On Maximizing Collaboration in Wireless Mesh Networks Without Monetary Incentives

Gabriel Popa, Eric Gourdin, Franck Legendre, Merkouris Karaliopoulos

On Maximizing Collaboration in Wireless Mesh Networks Without Monetary Incentives

Gabriel Popa*, Eric Gourdin†, Franck Legendre* and Merkouris Karaliopoulos*
* Computer Engineering and Networks Laboratory, ETH Zurich,
Gloriastrasse 35, CH-8092 Zurich, Switzerland
Email: {gpopa,lfranck,karaliom}@tik.ee.ethz.ch

† France Télécom, Orange Labs, CORE/TPN/TRM
38-40 rue du Général Leclerc, 92794 Issy-Les-Moulineaux
Email: eric.gourdin@orange-ftgroup.com

Abstract—**In distributed network settings, where nodes are not under the control of a single administrative entity, the fulfillment of fundamental network operations is heavily dependent on their cooperation. Nevertheless, individual interests in combination with resource constraints do not always encourage cooperative behavior. In this work, we focus on static Wireless Mesh Networks (WMNs) and address the issue of selfishness in packet forwarding. Firstly, we model the dependencies that emerge in these networks as a result of their topology, traffic demand matrix, and route selection and determine the conditions for the natural emergence of collaboration, without the need of (monetary) incentives. We then assess the achievable collaboration levels, i.e., percentage of traffic demands (flows) that can be served thanks to the emerging collaboration, in both synthetic and real-world WMN topologies under shortest-path routing. Our results show that the cooperation improves when the number of flows increases. Yet, certain topological characteristics (marginal nodes, node degree distributions) make full cooperation difficult to achieve for the average case and bound it asymptotically. Finally, and motivated by these results, we use our dependency model to drive the selection of routes in the network. We cast the routing problem as a mixed-integer programming problem, which tries to maximize the collaboration level in the network. Our study investigates the resulting tradeoff among network throughput, served traffic flows, and routing stretch factor.**

I. Introduction

As communication devices available to individuals have become more powerful and feature-rich, the communication process itself has evolved towards a user-centric paradigm. Perceived as a component of this trend, wireless mesh networks aim at providing a comprehensive set of services to users. They are characterized by lack of a central authority and offer more degrees of freedom when compared to conventional provider-operated networks. However, wireless mesh networks also introduce a new set of problems imposed by their de-facto distributed nature. In particular, routing algorithms require nodes on a route, acting as relays, to collaborate in forwarding data. These relay nodes, driven by pragmatic considerations (such as limited power on mobile devices), might silently drop packets, thus disrupting in a selfish way the network service. Even if we consider a static wireless mesh network without any energy constraints (e.g., community network), cooperation still cannot be taken for granted as a substantial bandwidth share has to be allocated to others to relay their traffic.

Researchers have used game-theoretic arguments to argue about the potential for cooperation in WMNs. Felegyhazi *et al.*[1] used a model based on node dependencies to show that the probability of obtaining a poor Nash equilibrium state for cooperation in the network is high, if no incentive-giving mechanisms are in place. Several protocols inspired by game theoretic mechanisms offer monetary incentives to nodes in exchange for their cooperation in forwarding data packets [2], [3], [4]. Most of them adapt VCG-based self-enforcing mechanisms to the peculiarities of wireless networks, in order to avoid the potential wrongful signals from selfish nodes. When acting as relays, nodes collect monetary units, which in turn can be used to request the service of other relays for forwarding their own data.

In our work, we take one step back with respect to these studies, to analyze achievable cooperation when monetary incentives are not available. Similar to the work of Felegyhazi *et al.* [1], we construct a model that will let us assess the potential for cooperation in the network. Nevertheless, we switch from the node space to the traffic demand space; this has been done in order to allow nodes which are sources (or destinations) for different demands to respond individually, on every route, to the perceived presence of non-cooperative behavior on the routes corresponding to these demands. Our *demand dependency model* uses the fact that both ends of a demand involved in communication are interested in packet forwarding services provided by the relay nodes. We use this model to introduce a metric for the achievable cooperation under given network topology, traffic demand matrix and routing policy.

We then go one step further than prior work in that we actively seek to increase cooperation in the network via routing. The routing function is formulated as a mixed-integer optimization problem, which aims at maximizing the dependencies among traffic flows in the network and, hence, encourage cooperation as a natural choice. We present several optimization models which maximize cooperation while trying to reduce route lengths or to increase the amount of traffic that can be routed.

The paper is structured as follows: we first provide a model of cooperation based on dependency graphs in Section II and evaluate the cooperation levels on synthetic and real-world topologies under shortest-path routing in Section III. We then propose an optimization framework as a basis for the design of cooperation friendly routing in Section IV. We consider various alternative formulations for the routing problem, which can increase cooperation up to 100% by rearranging the routes in the network. We evaluate the tradeoffs introduced by this approach in Section V. Finally, we conclude the paper by outlining the next steps to be pursued in Section VI.

