \in \mathbf{\Lambda} \end{array} \qquad (2)$$

where the optimization variables are $\bar{x}_n^s$ and $\mathbf{\Lambda}$ represents the *network layer capacity region*, defined as the closure of the set of all data rates $\{\bar{x}_n^s | n \in \mathcal{N}, s \in \mathcal{S}_n\}$[2] that are stably[3] supportable by the network, considering all possible multi-hop routing and resource allocations [9, Def. 3.7].

A dynamic cross-layer control algorithm which achieves data rates $\bar{x}_n^s$ arbitrarily close to the optimal operating point has been introduced in [4]. Particularized to our network model, in every time slot t, the algorithm performs the following steps:

[1]To avoid complications that may arise which are extraneous to our problem, we assume that all commodities have infinite demand at the transport layer [9, Sec. 5.2]. Nevertheless, the algorithm proposed in this paper is still applicable when this assumption is relaxed.

[2]For convenience this notation is used to describe the rate variables either as a set or as a vector.

[3]A network is strongly stable if all individual queues of the network have bounded time average backlogs [9, Def. 3.2]

Algorithm 1: Dynamic Cross-Layer Control Algorithm [4, Sec. III.A].

1) *Flow Control*: for each node $n \in \mathcal{N}$, $\{x_n^s(t)\}_{s\in\mathcal{S}_n}$ is obtained as the set of rates $\{x_n^s\}_{s\in\mathcal{S}_n}$ which solves the following problem:
$$\begin{array}{ll}\text{maximize} & \sum_{s\in\mathcal{S}_n} V g_n^s(x_n^s) - x_n^s q_n^s(t) \\ \text{subject to} & \sum_{s\in\mathcal{S}_n} x_n^s \le R_n^{\max},\ x_n^s \ge 0,\end{array} \tag{3}$$
where $V > 0$ and $R_n^{\max} > 0$ are the algorithm's parameters as described in [4].
2) *Routing and Scheduling*: for each link l, let $\beta_l(t) = \max_s \left\{q^s_{tran(l)}(t) - q^s_{rec(l)}(t), 0\right\}$. If $\beta_l(t) > 0$, the commodity that maximizes the differential backlog is selected for potential routing over link l.
3) *Resource Allocation* : the power allocation $\mathbf{p}(t)$ is given by $\mathbf{p}$ whose entries p_l solve the following problem
$$\begin{array}{ll}\text{maximize} & \displaystyle\sum_{l\in\mathcal{L}} \beta_l(t)\log\left(1+\frac{g_{ll}(t)p_l}{\sigma^2+\sum_{j\neq l} g_{jl}(t)p_j}\right) \\ \text{subject to} & \sum_{l\in\mathcal{O}(n)} p_l \le p_n^{\max},\ n\in\mathcal{N} \\ & p_l \ge 0,\ l\in\mathcal{L}\ .\end{array} \tag{4}$$
Once the optimal power variables are determined the rate allocation is given by (1).

Note that this algorithm, like all algorithms that we present in the sequel, is fully centralized. The value of such centralized algorithms lies in the establishment of benchmarks for performance assessment. The quest for distributed implementations, although very important, lies outside the scope of the present paper and is left for future research.

III. Resource Allocation Subproblem

In this section we focus on resource allocation (RA) subproblem (4). Let us denote the objective function of problem (4) by $f_0(\mathbf{p})$. It can be expressed as
$$f_0(\mathbf{p}) = \sum_{l\in\mathcal{L}} \log\left(1+\frac{g_{ll}p_l}{\sigma^2+\sum_{j\neq l} g_{jl}p_j}\right)^{\beta_l} \tag{5}$$
$$= -\log\prod_{l\in\mathcal{L}} \left(1+\gamma_l\right)^{-\beta_l}, \tag{6}$$
where the time index t was dropped for the sake of notational simplicity, and γ_l represents the SINR of link l, i.e.,
$$\gamma_l = \frac{g_{ll}p_l}{\sigma^2+\sum_{j\neq l} g_{jl}p_j},\ l\in\mathcal{L}. \tag{7}$$

