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***Abstract*—We consider the resource allocation (RA) problem of maximizing the weighted sum of the minimal user rates (WSMR) of coordinated cells subject to a total power constraint at each base station (BS) in the downlink of a cellular orthogonal frequency division multi-access (OFDMA) system. In particular, the solution of this problem corresponds to a RA that guarantees similar rates to users in each cell. We propose a coordinate ascent (CA) based algorithm which alternatively updates the power allocation by solving a successive set of convex optimization problems with a duality-based numerical algorithm, and the subcarrier allocation by solving a mixed integer linear program for each cell. The effectiveness of the algorithm is illustrated by numerical experiments.**

***Index Terms*—Orthogonal frequency division multiplexing, resource allocation, cochannel interference mitigation, spectrum management.**

I. Introduction

Next-generation cellular systems envision full spectrum reuse to meet higher data-rate requirements. To this end, joint resource allocation (RA) among coordinated base stations (BSs) is a promising way to mitigate cochannel interference [1]. In this paper, we consider the joint RA based BS coordination in the downlink of a cellular orthogonal frequency division multi-access (OFDMA) system.

Lately, a few margin-adaptive joint RA algorithms have been proposed for the downlink of cellular OFDMA systems [2], [3]. In case that the joint RA is optimized for each subcarrier individually, the rate-adaptive algorithms in [4]–[6] can be adopted. However, a joint RA over all subcarriers is preferred in order to better exploit the frequency and multiuser diversity inherent in OFDMA systems. To this end, two rate-adaptive joint RA algorithms, which aim to maximize the weighted sum-rate of all users, have been proposed in [7] and [8]. In particular, the weight of each user represents the priority of this user's rate. However, it is not clear how to determine the weight of each user so as to fulfill a prescribed fairness criterion among users.

Motivated by the applications requiring the maximum fairness among users in each cell, we consider a novel RA problem which aims to maximize the weighted sum of the minimal user rates (WSMR) of coordinated cells subject to a total power constraint at each BS. Most interestingly, the optimum RA guarantees the maximum fairness in each cell, i.e., the users in each cell have similar rates. In a previous contribution, we proposed a coordinate ascent (CA) based iterative algorithm which optimizes the subcarrier allocation and the power allocation alternatively, so that the WSMR keeps increasing until convergence [9]. At each iteration, the subcarrier allocation is updated by solving a mixed integer linear program (MILP) for each cell, then the power allocation is updated by solving a successive set of convex-optimization problems. To solve these problems, we proposed a heuristic iterative method based on Karush-Kuhn-Tucker (KKT) conditions in [9]. Though simple and effective in case of convergence, this KKT-based method is not guaranteed to converge.

In this paper, we make improvement on the power allocation algorithm proposed in [9]. Specifically, our contributions lie in the following points:

- We add extra constraints to each convex-optimization problem involved, so as to ensure that its solution set is nonempty and the successive optimization process of the power allocation algorithm can proceed successfully.
- We propose a duality-based method to solve each convex-optimization problem involved. Specifically, this method particularizes the subgradient method to find the dual optimum, and its convergence is guaranteed provided that the stepsize is sufficiently small.

The rest of this paper is organized as follows. In the next Section, we present the system model as well as a concave lower bound of user rate. In Section III, we formulate the RA problem and describe the general framework of the proposed algorithm. In Section IV and V, we develop the methods for initialization and subcarrier allocation, as well as the power allocation algorithm, respectively. In Section VI, the effectiveness of the proposed algorithm is illustrated by numerical experiments. Finally, some conclusions wrap up this paper in Section VII.

II. System model and a lower bound of user rate

A. System model

We consider the downlink of a cellular OFDMA system with K BSs coordinated by a central controller running the proposed RA algorithm. Using OFDMA with N subcarriers, BS k serves a group of users belonging to the user set $\mathcal{U}_k$ for cell k. We denote the channel gain of subcarrier n from BS l to a user u in cell k by $g^l_{u,k,n}$, and we assume $g^l_{u,k,n} > 0$, $\forall u \in \mathcal{U}_k$, $\forall k, l$, $\forall n$. In addition, $\sigma^2_{u,k,n}$ represents the noise-plus-uncoordinated-interference power for subcarrier n at user u in cell k.

We make the following assumptions for the considered system. First, the number of users in each cell is smaller than N, and each subcarrier is allocated to only one user exclusively in each cell. Second, we assume the controller knows $\{g^l_{u,k,n}, \forall u, \forall k, l, \forall n\}$ and $\{\sigma^2_{u,k,n}, \forall u, \forall k, \forall n\}$ precisely. Third, we assume the channel between each BS-user pair remains unchanged for a sufficiently long duration, so that the RA is carried out accordingly for that duration.

We denote the total power available to BS k by $P_{k,\max}$. $P_{k,n}$ represents the proportion of $P_{k,\max}$ allocated to subcarrier n by BS k. We stack $\{P_{k,n}\}_{k=1}^K$ into a $K \times 1$ vector $\mathbf{p}_n = [P_{1,n}, \cdots, P_{K,n}]^T$, and then stack $\{\mathbf{p}_n\}_{n=1}^N$ into a $K \times N$ matrix $\mathbf{P}$ satisfying $[\mathbf{P}]_{k,n} = [\mathbf{p}_n]_{k,1} = P_{k,n}$. Note that every possible $\mathbf{P}$ belongs to the set $\boldsymbol{\mathcal{P}}_{\mathrm{f}} = \{\mathbf{P} | \forall k, \forall n, P_{k,n} \in [0,1]\}$. We define a binary variable $A_{u,k,n}$, which indicates that subcarrier n is allocated to user u in cell k if $A_{u,k,n} = 1$. We stack $\{A_{u,k,n}\}_{n=1}^N$ into the vector $\mathbf{A}_{u,k} = [A_{u,k,1}, \cdots, A_{u,k,N}]^T$, and then stack $\mathbf{A}_{u,k}$'s of all users in cell k to a matrix $\mathbf{A}_k$ column by column. Finally, we define a matrix $\mathbf{A} = [\mathbf{A}_1, \cdots, \mathbf{A}_K]$ which indicates how subcarriers are allocated to all users.