II. Modeling Cooperation with Demand Dependency Graphs

This section formulates a criterion for packet forwarding cooperation by looking into the dependencies data flows create among wireless nodes. Our explicit assumptions are that a) nodes have global knowledge of the network topology and available routes therein; and b) nodes cooperate during the route discovery phase.

A. Network Model and Problem Statement

We will represent the wireless mesh network by an undirected graph $G = (V, E)$, where V is the set of nodes and E is the set of radio links between nodes. The traffic demand set K includes all pairs of traffic origin/destination nodes, $(s^k, t^k)_{k \in K}$, where $s^k, t^k \in V$. Each node can serve as source/destination for one or more demands. The assumption is that a node would not want to offer forwarding services in the network unless it wants to send or receive data from it. We denote by $\mathcal{P}^k$ the set of feasible routes for demand k (joining s^k and t^k) and by $\mathcal{P} = \cup_{k \in K} \mathcal{P}^k$ the respective set for the full demand set K. A *route profile* is a set of paths $\mathcal{S} \subseteq \mathcal{P}$ containing exactly one route p^k for each demand k: $|\mathcal{S} \cap \mathcal{P}^k| = 1, (\forall) k \in K$. We consider that every demand transports a single unit of traffic load. We assume that a node does not have incentives to cooperate in serving a demand, unless all its own demands are fulfilled, *i.e.*, all flows for which this node is either source or destination are served by the network.

Our objective is three-fold. First of all, we want to obtain a concise description of the dependencies that traffic demands and routing generate among nodes in the network and define a metric for the cooperation level that naturally exists therein. We treat this in the next two subsections. Secondly, we intend to show the conditions under which conventional routing approaches, such as the shortest path routing policy, favour collaboration. This is covered in Section III. Note that, whereas the traffic demand matrix is an exogenous factor and presented as an input to the network, routes can be controlled and chosen by the latter. Therefore, and this is our main objective, we seek to find and assess route profiles $\mathcal{S}$, such that the number of functional demands (the ones whose routes contain only cooperating relays) is maximized. This is contrary to conventional routing approaches that aim at minimizing some notion of "cost"; therefore, we expect a penalty in terms of route length (routing stretch factor) when compared to shortest-path routing. We elaborate on these routing approaches in Sections IV and V.

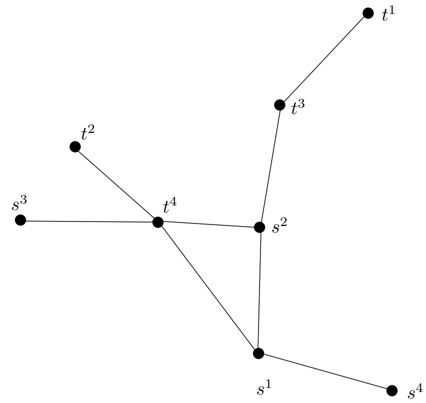
B. From Node Dependencies to Demand Dependency Graphs

We say that a given node $x \in V$ collaborates with a given demand $(s^k, t^k), k \in K$ if i) it is part of the route selected by the demand, $x \in p^k$, and, ii) it performs packet forwarding on p^k. Thus, we can say that nodes s^k and t^k depend on node x, once a route p^k containing x has been selected by demand k. Of course, x can also decide to be selfish. We hence seek to determine under which conditions collaboration emerges naturally because of inter-dependencies.

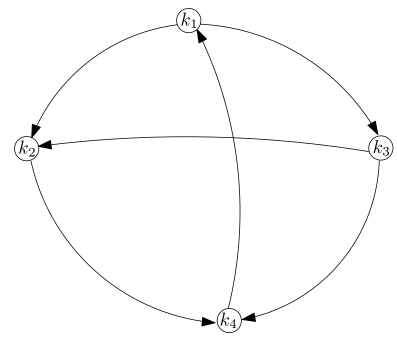
A route is *functional* as long as it obtains the collaboration of all the relays on the path. We will consider that the end nodes of a demand k are dependent on (the forwarding performed by) each of the relay nodes on the selected route p^k and, for this matter, on their cooperation.

These dependencies can be captured by the *demand dependency graph* that we will consider hereafter, denoted by G_{dep}. For each route profile $\mathcal{S}$, a directed dependency graph $G_{dep}(\mathcal{S}) = (K, A)$ is defined as follows: there is one node associated with each demand $k \in K$ and a directed arc $(k, k') \in A$ from demand node k to k' exists if path p^k is routed (in graph G) over $s^{k'}$ or $t^{k'}$ or both.

Consider Figure 1(a), which shows a simple network topology with eight nodes, through which four traffic demands need to be served. The demand set is $K = \{k_1, k_2, k_3, k_4\}$, where each demand is a tuple containing the source and destination nodes, $k_i = \{s^i, t^i\}, (\forall)i = \overline{1,4}$. The route profile under minimum-hop count routing would be $\mathcal{S} = \{p^1, p^2, p^3, p^4\}$, where $p^1 = \{s^2, t^3\}$, $p^2 = \{t^4\}$, $p^3 = \{s^2, t^4\}$ and $p^4 = \{s^1\}$. The dependency graph $G_{dep}(\mathcal{S})$ resulting from $\{K, \mathcal{S}\}$ is depicted in Figure 1(b).