Since the objective function increases with respect to each γ_l and since $\log(\cdot)$ is an increasing function, problem (4) can be reformulated equivalently as
$$\begin{array}{ll}\text{minimize} & \prod_{l\in\mathcal{L}} \left(1+\gamma_l\right)^{-\beta_l} \\ \text{subject to} & \gamma_l \le \dfrac{g_{ll}p_l}{\sigma^2+\sum_{j\neq l} g_{jl}p_j},\ l\in\mathcal{L} \\ & \sum_{l\in\mathcal{O}(n)} p_l \le p_n^{\max},\ n\in\mathcal{N} \\ & p_l \ge 0,\ l\in\mathcal{L}\ ,\end{array} \tag{8}$$
where the variables now are $\{p_l,\gamma_l\}_{l\in\mathcal{L}}$. Problem (8) is not convex, therefore it is very difficult to solve in general.

A. Successive approximation algorithm for RA in the absence of self interferers [35]

In [35] we proposed a successive approximation algorithm for RA inspired from SP/CGP solution methods [30], [31]. For reasons that will be clear soon in Sec. III-B, this algorithm works well in practice only in the absence of *self interference*. At each step, it solves an approximated version of problem (8) and the algorithm consists of repeating this step until convergence. This algorithm will be used as an auxiliary algorithm in the main iterative solution method that will be presented in Sec. III-B for RA in the presence of *self interference*. Here, we outline the auxiliary algorithm that will be useful in Sec. III-B.

Algorithm 2: Auxiliary Algorithm for RA

1) Given tolerance $\epsilon > 0$, a feasible power allocation $\mathbf{p}_0$; The initial SINR guess $\hat{\boldsymbol{\gamma}}$ is given by (7).
2) Solve the following GP,
$$\begin{array}{ll}\text{minimize} & \prod_{l\in\mathcal{L}} \gamma_l^{-\beta_l\frac{\hat{\gamma}_l}{1+\hat{\gamma}_l}} \\ \text{subject to} & \alpha^{-1}\hat{\gamma}_l \le \gamma_l \le \alpha\hat{\gamma}_l,\ l\in\mathcal{L} \\ & \sigma^2 g_{ll}^{-1}p_l^{-1}\gamma_l + \sum_{j\neq l} g_{ll}^{-1}g_{jl}p_jp_l^{-1}\gamma_l \le 1, \\ & \qquad l\in\mathcal{L} \\ & \sum_{l\in\mathcal{O}(n)} \left(p_n^{\max}\right)^{-1} p_l \le 1,\ n\in\mathcal{N}\end{array} \tag{9}$$
with the positive variables $\{p_l,\gamma_l\}_{l\in\mathcal{L}}$. Denote the solution by $\{p_l^\star,\gamma_l^\star\}_{l\in\mathcal{L}}$.
3) If $\max_{l\in\mathcal{L}} |\gamma_l^\star - \hat{\gamma}_l| > \epsilon$ set $\{\hat{\gamma}_l = \gamma_l^\star\}_{l\in\mathcal{L}}$ and go to Step 2; otherwise STOP.

Algorithm 2 can be used as such for solving the RA subproblem in a particular class of wireless networks, for which $\mathcal{A} = \emptyset$ (recall that $\mathcal{A} = \{(i,j)_{i,j\in\mathcal{L}} |\ tran(i) = rec(j)\}$). We refer to these networks as *bipartite* since the set of nodes can be divided into two distinct subsets, $\mathcal{T}$ and $\mathcal{R}$, such that $\mathcal{T}$ contains only transmitting nodes and $\mathcal{R}$ contains only receiving nodes. For such network topologies, there is no *self interference* problem, and a simple uniform power allocation can be used to initialize *Algorithm* 2.