The rate of user u in cell k, as a function of $\mathbf{A}_{u,k}$ and $\mathbf{P}$, is expressed by

$$R_{u,k}(\mathbf{A}_{u,k}, \mathbf{P}) = \sum_{n=1}^N A_{u,k,n} \cdot R_{u,k,n}(\mathbf{p}_n) \tag{1}$$

where $R_{u,k,n}(\mathbf{p}_n)$ stands for the rate of subcarrier n when allocated to user u in cell k. Note that $R_{u,k,n}(\mathbf{p}_n)$, as a function of $\mathbf{p}_n$, is calculated by

$$R_{u,k,n}(\mathbf{p}_n) = \ln\left(1 + \frac{\gamma_{u,k,n}(\mathbf{p}_n)}{\Gamma}\right) \tag{2}$$

in the unit of nats per OFDM symbol, where $\gamma_{u,k,n}(\mathbf{p}_n)$ is the signal-to-noise-plus-interference ratio, and Γ represents the signal-to-noise-ratio gap between the adopted modulation and coding scheme and the one achieving channel capacity. Particularly, $\gamma_{u,k,n}(\mathbf{p}_n)$ is evaluated by

$$\begin{aligned}\gamma_{u,k,n}(\mathbf{p}_n) &= \frac{P_{k,\max} P_{k,n} g^k_{u,k,n}}{\sigma^2_{u,k,n} + \sum_{l=1\backslash k}^K P_{l,\max} P_{l,n} g^l_{u,k,n}} \\ &= \frac{P_{k,n}}{I_{u,k,n}(\mathbf{p}_n)}\end{aligned} \tag{3}$$

where $I_{u,k,n}(\mathbf{p}_n) = x_{u,k,n} + \sum_{l=1\backslash k}^K P_{l,n} y^l_{u,k,n}$, $x_{u,k,n} = \frac{\sigma^2_{u,k,n}}{P_{k,\max} g^k_{u,k,n}}$, and $y^l_{u,k,n} = \frac{P_{l,\max} g^l_{u,k,n}}{P_{k,\max} g^k_{u,k,n}}$.

B. A concave lower bound of user rate

We first define

$$\underline{R}_{u,k,n}(e^{\mathbf{q}}, \mathbf{p}^*_n) = \alpha^*_{u,k,n} \cdot \ln(\gamma_{u,k,n}(e^{\mathbf{q}})) + \beta^*_{u,k,n} \tag{4}$$

$$\alpha^*_{u,k,n} = \frac{\gamma_{u,k,n}(\mathbf{p}^*_n)}{\Gamma + \gamma_{u,k,n}(\mathbf{p}^*_n)} \tag{5}$$

$$\beta^*_{u,k,n} = R_{u,k,n}(\mathbf{p}^*_n) - \alpha^*_{u,k,n} \cdot \ln(\gamma_{u,k,n}(\mathbf{p}^*_n)) \tag{6}$$

where $\mathbf{q}$ is a $K \times 1$ vector, $\mathbf{P}^*$ is a given power allocation with its n-th column denoted by $\mathbf{p}^*_n$. In particular, $e^{\mathbf{X}}$ is a matrix mapped from $\mathbf{X}$ entrywise through the function $y = e^x$. Note that $\mathbf{P}^*$ should belong to the set $\boldsymbol{\mathcal{P}}_+ = \{\mathbf{P} | \forall k, \forall n, P_{k,n} \in (0,1]\}$.

We now define a function

$$\underline{R}_{u,k}(\mathbf{A}_{u,k}, e^{\mathbf{Q}}, \mathbf{P}^*) = \sum_{n=1}^N A_{u,k,n} \underline{R}_{u,k,n}(e^{\mathbf{q}_n}, \mathbf{p}^*_n) \tag{7}$$

where $\mathbf{Q}$ is a $K \times N$ matrix with its n-th column denoted by $\mathbf{q}_n$. We have shown in [9] that $\underline{R}_{u,k}(\mathbf{A}_{u,k}, e^{\mathbf{Q}}, \mathbf{P}^*)$ is a concave function of $\mathbf{Q}$ as well as a lower bound of $R_{u,k}(\mathbf{A}_{u,k}, e^{\mathbf{Q}})$. Besides, this bound is tight when $e^{\mathbf{Q}} = \mathbf{P}^*$.