(a) Sample topology with 4 demands



(b) Dependency graph resulting from the network scenario shown in Figure (b)

Fig. 1. Constructing the dependency graph for a topology, a set of demands and the corresponding routes

C. Conditions for Cooperation

We outline the necessary and sufficient conditions for collaboration between arbitrary demands in G_{dep}. It follows from Section II-B that if two demands $k_1, k_2 \in K$ are dependent on each other then they will cooperate as long as none has other, unsatisfied dependencies. The assumption is again that the common good is being recognized by the two demands. By extension, we consider that if we have $k_1, k_2, k_3, \ldots, k_n \in K$ such that $(k_1, k_2), (k_2, k_3), \ldots, (k_{n-1}, k_n), (k_n, k_1) \in A$, but $(\forall)k \in K, (k_i, k) \notin A, k_i \neq k, (\forall)i = \overline{1,n}$, then demands $k_1, k_2, k_3, \ldots k_n$ collaborate fully (through direct and indirect dependencies). If the second condition is not fulfilled and $(\exists)i \in \{1, \ldots, n\}, k \in K, k \neq k_j, (\forall)j = \overline{1,n}$ such that $(k_i, k) \in A$, but this is not a part in a cycle of G_{dep}, then demand k will not cooperate with k_i (at least one node of p^k is selfish). Since k_i does not obtain the desired service, it will also decide to stop cooperating with k_{i-1}, the non-cooperating behavior spreading to all the demands in the cycle. It follows that two demands k', k'' cooperate as long as they are part of a cycle of G_{dep} and every dependency leaving the cycle is also included in a cycle of G_{dep}. For this, we formulate the following equivalent results, noting that overlapped dependency cycles form strongly connected components:

Proposition 2.1: We consider $k', k'' \in K, (k', k'') \in A$. k' and k'' cooperate if and only if the following three conditions are fulfilled simultaneously:

1. $(\exists)C$, a cycle of G_{dep}, such that $(k', k'') \in C$;
2. $(\exists)C_k$, a cycle of G_{dep}, such that $(k'', k) \in C_k, (\forall)(k'', k) \in A$.
3. condition 2. applies for any demand k'' reachable from k' in G_{dep}, recursively.

This can be reformulated as follows:

Proposition 2.2: Demands $k', k'' \in K$ cooperate (directly or indirectly) if and only if the following conditions are fulfilled simultaneously:

1. $k', k'' \in K$ are members of the same (maximal) strongly connected component of G_{dep};
2. the strongly connected component does not have any outgoing dependencies.

To measure the collaboration level in a given network scenario, we introduce the *cooperation ratio* $\mathcal{F}(G, D, S)$, as the average percentage of functional demands over various random demand sets K of fixed size in network G under given route policy S and demands-per-node ratio $D = \frac{|K|}{|V|}$. The numerator of $\mathcal{F}$ equals the number of vertices in the demand dependency graph that fulfill the requirements of Proposition 2.2.

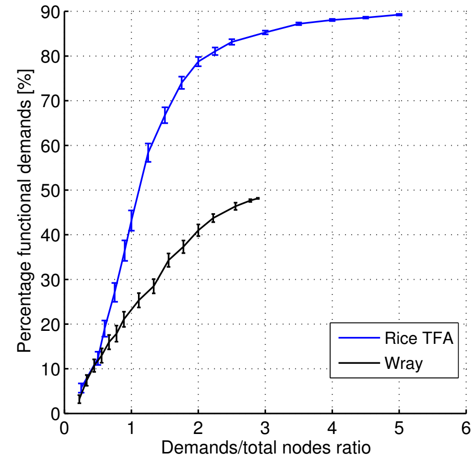
The actual players remain the nodes and for cases when the same node finds itself as either source or destination for multiple demands, then dependencies need to be added between these demands to ensure the cooperation between them when needed.

III. Cooperation Levels under Shortest Path Routing

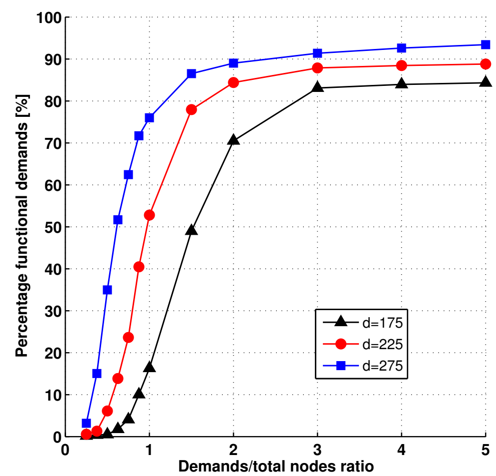
In this section, we compute the cooperation ratio in various WMN topologies, under shortest-path routing ($S \equiv SP$). We examine both synthetic and real-world WMN topologies, varying for each case the demands-per-node ratio, D, as measure of the traffic activity of the network nodes. Results for $\mathcal{F}(G, D, SP)$ are averages of 1000 random demand placement rounds, with D starting from 0.25.