B. Successive approximation algorithm for RA in the presence of self interferers

Let us now consider the general case where $\mathcal{A} \neq \emptyset$, and, consequently, the RA problem must also cope with the self interference problem. The difficulty comes from the fact that the self interference gains $\{g_{ij}\}_{(i,j)\in\mathcal{A}}$ are typically few order of magnitude larger than the power gains between distinct network nodes $\{g_{jj}\}_{j\in\mathcal{L}}$. Thus, the SINR values at the incoming links of a node that simultaneously transmits in the same channel are very small and the convergence of *Algorithm* 2 becomes very slow if it starts with an initial SINR guess $\hat{\boldsymbol{\gamma}}$ containing entries with nearly zero values.

A standard way to deal with the self interference problem consists of adding a supplementary combinatorial constraint in the RA subproblem which does not allow any node in the network to transmit and receive simultaneously in the same channel [15], [16], [19]. We will refer to a power allocation which satisfies this constraint as *admissible*. Note that this approach would require solving a power optimization problem for each possible subsets of links that can be simultaneously activated. As the complexity of this approach grows exponentially with the number of links, this solution become quickly impractical. To avoid such enormous complexity we propose an iterative method, which runs *Algorithm* 2 for incrementally increasing values of self interference gains.

The outline of the proposed algorithm is as follows. It alternates between two steps: increasing the value of self interference gains, and running *Algorithm* 2 for updated values of self interference gains (the last point found is used as the initial point for *Algorithm* 2 in the next iteration). The algorithm consists of repeating these steps until a stopping criterion is satisfied.

Algorithm 3: Successive approximation algorithm for RA in the presence of self interferers

1) Given an initial self interference gain $g_0 < 1$, $\rho > 1$, an initial (feasible) power allocation $\mathbf{p}_0$; Let $g = g_0$ and $\mathbf{p} = \mathbf{p}_0$.
2) Set $g_{ij} = g$ for all $(i,j) \in \mathcal{A}$.
3) Update SINR guess $\hat{\gamma}$ by using (7) and perform steps 2) and 3) of *Algorithm* 2.
4) If $\exists(i,j) \in \mathcal{A}$ such that $p_i p_j > 0$, then set $g = \min\{\rho g, 1\}$ and go to Step 4, otherwise STOP.
5) If $g < 1$, go to Step 2, otherwise STOP.

The initial self interference gain g_0 is chosen in the same range of values as the power gains between distinct nodes. Specifically, in our simulations we select $g_0 = \max_{j\in\mathcal{L}}\{g_{jj}\}$. For any feasible power allocations $\mathbf{p}_0$, the initial SINR guess $\hat{\gamma}$ is given by (7) where all self interference gains, i.e., $\{g_{ij}|(i,j) \in \mathcal{A}\}$, are replaced by g and gradually increased in each iteration. Note that *Algorithm* 3 terminates either when the power allocation obtained at Step 2 is *admissible* or when the self interference gains $g = 1$ (i.e., the actual value of the self interference gains). Terminating *Algorithm* 3 if the solution is *admissible* is intuitively obvious for the following reason. The data associated with problem (9) become independent of self interference gains and therefore further increase in g after having an *admissible* solution has no effect on the results. Our computational experience suggests that *Algorithm* 3 yields an *admissible* solution way before g reaches value 1 (e.g., by selecting $\rho = 2$, in all our simulations an admissible power allocation is achieved in about $1-4$ iterations).

A simple extension on the method can be used to dramatically decrease the complexity per GP in (9). Here, we eliminate the power variables p_l and the associated SINR variables γ_l from problem (9) when they have relatively very small contributions to the overall objective value of (9). Specifically, the exponent term $\beta_l \frac{\hat{\gamma}_l}{1+\hat{\gamma}_l}$ in the objective of (9) is evaluated for all $l \in \mathcal{L}$ and if $\beta_l \frac{\hat{\gamma}_l}{1+\hat{\gamma}_l} \ll \max_{l'\in\mathcal{L}}\left(\beta_{l'} \frac{\hat{\gamma}_{l'}}{1+\hat{\gamma}_{l'}}\right)$ then p_l's and the associated γ_l's are eliminated in successive GPs.