III. Problem formulation and overall algorithm

We consider the RA problem that maximizes the WSMR subject to a total power constraint for each BS, namely

$$\begin{aligned}\max_{\mathbf{A},\mathbf{P}} \quad & f(\mathbf{A}, \mathbf{P}) = \textstyle\sum_{k=1}^K w_k \cdot \min_{u \in \mathcal{U}_k} R_{u,k}(\mathbf{A}_{u,k}, \mathbf{P}) \\ \text{s.t.} \quad & \mathbf{P} \in \boldsymbol{\mathcal{P}}_{\mathrm{f}}, \\ & \sum_{n=1}^N P_{k,n} \le 1 \qquad \forall\, k, \\ & \sum_{u \in \mathcal{U}_k} A_{u,k,n} \le 1 \qquad \forall\, k, \forall\, n, \\ & A_{u,k,n} \in \{0,1\} \qquad \forall\, u \in \mathcal{U}_k, \forall\, k, \forall\, n.\end{aligned} \tag{8}$$

where $w_k > 0$ represents the weight assigned to cell k's minimal user rate. In particular, increasing w_k leads to a higher priority assigned to the users in cell k.

Algorithm 1 Overall RA optimization algorithm

Initialize: $m = 0$, compute $\mathbf{P}^0$ and $\mathbf{A}^0$ using the initialization algorithm in Subsection IV-A;
repeat
Use the power allocation algorithm proposed in Subsection V-A to find a suboptimal solution for (8) with $\mathbf{A} = \mathbf{A}^m$, and assign it to $\mathbf{P}^{m+1}$;
Use the subcarrier allocation algorithm proposed in Subsection IV-B to solve (8) with $\mathbf{P} = \mathbf{P}^{m+1}$ for the optimal $\mathbf{A}$, and assign it to $\mathbf{A}^{m+1}$;
$m = m + 1$;
until $||\mathrm{vec}(\mathbf{P}^m - \mathbf{P}^{m-1})|| \le \Omega_1$ **or** $m = M$
Output: $\mathbf{P}^m$ and $\mathbf{A}^m$ are adopted for the considered system.

We propose a CA based iterative algorithm which optimizes the subcarrier allocation and the power allocation alternatively. This algorithm is depicted in **Algorithm 1**, where the superscript m indicates the associated variable is produced after the m-th iteration. Note that the algorithm yields non-decreasing WSMRs as the iterations continue. Since each BS's total power is upper bounded, the optimal WSMR is upper bounded as well, and therefore the WSMR yielded will not increase infinitely as the iterations proceed. We stop the iteration when $||\mathrm{vec}(\mathbf{P}^m - \mathbf{P}^{m-1})||$ is smaller than a prescribe small positive value Ω_1, or when the number of iterations exceeds a prescribed value M. Here $||\cdot||$ and $\mathrm{vec}(\cdot)$ represent the Euclidean norm and the vectorization operator, respectively. Finally, $\mathbf{P}^m$ and $\mathbf{A}^m$ produced by the last iteration are adopted as a suboptimal solution to (8).

IV. Initialization and subcarrier allocation algorithm

A. Initialization methods

We use the uniform power allocation (UPA) for initialization, i.e., $[\mathbf{P}^0]_{k,n} = \frac{1}{N}$, $\forall n$, $\forall k$. We consider two different methods for computing $\mathbf{A}^0$. One is to let users in each cell to choose in turn the subcarrier that is unassigned and corresponds to the highest SNIR. In this way, each user will be allocated almost the same number of subcarriers, and we refer to this method as the even subcarrier allocation (ESA). The other one is to compute $\mathbf{A}^0$ as the best subcarrier allocation (BSA) that maximizes the WSMR when $\mathbf{P} = \mathbf{P}^0$. Note that this subcarrier allocation can be obtained by solving a MILP problem for each cell, which will be elaborated in Section IV-B.

B. Subcarrier allocation optimization

The problem of finding the optimal subcarrier allocation $\mathbf{A}^{m+1}$ when $\mathbf{P} = \mathbf{P}^{m+1}$ can be decomposed into K subproblems, the k-th of which is to find the optimal subcarrier allocation for cell k:

$$\begin{array}{lll} \max\limits_{\mathbf{A}_k} & \min\limits_{u\in\mathcal{U}_k} R_{u,k}(\mathbf{A}_{u,k}, \mathbf{P}^{m+1}) & \\ \text{s.t.} & \sum\limits_{u\in\mathcal{U}_k} A_{u,k,n} \le 1, & \forall n, \\ & A_{u,k,n} \in \{0,1\}, & \forall u \in \mathcal{U}_k, \forall n. \end{array} \tag{9}$$

Let's denote the optimal $\mathbf{A}_k$ for (9) by $\mathbf{A}_k^{m+1}$. Then the optimal subcarrier allocation is constructed by $\mathbf{A}^{m+1} = [\mathbf{A}_1^{m+1}, \cdots, \mathbf{A}_K^{m+1}]$. In fact, (9) can be reformulated into an equivalent form:

$$\begin{array}{lll} \max\limits_{\mathbf{A}_k, S_k} & S_k & \\ \text{s.t.} & \sum\limits_{n=1}^{N} A_{u,k,n}\cdot R_{u,k,n}(\mathbf{p}_n^{m+1}) \ge S_k, \ \forall u \in \mathcal{U}_k, & \\ & \sum\limits_{u\in\mathcal{U}_k} A_{u,k,n} \le 1, & \forall n, \\ & A_{u,k,n} \in \{0,1\}, & \forall u \in \mathcal{U}_k, \forall n. \end{array} \tag{10}$$

where $\mathbf{p}_n^{m+1}$ is the n-th column of $\mathbf{P}^{m+1}$, and S_k is the extra optimization variable introduced to guarantee the equivalence between (9) and (10). Note that (10) is a MILP problem involving both integer and continuous variables. In practice, we can solve it by using a commercial optimization software such as CPLEX.