We first study the wireless mesh network used as backhaul for the Rice TFA infrastructure [5], a large scale community network composed of 21 nodes that serves areas with poor Internet connectivity in Houston, TX, United States. Figure 2(a) plots the cooperation ratio with 95% confidence intervals. We observe that the cooperation increases fast for small values of D and then slower as D increases, approaching 90%. As one would expect, as D increases, more dependencies give rise to a denser $G_{dep}(\mathcal{S})$, thus increasing the probability of obtaining fewer larger strongly connected components and decreasing the number of dependencies between them. Nevertheless, when D grows beyond some value, both the functional and non-functional demands increase to similar extent. This is partially an effect of marginal nodes, which have a single link to the rest of the network. Demands from a marginal end node have a lower probability of obtaining collaboration since marginal nodes cannot act as relays on any route. In other words, no other demand can have a route going through marginal nodes under shortest-path routing and, hence, it is not possible for those marginal nodes to give rise to demand dependencies; dependencies can instead be generated by the other end node of the traffic demand, unless that is also a marginal node.

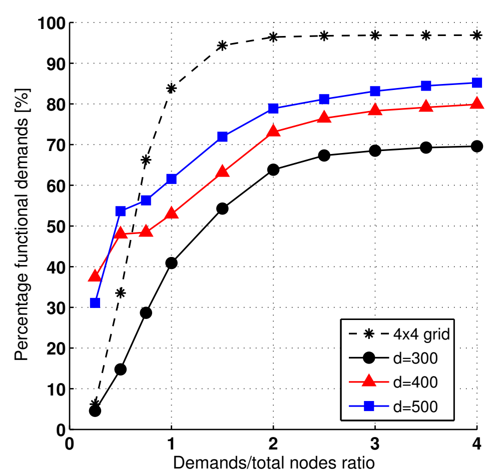
We make similar investigations for the Wray village wireless mesh network [6] composed of 9 nodes, set up by Lancaster



(a) Wray and Rice TFA topologies



(b) Random graphs (with marginal nodes)



(c) Random and grid graphs (w/o marginal nodes)

Fig. 2. Cooperation curves for various mesh topologies as a function of D

University for providing Internet access to a remote rural area. For D close to maximum, $\mathcal{F}(G_{Wray}, D, SP)$ still does not achieve 50% (Figure 2(a)). We observe in Figure 2(a), first, that $\mathcal{F}(G_{Wray}, D, SP)$ is bounded to 50% and, second, that the rate of increase is lower than for the previous network. Looking closely at the Wray topology, we notice that the proportion of marginal nodes is more than three times higher than in the Rice TFA network. This reinforces our previous explanation on the impact of marginal nodes on collaboration. It also appears to outweigh the impact of the network topology density: since the Wray topology is actually much sparser than the Rice topology and results in more overlapping flows, one would expect more demand dependencies.

We now turn to assessing the achievable cooperation for synthetic network topologies. We first use geometric random graphs with 40 nodes spread randomly on a 1000m $\times$ 1000m square area, with radio ranges set to 175, 225 and 275m. Figure 2(b) plots $\mathcal{F}(G_{GRG}, D, SP)$ for each of these ranges. An increased transmission range decreases the number of marginal nodes and decreases the expected route length, resulting in a decreasing probability of encountering a non-cooperative node. This gives better chances, even to demands with a marginal end node, to find useful dependencies. For each radio range, results are shown for one geometric random graph, but we obtain similar results for other instances of geometric random graphs.

Eventually, we consider four topologies without marginal nodes, which we will also use in Section V for the evaluation of our optimized routing approach. These are three geometric random graph topologies (with 16 nodes spread randomly over a 1000m $\times$ 1000m area and radio ranges of 300m, 400m and 500m, respectively) and a 4×4 grid topology (i.e., 16 nodes). We observe that a similar asymptotic behavior is obtained even without marginal nodes (Figure 2(c)). We conclude that marginal nodes are not solely responsible for this asymptotic behavior. The explanation is that the increased average node degree, resulting from the increasing transmission ranges, leads to shorter routes. Since a lower number of relays is required, the cooperation increases. However, it also leads to an increased number of possible paths for each demand (i.e., many shortest-path with same weight), thus decreasing the probability that routes will intersect in a way that leads to a better connected $G_{dep}(\mathcal{S})$. It is reasonable to believe that the interplay of these two factors ultimately dictates the level of achieved collaboration. We plan to examine analytically the effect of connectivity graph density and node degree distribution in future work.

IV. Cooperation-friendly Routing as an Optimization Problem

Having developed the demand dependency model in Section II as the starting point, we now look into cooperation-friendly routing as a mixed integer optimization problem (MIP). The objective is to maximize cooperation, even at the expense of reasonable route lengths, in a way that satisfies most demands and increases the overall network throughput. The optimization problem formulations presented in the next paragraph minimize the aggregate route length, maximize the routable throughput (both while enforcing full cooperation) or maximize cooperation even in the presence of marginal nodes.