IV. Numerical Results

In this section, we investigate quantitatively the gains achieved at the network layer when the proposed algorithm is used to solve the RA subproblem of the Dynamic Cross-Layer Control Algorithm (i.e., *Algorithm* 1, Sec. II). Specifically, in every time slot t, the rate allocation at Step 3 of *Algorithm* 1 is obtained by using the proposed RA *Algorithm* 3 described in Sec. III. We assume a block fading Rayleigh channel model, where the channel coefficients are constant during each time slot and change independently from slot-to-slot.

A fully connected multi-hop, multi-commodity wireless network as shown in Figure 2 is considered which is similar to the grid topology considered in [3, Sec. IV]. There are $N = 9$ nodes and $S = 3$ commodities. The commodities arrive exogenously at different nodes in the network as described in Table I. Thus we have $\mathcal{S}_1 = \{2\}, \mathcal{S}_2 = \{3\}, \mathcal{S}_3 = \{3\}, \mathcal{S}_5 = \{2\}, \mathcal{S}_7 = \{1,3\}$, and $\mathcal{S}_i = \emptyset$ for all $i \in \{4,6,8,9\}$. We assume that the average rates $\bar{x}_n^s$ corresponding to all node-commodity pairs $(n,s)_{s\in\mathcal{S}_n}, n \in \mathcal{N}$ are subject to proportional fairness, therefore we select the utility functions $g_n^s(\bar{x}_n^s) = \ln(\bar{x}_n^s)$.

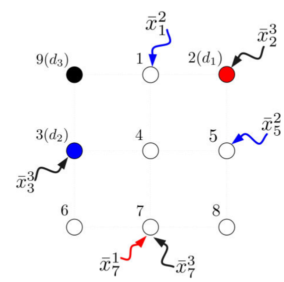

Fig. 2. Multi-hop wireless network with $N = 9$ nodes and $S = 3$ commodities.

The nodes are located in a rectangular grid such that the horizontal and vertical distances between adjacent nodes are D_0 meters [m]. We assume an exponential path loss model, where the channel power gains $g_{ij}(t)$, between distinct nodes are given by $g_{ij}(t) = \left(\frac{d_{ij}}{d_0}\right)^{-\eta} c_{ij}(t)$, where d_{ij} is the distance from the transmitter of link i to the receiver of link j, d_0 is the far field reference distance [37], η is the path loss exponent, and $c_{ij}(t)$ are exponentially distributed random variables with unit mean, independent over the time slots as well as over the links. The first term of $g_{ij}(t)$ represents the path loss factor and the second term models the Rayleigh small-scale fading. We selected $\eta = 4$, $d_0 = 1$m, and $D_0 = 10$m for all simulations. The maximum power constraint is assumed the same for all nodes, i.e., $p_n^{\max} = p_0^{\max}$ for all $n \in \mathcal{N}$. The SNR operating point is defined as $\mathrm{SNR} = \frac{p_0^{\max}}{\sigma^2}\left(\frac{D_0}{d_0}\right)^{-\eta}$.