V. Power allocation algorithm

A. Successive convex optimization algorithm for power allocation

When $\mathbf{A} = \mathbf{A}^m$, we use $R_{u,k}^m(\mathbf{P})$ and $\underline{R}_{u,k}^m(e^{\mathbf{Q}}, \mathbf{P}^*)$ to represent respectively the rate of user u in cell k and $\underline{R}_{u,k}(\mathbf{A}_{u,k}^m, e^{\mathbf{Q}}, \mathbf{P}^*)$ for notation simplicity. Here, $\mathbf{A}_{u,k}^m$ represents the $\mathbf{A}_{u,k}$ contained in $\mathbf{A}^m$. In the following, we denote the user in cell k scheduled for subcarrier n by $u_{k,n}^m$, and we define the sets $\boldsymbol{\mathcal{P}}_\epsilon = \{\mathbf{P}|\forall k, \forall n, P_{k,n} \in [e^\epsilon, 1]\}$ and $\boldsymbol{\mathcal{Q}}_\epsilon = \{\mathbf{Q}|\forall k, \forall n, q_{k,n} \in [\epsilon, 0]\}$, where $q_{k,n} = [\mathbf{Q}]_{k,n}$ and ϵ is a prescribed negative value.

To develop a method for optimizing the power allocation, let's consider a parameterized convex optimization problem

$$\begin{array}{lll} \min\limits_{\mathbf{Q}, \{C_k\}_{k=1}^K} & -\sum\limits_{k=1}^{N} w_k \cdot C_k & \\ \text{s.t.} & \mathbf{Q} \in \boldsymbol{\mathcal{Q}}_\epsilon, & \\ & C_k - \underline{R}_{u,k}^m(e^{\mathbf{Q}}, \mathbf{P}^*) \le 0, & \forall u \in \mathcal{U}_k, \forall k, \\ & \sum\limits_{n=1}^{N} e^{q_{k,n}} - 1 \le 0, & \forall k \end{array} \tag{11}$$

According to Theorem 2 in [10], the solution set to (11) is nonempty. Let's denote a solution to (11) by $\mathbf{Q}_\mathrm{o}$ and suppose $\mathbf{P}^*$ is feasible for (8) and $\mathbf{P}^* \in \boldsymbol{\mathcal{P}}_\epsilon$, $\mathbf{P}_\mathrm{o} = e^{\mathbf{Q}_\mathrm{o}}$ is feasible for (8) and $f(\mathbf{A}^m, \mathbf{P}_\mathrm{o}) \ge f(\mathbf{A}^m, \mathbf{P}^*)$ holds. Capitalizing on these facts, we propose an iterative power-allocation optimization algorithm depicted in **Algorithm 2**, which introduces an inner loop nested in the overall algorithm, to find a power allocation no worse than $\mathbf{P}^m$ when $\mathbf{A} = \mathbf{A}^m$. Note that T and Ω_2 are a prescribed integer and a small positive value, respectively.

Algorithm 2 Power allocation optimization algorithm

Initialize: $t = 0$, $\mathbf{P}^{m,0} = \mathbf{P}^m$;
  repeat
    Solve (11) with $\mathbf{P}^* = \mathbf{P}^{m,t}$ for $\mathbf{Q}_\mathrm{o}$ using **Algorithm 3 proposed in Section V-B**;
    $\mathbf{P}^{m,t+1} = e^{\mathbf{Q}_\mathrm{o}}$;
    $t = t + 1$;
  until $||\mathrm{vec}(\mathbf{Q}_\mathrm{o} - \mathbf{Q}^*)|| \le \Omega_2$ **or** $t = T$
Output: $\mathbf{P}^{m+1} = \mathbf{P}^{m,T}$.

We can see that **Algorithm 2** produces a set of power allocations yielding nondecreasing WSMRs. When the WSMR stops increasing, $f(\mathbf{A}^m, \mathbf{P}_\mathrm{o}) = f(\mathbf{A}^m, \mathbf{P}^*)$ holds. In this case, $\mathbf{P}^*$ might be a local minimum of (8) with $\mathbf{A} = \mathbf{A}^m$ and $\mathbf{P} \in \boldsymbol{\mathcal{P}}_\epsilon$, since the KKT conditions are satisfied by $\mathbf{P}^*$ according to Theorem 3 in [10].

In fact, **Algorithm 2** iteratively looks for a suboptimal power allocation in $\boldsymbol{\mathcal{P}}_\epsilon$, while a possible $\mathbf{P}$ actually lies in a bigger set $\boldsymbol{\mathcal{P}}_\mathrm{f} \supset \boldsymbol{\mathcal{P}}_\epsilon$. However, we can not replace $\boldsymbol{\mathcal{P}}_\epsilon$ by $\boldsymbol{\mathcal{P}}_\mathrm{f}$, because we can not construct (11) for a $\mathbf{P}^* \in \boldsymbol{\mathcal{P}}_\mathrm{f}$ with zero entries as explained in Section II-B. We can not replace $\boldsymbol{\mathcal{P}}_\epsilon$ by $\boldsymbol{\mathcal{P}}_+$ either, since when $\boldsymbol{\mathcal{Q}}_+$ is used the set of feasible $\mathbf{Q}$'s is unbounded, and the existence of an optimal solution to (11) is not guaranteed [11]. On one hand, e^ϵ should not be too small to be rounded to zero by the central controller. On the other hand, ϵ should be much smaller than 0, so that $\boldsymbol{\mathcal{P}}_\epsilon$ is a good approximation to $\boldsymbol{\mathcal{P}}_+$ and **Algorithm 1** is able to start with $\mathbf{P}^0 \in \boldsymbol{\mathcal{P}}_\epsilon$.