A. A First Model of Full Collaboration

For routing purposes we define $G_d = (V, E_d)$, where E_d is the set of arcs obtained by the bidirection of every edge in E. For each routing solution, we have a directed dependency graph $G_{dep}(\mathcal{S}) = (K, A)$. As a first approach to the problem, we aim to find $\mathcal{S}$ such that $G_{dep}(\mathcal{S})$ is *complete* (fully meshed). To model this problem as MIP, we define the following variables: the binary routing variable $r_{ij}^k \in \{0,1\}$ is equal to 1 iff the demand k is routed on the edge $\{i,j\} \in E$ from i to j (in other words, the arc (i,j) belongs to the uniquely selected path from s^k to t^k). Similarly, the binary variable $y_i^k \in \{0,1\}$ is equal to 1 iff the node i also belongs to the same path. The problem we consider can be modeled as follows:

$$\sum_{i \in V^-(\ell)} r_{i\ell}^k - \sum_{j \in V^+(\ell)} r_{\ell j}^k = 0, \quad k \in K, \ell \in V^k \tag{1}$$

$$\sum_{j \in V^+(s^k)} r_{s^k j}^k = 1, \quad k \in K \tag{2}$$

$$\sum_{i \in V^-(t^k)} r_{i t^k}^k = 1, \quad k \in K \tag{3}$$

$$\sum_{j \in V^+(\ell)} r_{\ell j}^k = y_\ell^k, \quad k \in K, \ell \in V^k \tag{4}$$

$$\sum_{i \in V^-(\ell)} r_{i\ell}^k = y_\ell^k, \quad k \in K, \ell \in V^k \tag{5}$$

$$y_{s^k}^k = y_{t^k}^k = 1, \quad k \in K \tag{6}$$

$$y_{s^{k'}}^k + y_{t^{k'}}^k \geq 1, \quad k, k' \in K \tag{7}$$

$$r_{ij}^k, y_i^k \in \{0,1\}, \quad (i,j) \in E_d, k \in K \tag{8}$$

for all $k, k' \in K$, $\ell \in V^k$, $(i,j) \in E_d$, where $V^-(\ell) = \{i \in V : (i,\ell) \in E_d\}$, $V^+(\ell) = \{j \in V : (\ell, j) \in E_d\}$, $V^k = V \setminus \{s^k, t^k\}$. The constraints (1), (2), (3) build a valid path between s^k and t^k for each $k \in K$. Constraints (4), (5) relate the variables defining the paths with arcs with the variables defining the paths with nodes. Constraints (6) simply enforce the use of origin and destination nodes in each demands path. Finally, constraints (7) are the main design variables imposing that the route for demand k should be routed through the origin or the destination of demand k' (or both). To discriminate among a potentially large number of solutions, we include an objective function aiming at global route length reduction:

$$\min \sum_{a \in E_d} \sum_{k \in K} r_a^k \tag{9}$$

If one wishes to impose specific path lengths restrictions on certain (or all) paths, the following additional constraints should be included in the model:

$$\sum_{a \in E_d} r_a^k \leq L^k, k \in K \tag{10}$$

where L^k is the imposed upper bound on the length of chosen path p^k. Note that this model is the most restrictive. It imposes *direct dependencies between all demands*, which may result in long routes.

B. Eliminating Model Limitations

We now relax the restrictive requirement for completeness of $G_{dep}(\mathcal{S})$. To obtain a strongly connected $G_{dep}(\mathcal{S})$, in line with Proposition 2.2, we introduce additional variables $x^{kk'} \in \{0, 1\}$, $x^{kk'} = 1$ iff $(k, k') \in A$. These new variables are linked to the previous ones by the following constraints:

$$x^{kk'} \geq y_{s^{k'}}^k, \quad k, k' \in K \tag{11}$$

$$x^{kk'} \geq y_{t^{k'}}^k, \quad k, k' \in K \tag{12}$$

$$x^{kk'} \leq y_{s^{k'}}^k + y_{t^{k'}}^k, \quad k, k' \in K \tag{13}$$

They state that demand k is routed over the origin or the destination of demand k' iff $(k, k') \in A$. To enforce the strong connectivity of the dependency graph $G_{dep}(\mathcal{S})$, we require this graph to contain at least one arc in each set $\delta^+(Z)$ (outgoing dependencies from demands in Z) and $\delta^-(Z)$ (dependencies pointing to demands in Z) for each non-empty set $Z \subset K$:

$$\sum_{a \in \delta^+(Z)} x^a \geq 1, \quad Z \subset K, Z \neq \emptyset \tag{14}$$

$$\sum_{a \in \delta^-(Z)} x^a \geq 1, \quad Z \subset K, Z \neq \emptyset \tag{15}$$

Of course, there is an exponential number of such constraints, but violated connectivity constraints are separated in polynomial time and iteratively introduced in the model.