For comparing different RA algorithms, we consider the

TABLE I
Network commodities, destination nodes, and source nodes

Commodity (s)	Destination node (d_s)	Source nodes
1	2	7
2	3	1, 5
3	9	2, 3, 7

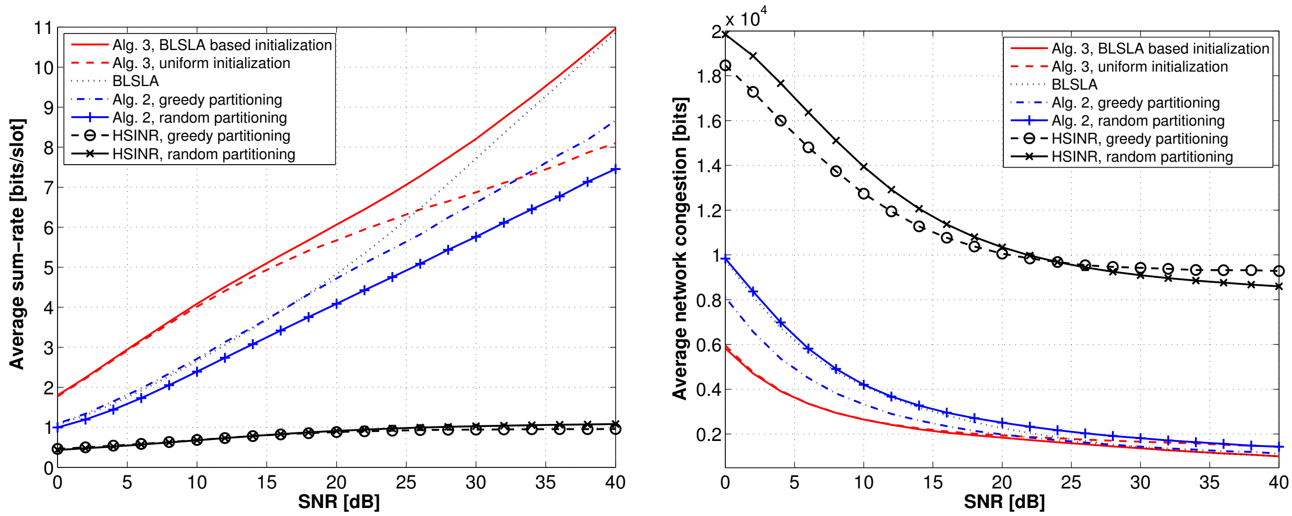

Fig. 3. Dependence of the average sum-rate $\sum_{n=1}^{9}\sum_{s\in\mathcal{S}_n}\bar{x}_n^s$ (left) and of the average network congestion $\sum_{n=1}^{9}\sum_{s=1}^{3}\bar{q}_n^s$ (right) on the SNR.

following two performance metrics: 1) *the average sum-rate* $\sum_{n=1}^{9}\sum_{s\in\mathcal{S}_n}\bar{x}_n^s$ and 2) *the average network congestion* $\sum_{n\in\mathcal{N}}\sum_{s=1}^{S}\bar{q}_n^s$. The Dynamic Cross-Layer Control Algorithm (i.e., *Algorithm* 1) is simulated for $T = 10000$ time slots, and the average rates $\bar{x}_n^s$ and queue sizes $\bar{q}_n^s$ are computed by averaging the last $t_0 = 3000$ time slots, i.e., $\bar{x}_n^s = 1/t_0\sum_{t=T-t_0}^{T}x_n^s(t)$ and $\bar{q}_n^s = 1/t_0\sum_{t=T-t_0}^{T}q_n^s(t)$. In all considered setups, we selected $V = 100$ (in eq. (3)) and the parameters $R_n^{\max}$ (in eq. (3)) were chosen such that all conditions presented in [4, Sec. III-D] were satisfied.

Figure 3 shows the dependence of the average sum-rate (left) and of the average network congestion (right) on the SNR for several RA algorithms. First we have considered the optimal *base line single link activation* (BLSLA) policy[4] and the proposed *Algorithm* 3 with two initialization methods: 1) Uniform initialization and 2) BLSLA based initialization. In the case of uniform initialization the initial power allocation $\mathbf{p}_0$ is chosen such that $[\mathbf{p}_0]_l = p_0^{\max}/(|\mathcal{O}_{tran(l)}|)$. In the case of BLSLA based initialization the initial power allocation $\mathbf{p}_0$ is chosen such that $[\mathbf{p}_0]_{l^\star} : [\mathbf{p}_0]_j = M : 1$ for all $j \in \mathcal{L},\ j \neq l^\star$ where $l^\star$ is the index of the active link obtained based on the optimal BLSLA policy and $M \gg 1$ is a real number.