B. A duality-based algorithm to solve (11)

We denote the Lagrange multipliers related to the rate constraint of user u in cell k and the power constraint of BS k by $\lambda_{u,k}$ and μ_k, respectively. We stack the $\lambda_{u,k}$'s related to all users in cell k into a row vector $\boldsymbol{\lambda}_k$, and stack the μ_k's related to all BSs into a row vector $\boldsymbol{\mu} = [\mu_1, \cdots, \mu_K]$. We then stack all $\boldsymbol{\lambda}_k$'s and $\boldsymbol{\mu}$ into a row vector $\boldsymbol{\rho} = [\boldsymbol{\lambda}_1, \cdots, \boldsymbol{\lambda}_K, \boldsymbol{\mu}]$.

We define the Lagrangian of (11) as

$$D(\boldsymbol{\rho}) = \min_{\mathbf{Q}\in\boldsymbol{\mathcal{Q}}_\epsilon, \mathbf{C}} \left[h(\mathbf{Q},\boldsymbol{\rho}) + \sum_{k=1}^{K}(\sum_{u\in\mathcal{U}_k}\lambda_{u,k} - w_k)C_k\right] \tag{12}$$

where $\mathbf{C} = [C_1, \cdots, C_K]$ and

$$h(\mathbf{Q},\boldsymbol{\rho}) = \sum_{k=1}^{K}\mu_k(\sum_{n=1}^{N} e^{q_{k,n}} - 1) - \sum_{k=1}^{K}\sum_{u\in\mathcal{U}_k}\lambda_{u,k}\underline{R}_{u,k}^m(e^{\mathbf{Q}}, \mathbf{P}^*) \tag{13}$$

Note that the domain of $D(\boldsymbol{\rho})$ is $\boldsymbol{\mathcal{D}}_{\boldsymbol{\rho}} = \{\boldsymbol{\rho}|D(\boldsymbol{\rho}) > -\infty; \forall k, \mu_k \geq 0; \forall u \in \mathcal{U}_k, \forall k, \lambda_{u,k} \geq 0\}$ [11]. Since $\forall k$ C_k can take any real value for the minimization operator in (12), $\sum_{u\in\mathcal{U}_k}\lambda_{u,k} = w_k, \forall k$ must be fulfilled so as to guarantee that $D(\boldsymbol{\rho}) > -\infty$. As a result, $D(\boldsymbol{\rho})$ can be reduced to

$$D(\boldsymbol{\rho}) = \min_{\mathbf{Q}\in\boldsymbol{\mathcal{Q}}_\epsilon} h(\mathbf{Q},\boldsymbol{\rho}) \tag{14}$$

and $\boldsymbol{\mathcal{D}}_{\boldsymbol{\rho}}$ is reduced to $\{\boldsymbol{\rho}|\forall k, \boldsymbol{\lambda}_k \in \boldsymbol{\mathcal{D}}_k; \boldsymbol{\mu} \in \boldsymbol{\mathcal{D}}_{\boldsymbol{\mu}}\}$, where $\boldsymbol{\mathcal{D}}_{\boldsymbol{\mu}} = \{\boldsymbol{\mu}|\forall k, \mu_k \geq 0\}$, and $\boldsymbol{\mathcal{D}}_k = \{\boldsymbol{\lambda}_k|\sum_{u\in\mathcal{U}_k}\lambda_{u,k} = w_k; \forall u \in \mathcal{U}_k, \lambda_{u,k} \geq 0\}$. Apparently, $\boldsymbol{\mathcal{D}}_{\boldsymbol{\mu}}$ and $\boldsymbol{\mathcal{D}}_k$ are both convex, and thus $\boldsymbol{\mathcal{D}}_{\boldsymbol{\rho}}$ is also a convex set.

Let's define $\mathbf{Q}_{\boldsymbol{\rho}} = \arg\min_{\mathbf{Q}\in\boldsymbol{\mathcal{Q}}_\epsilon} h(\mathbf{Q},\boldsymbol{\rho})$. Specifically, the duality-based algorithm to solve (11) is to first find the optimal dual variable $\boldsymbol{\rho}^\star = \arg\max_{\boldsymbol{\rho}\in\boldsymbol{\mathcal{D}}_{\boldsymbol{\rho}}} D(\boldsymbol{\rho})$, and then compute $\mathbf{Q}_{\boldsymbol{\rho}^\star}$ as the optimal solution to (11). We propose **Algorithm 3** which particularizes the subgradient method in [11] to find $\boldsymbol{\rho}^\star$. Here, $\boldsymbol{\rho}^i$ represents the dual variable produced after the i-th iteration, $\boldsymbol{\eta}^i$ denotes the subgradient of $D(\boldsymbol{\rho})$ when $\boldsymbol{\rho} = \boldsymbol{\rho}^i$, and δ and Δ_1 are both prescribed small positive values.

Algorithm 3 The duality-based algorithm to solve (11).