Observe that the objective tends to limit the length of paths whereas the strong connectedness of $G_{dep}(\mathcal{S})$ on the other hand, tends to increase the length of certain paths. As a result, the optimization can return routes, which are not simple any more and contain cycles separated from the actual established route. To remove such unwanted cycles, it is necessary to introduce additional constraints. Assuming that at a given step the optimal routing for demand $k \in K$ contains a cycle in the subset $X \subset V$ of vertices, $X \cap \{s^k, t^k\} = \emptyset$, the following constraint remove cycles routed on each of the vertices of X:

$$\sum_{a \in A(X)} r_a^k \leq |X| - 1, \quad X \subset V \tag{16}$$

where $A(X) \subseteq E_d$ is the subset of arcs induced by X. Note that again there is an exponential number of such constraints, but the violated ones can be separated in polynomial time.

Full collaboration can also result if $G_{dep}(\mathcal{S})$ is partitioned into strongly connected components without dependencies between them (necessary and sufficient condition from Proposition 2.2). Our optimization achieves strongly connected $G_{dep}(\mathcal{S})$, where possible, i.e., a stronger (sufficient) condition that may result in slightly longer routes. Therefore, running the associated MIP optimization returns an upper bound of total route length. We plan to investigate the applicability of this model at local level in future work.

C. Adding Capacities to the Model

The optimization presented in Section IV-B only requires that flows are functional. This means that an arbitrarily chosen route will be able to transport data, abstracting away throughput requirements that demands may have. While the objective of minimizing overall route length is useful, we are in fact interested in maximizing the throughput of flows in the network, while preserving fairness as much as possible. To address this issue, we are using a modified version of this optimization, based on the MCF (Maximum Concurrent Flow) model.

We consider fixed throughput demands $d_k > 0$, $(\forall)k \in K$ and edge capacities C_e, $(\forall)e \in E$ for each ratio link. Given the edge capacities, demand throughputs, and the requirement that the chosen routes $\mathcal{S}$ should create a strongly connected $G_{dep}(\mathcal{S})$, it may be that the network is not able to support the requested throughputs. We therefore set our objective as *determining the maximum fraction* $\lambda \in [0, 1]$ such that $\lambda \times$

d_k units of each demand can be routed in the graph without exceeding any edge capacity, while observing the constraints mentioned above. The following constraints need to be added in consideration of this fact:

$$\lambda \times \sum_{k \in K} d_k \times (r_{ij}^k + r_{ji}^k) \leq C_e, \quad (\forall) e \in E \tag{17}$$

where i and j are the two ends of edge e. In implementation, in order to shorten the computation time, we use the following equivalent constraint where we minimize μ:

$$\sum_{k \in K} d_k \times (r_{ij}^k + r_{ji}^k) \leq \mu C_e, \quad (\forall) e \in E \tag{18}$$

Finally, our problem can be completely modelled by the following:
max $\{\lambda$: (1), (2), (3), (4), (5), (6), (11), (12), (13), (14), (15), (16), (17)$\}$.

D. Maximizing the Number of Functional Demands

Sometimes it is not possible to achieve a solution where all the demands are functional. This happens for instance when both ends of a flow are assigned to marginal nodes. For connectivity graphs G where these situations arise, the model in Section IV-B is not able to maximize the number of functional demands and returns no solution, since it requires that all of them are functional. To make our model robust to such cases, it is necessary to introduce additional variables $t_k \in \{0, 1\}, (\forall) k \in K$, where t_k is 1 if demand k is functional and 0 otherwise. Equations (2), (3), (6) have to be modified accordingly:

$$\sum_{j \in V^+(s^k)} r_{s^k j}^k = t_k, \quad k \in K \tag{19}$$

$$\sum_{i \in V^-(t^k)} r_{i t^k}^k = t_k, \quad k \in K \tag{20}$$

$$y_{s^k}^k = y_{t^k}^k = t_k, \qquad k \in K \tag{21}$$

for every $k \in K$. The objective is also adjusted and aims at maximizing the number of functional routes:

$$\max \sum_{k \in K} t_k \tag{22}$$

Once completed, a preliminary optimization run provides us with the set of demands that cannot be routed. However, route lengths are not being minimized by this model. It is therefore necessary to run afterwards the optimization presented in Section IV-B on the same connectivity graph, but by removing the unroutable demands from the initial demand set. By modifying our initial model we obtain an optimization that maximizes the number of functional demands, while keeping the associated routes' size at a minimum.

TABLE I
FRACTION OF FUNCTIONAL DEMANDS FOR EACH ROUTE LENGTH – 4×4 GRID NETWORK

Route length	Fraction functional demands
2	0.72
3	0.66
4	0.60
5	0.48
6	0.14

V. Numerical Results

This section presents results from the previous optimization models on various networks. The aim is to offer an assessment of how well is the shortest path approach behaving and what are the performance tradeoffs introduced by our cooperation criterion. The results have been obtained using FICO Xpress Optimization Suite [7].

A. Route Stretch

We now show representative numerical results for the optimization problem formulated in Section IV-B. The objective therein was the minimization of aggregate route length, while achieving full cooperation. Better cooperation implies on average longer paths and we therefore study the tradeoff between path lengths and cooperation on selected topologies.