For comparison, we also plot the results for the low complex approaches where the set of nodes $\mathcal{N}$ is first partitioned into two disjoint subsets, the set of transmitting nodes $\mathcal{T}$ and the set of receiving nodes $\mathcal{R}$ and then either *Algorithm* 2 or a commonly used high SINR (HSINR) approximation[5] [27] is used for the RA subproblem. The partitioning the set of nodes $\mathcal{N}$ in to two disjoint subsets is performed using two simple methods: 1) random partitioning and 2) greedy partitioning based on differential backlogs. In random partitioning, each node is allocated either to $\mathcal{T}$ or to $\mathcal{R}$ with equal probabilities. The greedy partitioning is performed as follows: we start with an empty set of links $\bar{\mathcal{L}} = \emptyset$. At each step, the link $l^\star$ from the set $\mathcal{L}\setminus\bar{\mathcal{L}}$ which has the largest differential backlog β_l (i.e., $l^\star = \arg\max_{l\in\mathcal{L}\setminus\bar{\mathcal{L}}}\beta_l$) is added to the set $\bar{\mathcal{L}}$. Then all links outgoing from $rec(l^\star)$ and all links incoming to $tran(l^\star)$ are deleted from $\mathcal{L}$. This procedure continues until there are no links left in $\mathcal{L}\setminus\bar{\mathcal{L}}$. The sets $\mathcal{T}$ and $\mathcal{R}$ can be found as $\mathcal{T} = \{tran(l)|l \in \bar{\mathcal{L}}\}$ and $\mathcal{R} = \{rec(l)|l \in \bar{\mathcal{L}}\}$.

From Figure 3 we make the following observations. First, *Algorithm* 3 with BLSLA based initialization yields results better than any other counterpart. In contrast, *Algorithm* 3 with uniform initialization exhibits significant deviations from the BLSLA solution at high SNR especially in the terms of average sum-rate. This type of behavior is indeed expected due to the nonconvexity of the RA problem. Moreover, it is important to remark that at low and moderate values of SNR, results due to *Algorithm* 3 are not significantly affected by the initialization method. We also observe that, *Algorithm* 3 with a proper initialization can significantly outperforms *Algorithm* 2 in conjunction with either random or greedy partitioning. This elaborates the importance of gradual self interference gain increments (i.e., Step 4 of *Algorithm* 3) in finding a better RA compared to the direct application of *Algorithm* 2 with a heuristic partitioning. In most cases there is no advantage to using HSINR approximation. These observations are very useful in practice since they illustrate that *Algorithm* 3 often work well when initialized with a reasonable starting point (e.g., BLSLA based initialization). In addition, we note that even with a very simple initialization, e.g., uniform initialization, *Algorithm* 3 yields substantial gains especially at small and moderate SNR values (e.g., 0dB - 20dB). Results suggest that the proposed algorithm deals well with the self interference. It often yields designs that are far superior to those obtained by employing simple extensions of certain RA techniques which are not specifically designed to handle the self interference problem (e.g., combinations of *Algorithm* 2 or HSINR approximation with heuristic node partition methods).

[4]A channel access policy where during each time slot only one link is activated is called BLSLA policy. The optimal BLSLA policy can be found easily: it consists of activating during each time slot only the link which achieves the maximum weighted rate [38].

[5]The achievable rates $\log(1+\gamma_l)$ are approximated by $\log(\gamma_l)$. Hence, the objective function of problem (8) is approximated by $\prod_{l\in\mathcal{L}}\gamma_l^{-\beta_l}$. This results in a convex approximation (i.e., a GP) of problem (8).

V. Conclusions

We have considered the power and rate control problem for multi-hop wireless networks in conjunction with the next-hop routing / scheduling and flow control problem. Thus, although our focus lies on the so-called resource allocation problem in the presence of self interference that is confined to the physical / MAC layers, its formulation captures the interactions with the higher-layers in a manner similar to the one employed in [4]. The result is a cross layer formulation. The problem, unfortunately, is very complex due to the lack of convexity and due to the combinatorial nature of the transmitter nodes selection in the case of multi-hop networks. Based on our previous work [35], we offered a new optimization methodology which is capable to handle the self interference problem and does not rely on combinatorial constraints for finding the set of links which can be simultaneously activated.

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