Initialize: $i = 0$; compute $\boldsymbol{\rho}^0$ in which $\forall u \in \mathcal{U}_k, \forall k, {\lambda_{u,k}}^0 = \frac{w_k}{|\mathcal{U}_k|}$ and $\forall k, {\mu_k}^0 = 1$.
  repeat
    Compute $\mathbf{Q}_{\boldsymbol{\rho}^i}$ using the algorithm proposed in Section V-C;
    Compute $\boldsymbol{\eta}^i$ and $\boldsymbol{\rho}^{i+1} = \arg\min_{\boldsymbol{\rho}\in\boldsymbol{\mathcal{D}}_{\boldsymbol{\rho}}} ||\boldsymbol{\rho}^i + \delta\cdot\boldsymbol{\eta}^i - \boldsymbol{\rho}||^2$.
    $i = i + 1$;
  until $||\boldsymbol{\rho}^i - \boldsymbol{\rho}^{i-1}|| \leq \Delta_1$
Output: $\mathbf{Q}_{\boldsymbol{\rho}^i}$ is the optimal solution to (11).

Note that $|\mathcal{U}_k|$ is the number of users in cell k, $\boldsymbol{\eta}^i$ has the same size as $\boldsymbol{\rho}$, and the entries in $\boldsymbol{\eta}^i$ associated with $\lambda_{u,k}$ and μ_k are equal to $-\underline{R}_{u,k}^m(e^{\mathbf{Q}_{\boldsymbol{\rho}^i}}, \mathbf{P}^*)$ and $\sum_{n=1}^{N} e^{[\mathbf{Q}_{\boldsymbol{\rho}^i}]_{k,n}} - 1$, respectively. Using the associated KKT conditions, we can compute $\boldsymbol{\rho}^{i+1}$ by

$$\lambda_{u,k}^{i+1} = [\lambda_{u,k}^i - \delta\cdot\underline{R}_{u,k}^m(e^{\mathbf{Q}_{\boldsymbol{\rho}^i}}, \mathbf{P}^*) + \xi_k^i]^+, \forall u \in \mathcal{U}_k, \forall k \tag{15}$$

$$\mu_k^{i+1} = [\mu_k^i + \delta\cdot(\sum_{n=1}^{N} e^{[\mathbf{Q}_{\boldsymbol{\rho}^i}]_{k,n}} - 1)]^+, \forall k \tag{16}$$

where the superscript i indicates the associated dual variable is an entry in $\boldsymbol{\rho}^i$, $[x]^+ = \max\{0, x\}$, and ξ_k^i is a real value that satisfies $\sum_{u\in\mathcal{U}_k}\lambda_{u,k}^{i+1} = w_k$. We find the value of ξ_k^i by the bisection method, since $\sum_{u\in\mathcal{U}_k}\lambda_{u,k}^{i+1}$ increases monotonically with respect to ξ_k^i.

Provided that δ is sufficiently small, the convergence of **Algorithm 3** is guaranteed [11]. We terminate the iterations when $||\boldsymbol{\rho}^i - \boldsymbol{\rho}^{i-1}||$ is smaller than a prescribed small positive value Δ_1, then $\mathbf{Q}_{\boldsymbol{\rho}^i}$ produced in the last iteration is taken as the optimal solution to (11).

C. An algorithm to compute $\mathbf{Q}_{\boldsymbol{\rho}^i}$

Thanks to the convexity of $h(\mathbf{Q},\boldsymbol{\rho}^i)$, we resort to a gradient-projection based iterative algorithm to find $\mathbf{Q}_{\boldsymbol{\rho}^i}$. This method begins with initializing $\mathbf{Q}$ by $\mathbf{Q}^*$ if $i = 0$, otherwise by $\mathbf{Q}_{\boldsymbol{\rho}^{i-1}}$. Then, $\mathbf{Q}$ is iteratively updated by $\mathbf{Q} = [\mathbf{Q} - \tau\cdot\mathbf{B}]_{\boldsymbol{\mathcal{Q}}_\epsilon}$, where $\mathbf{B}$ is a matrix containing the gradients of $h(\mathbf{Q},\boldsymbol{\rho}^i)$ with respect to every entry of $\mathbf{Q}$, $[\cdot]_{\boldsymbol{\mathcal{Q}}_\epsilon}$ is the operator of projection into $\boldsymbol{\mathcal{Q}}_\epsilon$, and τ is a prescribed small positive value that guarantees convergence. We terminate the iterations when $||\text{vec}([\mathbf{Q} - \tau\cdot\mathbf{B}]_{\boldsymbol{\mathcal{Q}}_\epsilon} - \mathbf{Q})||$ is smaller than a prescribed small value Δ_2. Then $\mathbf{Q}$ produced by the last iteration is taken as $\mathbf{Q}_{\boldsymbol{\rho}^i}$.

In fact, every entry of $\mathbf{Q}$ is updated by

$$\forall k, \forall n, q_{k,n} = \begin{cases} \epsilon & \text{if } q_{k,n} - \tau\cdot[\mathbf{B}]_{k,n} \leq \epsilon \\ 0 & \text{if } q_{k,n} - \tau\cdot[\mathbf{B}]_{k,n} \geq 0 \\ q_{k,n} - \tau\cdot[\mathbf{B}]_{k,n} & \text{otherwise} \end{cases} \tag{17}$$

where $[\mathbf{B}]_{k,n} = \frac{\partial h(\mathbf{Q},\boldsymbol{\rho}^i)}{\partial q_{k,n}}$ is derived as

$$[\mathbf{B}]_{k,n} = -\lambda_{u_{k,n}^m,k}^i\alpha_{u_{k,n}^m,k,n}^* + e^{q_{k,n}}\big(\mu_k^i + \tag{18}$$

$$\sum_{l\neq k}\lambda_{u_{l,n}^m,l}^i\alpha_{u_{l,n}^m,l,n}^*\frac{y_{u_{l,n}^m,l,n}^k}{I_{u_{l,n}^m,l,n}(e^{\mathbf{q}_n})}\big). \tag{19}$$