For low demands/nodes ratio D, $\mathcal{F}(G, D, SP)$ tends to be also small, regardless of the actual topology G and that is why our study focuses on low demands/nodes ratios. All the provided figures are averaged over 100 random multi-hop demands, such that every node is either source or destination (but not both) for exactly one demand.

We first choose a 4×4 grid topology with 8 demands. The average cooperation ratio reaches 65% without optimization, which is quite high when compared to complete random assignments. This can be attributed to the absence of inactive and marginal nodes. The average route length for each demand, whether the simulation links it to a functional route or not, is 3.05; it becomes 2.91, if measured only for functional paths. Table I shows that, as expected, the longer routes are less likely to be functional.

By applying the optimization on the same set of randomly generated demands, the average route length increases to 4.88. Therefore, an increase by a factor of at most 1.67 (for functional routes) is required to raise the cooperation ratio from 65% to 100%. Quite intuitively, for demand profiles with poor cooperation, the paths need to be lengthened more to obtain full cooperation and vice-versa.

We then consider the same random geometric connected topologies without marginal nodes from Section III. For radio ranges of 300, 400 and 500, the cooperation ratios are 37%, 35.5% and 45.25%, respectively, under shortest path routing. The edges/nodes ratios for these graphs are 1.93, 3.43 and 3.06, respectively.

The average route lengths of functional demands for the three networks are 2.51, 2.03 and 2.24, respectively. In order to achieve full cooperation, the average paths need to be lengthened to at most, 5.61, 4.72 and 4.67, respectively. This means an increase by a factor of at most 2.23, 2.32 and 2.08 in the average path length of the networks.

B. Impact of Cooperation on Demand Throughputs

Our model for optimizing cooperation increases route lengths and is therefore expected to reduce the demands' throughputs. This is in fact the price that has to be paid in exchange for full cooperation in the network. Based on the model proposed in Section IV-C, we now evaluate the performance penalty stemming from our cooperation-maximizing approach.

There are two factors which contribute to a drop in performance, namely: i) increase in intra-flow and inter-flow interference and ii) partially overlapping routes. Since evaluating factors in i) goes outside the scope of the current exploration, we study solely the influence of overlapping routes. For this, we use a 4×4 grid topology as before, with 8 randomly selected demands whose end nodes are always distinct, such that a node is either source or destination for exactly one demand. We assign a capacity of 10 units to all edges, and set the throughput of all the 8 demands to 1, 2 and 4 units, analyzing for each case λ (over 15 random demand placements). The reason for choosing these values for demand throughputs is that routes will be anyway affected by intra-flow interference and, since the large majority of them will have anyway a route length of at least three hops, the throughput cannot exceed $\frac{10}{3}$ in any case for these routes.

Running the optimization presented in Section IV-C, we obtain $\lambda = 1$ for all the optimization rounds where demand throughputs were set to 1 and 2, which means that the entire traffic can be routed without performance penalties in these cases. In other words, the network is able to transport the data as requested, under the stated assumptions. When the demand throughput is set to 4, in 40% of the cases we obtain $\lambda = 0.83$, while for the remaining 60% $\lambda = 1$. This means that in 40% of the cases, the throughputs set by demands cannot be sustained by the network and that each of them is to be reduced by approximately 17%. It also implies that the edge with the maximum number of overlapping routes (number denoted by N_{max}) is crossed by 2 demands in 60% of the cases ($N_{max} = 2$) and 3 demands in 40% of the cases ($N_{max} = 3$). It is only after setting demand throughputs to 6 that λ remains below 1 for all the random rounds. By analyzing the data, we obtain that for most of the demand placements $N_{max} = 2$, while for a smaller fraction of cases $N_{max} = 3$. These results point to the fact that demand overlapping remains moderate.

C. Wray Topology: Maximizing the Number of Functional Demands

As we have seen previously, the cooperation on the Wray topology is relatively low (smaller number of functional demands relative to other topologies), when we use shortest path routing. This happens partly because this topology is similar to a tree, containing a single cycle and a larger number of marginal nodes.

Our exploration will be using the optimization model presented in Section IV-D, which minimizes the number of non-functional demands. We aim at understanding how far from optimal were the simulation results based on shortest-path routing and, in general, what is the improvement that the aforementioned optimization model can achieve, thereby characterizing how much cooperation are networks of this type able to support.

We let demands/nodes ratio D vary between $\frac{1}{3}$ and 1, while observing the resulting cooperation ratio $\mathcal{F}(G_{Wray}, D, S_o)$, where S_o is the route placement policy resulting from our optimization model. For each D, 100 random demand placements are created and the corresponding optimization is run. Therefore, the results presented in Figure 3 are the average of 100 random rounds. When the optimized and unoptimized results (Figure 2(a)) are compared, it can be noticed that the optimized routing offers a much better cooperation ratio, but the improvement tends to be smaller as more and more demands are added to the network. It can be concluded that shortest path routing keeps cooperation to low levels when applied to sparse topologies, such as Wray.

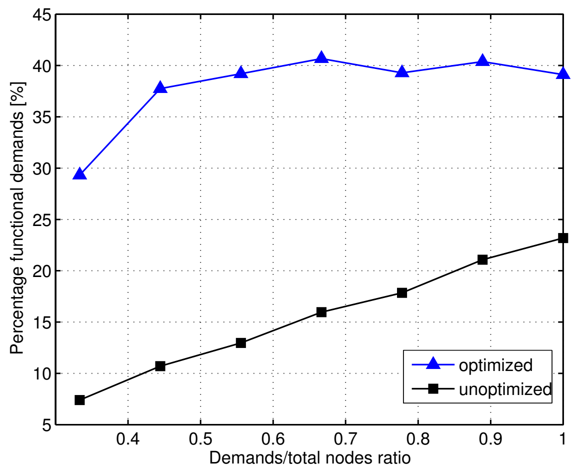


Fig. 3. Cooperation curves for the Wray village topology

VI. Conclusion and Future Work

In our work, we extend the study of Felegyhazi *et al.*[1], where authors argue that cooperation in wireless mesh networks remains low unless an incentive scheme is introduced (using, for instance, virtual currencies). Nevertheless, by taking a different approach we elaborate on a number of questions associated with the assessment of the level of achievable cooperation when monetary incentives are not present.

As a first contribution, we formalize the implicit dependencies among network nodes, generated by given route profiles and traffic matrices. We have derived a graph theoretic criterion for identifying the sets of mutually collaborating nodes, which we have then used in assessing the achievable collaboration for both synthetic (lattice and geometric random graphs) and real-world WMN topologies (Rice TFA, Wray village) under shortest-path routing. We first address the question of how much collaboration can emerge in the network when incentive schemes are absent and how is it affected by the network topology, routing policy, and traffic demand matrix. Our results show that uniformly spread, higher density traffic fosters cooperation. Yet, the achievable cooperation is ultimately bounded by the routing policy and topological characteristics (marginal nodes, node degree distribution).

In the second part we analyze the problem in a more constructive manner, by examining the question of *how could the achieved cooperation be improved*. In this work, we have realized this by modifying the routing policy such that cooperation is maximized. We first propose an optimization method for obtaining a minimal length route profile that achieves full cooperation, where possible. We then extend this model to account for unroutable demands and present an optimization model based on MCF which shows that although routes are longer when full cooperation is obtained, their overlapping remains within reasonable limits.

Numerical results for cooperation-friendly routing policies emphasize a number of related tradeoffs. In particular, to compensate for the selfishness of some nodes, the routes need to be lengthened. Moreover, routes are less disjoint and contention on network resources increases. In this paper we have only analyzed the route overlap, but assessing the additional interference introduced by the cooperation criterion is also very relevant to this exploration. We therefore plan to incorporate interference in our models, which would then give a complete image of the tradeoffs involved. One way to achieve this would be by extending problem formulations in Section IV while drawing on previous studies regarding interference [8].

Future work will also investigate analytically dynamic demand profiles, where demand profiles may vary over time, and consider the requirements that arise when adding throughput constraints.

Finally, on a more practical level, we would like to address the problem of a distributed algorithm appropriate for cooperation-maximizing routing.

Acknowledgment

The work presented in this paper is supported by the European Commission, under Grant No. FP7-224619 (the ResumeNet project).

References

[1] M. Felegyhazi, L. Buttyan, and J.-P. Hubaux, “Nash Equilibria of Packet Forwarding Strategies in Wireless Ad Hoc Networks,” *IEEE Transactions on Mobile Computing*, vol. 5, no. 5, pp. 463–476, 2006.

[2] L. Anderegg and S. Eidenbenz, “Ad hoc-VCG: a truthful and cost-efficient routing protocol for mobile ad hoc networks with selfish agents,” in *MobiCom ’03: Proceedings of the 9th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM, 2003, pp. 245–259.

[3] S. Zhong, L. E. Li, Y. G. Liu, and Y. R. Yang, “On designing incentive-compatible routing and forwarding protocols in wireless ad-hoc networks: an integrated approach using game theoretical and cryptographic techniques,” in *MobiCom ’05: Proceedings of the 11th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM, 2005, pp. 117–131.

[4] X.-Y. Li, Y. Wu, P. Xu, G. Chen, and M. Li, "Hidden information and actions in multi-hop wireless ad hoc networks," in *MobiHoc '08: Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing*. New York, NY, USA: ACM, 2008, pp. 283–292.

[5] " Rice TFA Wireles Network ," http://tfa.rice.edu/.

[6] J. Ishmael, S. Bury, D. Pezaros, and N. Race, "Deploying Rural Community Wireless Mesh Networks," *IEEE Internet Computing*, vol. 12, no. 4, pp. 22–29, 2008.

[7] " FICO Xpress ," http://www.fico.com/.

[8] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, "Impact of interference on multi-hop wireless network performance," *Wirel. Netw.*, vol. 11, no. 4, pp. 471–487, 2005.