VI. Numerical Experiments

For illustration purposes, we consider the downlink of a cellular OFDMA system with $K = 3$ coordinated cells and $N = 8$ subcarriers, as shown in Figure 1. Note that $\mathcal{U}_1 = \{u_1, u_2\}$, $\mathcal{U}_2 = \{u_3, u_4\}$, and $\mathcal{U}_3 = \{u_5, u_6\}$. We set $w_1 = w_2 = w_3 = 1$, $\Gamma = 1$, $\sigma_{u,k,n}^2 = 1, \forall u \in \mathcal{U}_k, \forall k, \forall n$, $P_{k,\max} = P_M, \forall k$, $\epsilon = -10$ which corresponds to $e^\epsilon = 4.54\times10^{-5}$, $\delta = \tau = 0.01$, $\Omega_1 = \Omega_2 = \Delta_1 = \Delta_2 = 0.001$, $M = 4$, and $T = 5$ in numerical experiments. We conducted the following numerical experiments with Matlab v7.1 on a laptop equipped with an AMD Turion CPU of speed 2.2 GHz and a memory of 2 GBytes. In particular, we used the TOMLAB/CPLEX toolbox v12.1 to solve (10).

We generate the channel between each BS-user pair according to the following assumptions. First, it is modeled by a 6-tap delay line, and the average received power decays exponentially with distance. In particular, the propagation exponent is equal to 3. Second, we assume the amplitude of the i-th tap is a circularly symmetric complex Gaussian random variable with zero mean and variance as σ_i^2. Finally,

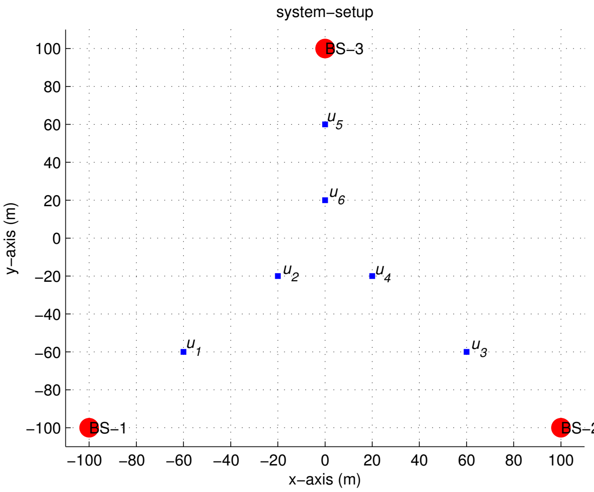

Fig. 1. An exemplary system setup.

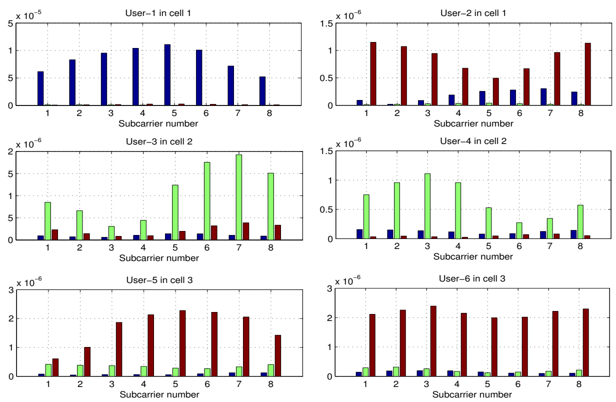

Fig. 2. A random realization of channel gains, where the blue, green, and red bars represent the channel gains from BS 1, 2, and 3, respectively.

we assume $\frac{\sigma_{i+1}^2}{\sigma_i^2} = e^3$, i.e., the tap power decays exponentially with the coefficient equal to 3.

We have tested the proposed algorithms on a random realization of the BS-user channel gains shown in Figure 2 when $P_M = 90$ dBW. To facilitate the following explanation, we define a 3×8 user-scheduling matrix $\mathbf{U}^m$ which contains $u_{k,n}^m$ at the k-th row and the n-column. Apparently, there exists an one-to-one mapping between $\mathbf{U}^m$ and $\mathbf{A}^m$. When the UPA as well as the BSA and the ESA are used for initialization, the corresponding $\mathbf{U}^0$'s are evaluated respectively as

$$\mathbf{U}^0 = \begin{bmatrix} 2 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 \\ 6 & 6 & 5 & 5 & 5 & 5 & 6 & 6 \end{bmatrix} \tag{20}$$

and

$$\mathbf{U}^0 = \begin{bmatrix} 2 & 1 & 2 & 2 & 2 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 3 & 3 & 3 & 3 \\ 6 & 5 & 6 & 5 & 5 & 5 & 6 & 6 \end{bmatrix}. \tag{21}$$

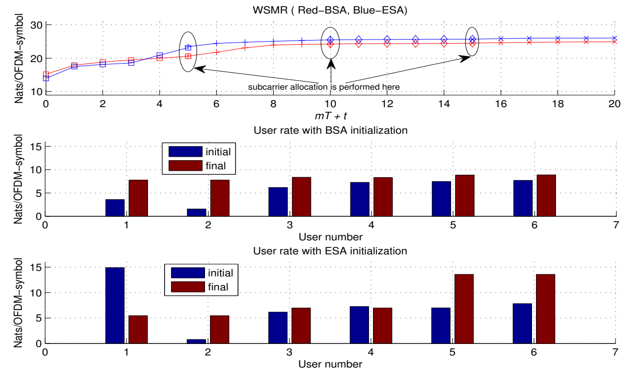

Fig. 3. The computed results for the random realization of channel gains.

After **Algorithm 1** terminates, the computed WSMR's with respect to $mT + t$, as well as user rates at initialization and after optimization are shown in Figure 3. In particular, each line segment with the same mark shows the WSMR corresponding to iterations of **Algorithm 2** with the subcarrier allocation fixed. After the algorithm terminates, $\mathbf{P}^4$ and $\mathbf{U}^4$ corresponding to the BSA and ESA initialization are feasible for (8), and they are computed respectively as

$$\mathbf{P}^4 = \begin{bmatrix} 0.0066 & 0.0008 & 0.3626 \\ 0.1075 & 0.0063 & 0.0553 \\ 0.0005 & 0.4094 & 0.0053 \\ 0.0152 & 0.0380 & 0.0905 \\ 0.0018 & 0.0003 & 0.4443 \\ 0.1497 & 0.0701 & 0.0003 \\ 0.6557 & 0.0115 & e^{\epsilon} \\ 0.0039 & 0.4678 & 0.0004 \end{bmatrix}, \tag{22}$$

$$\mathbf{U}^4 = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 & 4 & 3 & 3 & 3 \\ 6 & 5 & 5 & 6 & 5 & 6 & 6 & 5 \end{bmatrix} \tag{23}$$

and

$$\mathbf{P}^4 = \begin{bmatrix} 0.0002 & 0.0001 & 0.2454 \\ 0.0049 & 0.0001 & 0.2229 \\ 0.0047 & 0.1574 & 0.0482 \\ 0.0056 & 0.0681 & 0.0770 \\ 0.9454 & 0.0004 & e^{\epsilon} \\ 0.0004 & 0.0001 & 0.2729 \\ 0.0035 & 0.7671 & e^{\epsilon} \\ 0.0013 & 0.0002 & 0.2263 \end{bmatrix}, \tag{24}$$

$$\mathbf{U}^4 = \begin{bmatrix} 2 & 1 & 1 & 1 & 2 & 2 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 & 4 & 3 & 4 \\ 6 & 5 & 6 & 5 & 5 & 5 & 5 & 6 \end{bmatrix}. \tag{25}$$

We can see that both initialization methods lead to the similar WSMR after optimization, even though the BSA initialization results in a higher WSMR at the beginning. Besides, the power and subcarrier allocation after optimization results in a WSMR about 10 nats/OFDM-symbol higher than the initial UPA and BSA/ESA. Despite the initialization method, the following points can be observed. First, the WSMR is

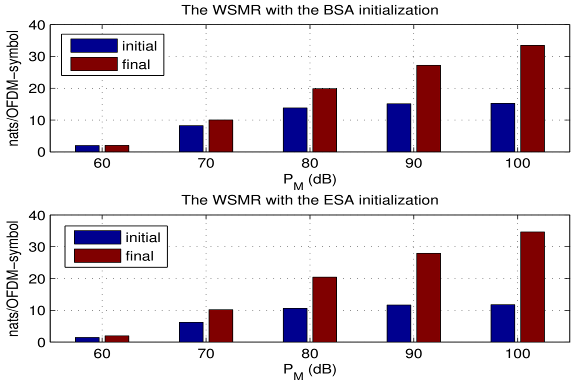

Fig. 4. Average WSMR over 100 random realizations of channel gains.

increased significantly by the power allocation optimization at the beginning, while the subcarrier allocation optimization leads to a slight increase of the WSMR. Second, the power allocation after optimization shows that each BS should allocate the dominant part of its power to only a few subcarriers. Once a BS allocates a significant part of its power to a subcarrier, the other BSs should allocate only a small part of its power to this subcarrier, so as to reduce CCI. Third, the minimal user rate in each cell is enhanced, and the users in each cell have similar rates after optimization. These observations show that the power and subcarrier allocation yielded by the proposed algorithm leads to a significantly better WSMR performance than the UPA and the BSA/ESA for the considered random realization of channel gains.

We have also tested the proposed algorithm on 100 random realizations of the BS-user channel gains, when P_M varies from 60 to 100 dBW. The average value of the WSMR at initialization and after optimization are shown in Figure 4. We can see that two initialization methods result in the similar average WSMR after optimization, although the BSA initialization leads to a higher average initial WSMR. Regardless of the initialization method, the average WSMR after optimization is slightly higher than the average initial WSMR when P_M is below 80 dB. This is because the CCI is relatively low due to the relatively low total power of each BS, and therefore the UPA is nearly optimal as explained in Section IV-A. However, the average initial WSMR increases slightly, while the average WSMR after optimization increases significantly, when P_M increases from 80 dB to 100 dB. Besides, the average WSMR after optimization is significantly higher than the average initial WSMR when P_M is greater than 80 dB. These observations show that the proposed algorithm can effectively produce the power and subcarrier allocations that lead to the significantly better WSMR performance in average than the UPA and the BSA/ESA, especially when the total power of each BS is high.

VII. Conclusion

We have considered the downlink of a cellular OFDMA system, in which multiple BSs are coordinated by a centralized resource allocation algorithm. We have proposed an iterative algorithm to maximize the WSMR subject to a total power constraint at each BS, in terms of jointly optimizing coordinated BSs' subcarrier and power allocation. The effectiveness of the algorithm has been illustrated by numerical experiments.

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