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***Abstract*—We consider the problem of sum rate maximization with joint resource allocation and interference mitigation by multiantenna processing in wireless networks. The denominators in the users' signal-to-interference-plus-noise expressions are assumed to be representable in the form of matrix-based, concave interference functions. It is shown that the problem of interest for this system model can be readily rewritten as a minimization of a difference of convex functions. Based on this representation, an iterative algorithm with guaranteed convergence is employed to calculate possibly suboptimal solutions of the main problem, which is known to be NP-hard. The proposed technique enables achieving a large portion of the globally optimal sum rate. It is also very efficient and rather general in terms of allowing interesting extensions, compared with the related results from the literature.**

***Index Terms*—DC programming, interference mitigation, multiantenna systems, resource management, power control.**

I. Introduction

A trend in wireless communications is towards smaller cell sizes (e.g., pico/femto cells) and higher data rates. This raises the question of how to deal with potential interference between users and cells. Interference is recognized as a key issue in current standardization activities (e.g., 3GPP LTE-Advanced [1]). Promising techniques for dealing with this problem are multiantenna processing and dynamic resource allocation.

However, both techniques are highly interdependent, so a joint optimization strategy is required in order to fully exploit the available resources. This turns out to be difficult, because of the many degrees of freedom involved. Some special cases are well understood, like the multiple access, multiple-input, multiple-output channel (MIMO-MAC) with successive interference cancellation, and the corresponding broadcast channel (BC) [2]. But there are many other practically relevant cases, which still remain open. Some of the examples are dynamic resource allocation with linear MIMO processing [3], [4], [5], [6], robust transmission [7], or multi-cellular transmission (interference channel) [8]. All these problems are still considered as intricate.

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In this paper, we present a unifying optimization framework, which contains some of the previously mentioned, involved problems as special cases. The key idea is based on using the standard mapping of signal-to-interference-plus-noise ratios (SINRs) to rates, with the SINRs having the form

$$\mathrm{SINR}_l(\boldsymbol{p}) = \frac{p_l}{Y_l(\boldsymbol{p})}, \quad \boldsymbol{p} = [p_1, \dots, p_L]^T \tag{1}$$

where $l = 1, \dots, L$ is the link index, p_l is the power for link l, and $Y_l(\boldsymbol{p})$ is a concave, axiomatic interference function [9], [10] (a detailed explanation of the system model is provided in Section II). The concavity of the interference functions will enable us to include adaptive strategies such as beamforming in our framework. In this way, if the channel knowledge is provided, the interference can be additionally reduced.

A. Previous Work and Contributions

The problem of sum rate maximization has already been studied for linear interference functions in (1). Even for this model, which is simpler compared with our scenario, the problem turns out to be NP-hard [11]. Note that the assumed model with concave interference functions also includes the case of precoder optimization for the broadcast channel. By exploiting the MAC/BC duality [12], [13], the precoder design can be represented as an equivalent problem involving "virtual" receive beamformers, for which the SINRs are of the form (1). Suboptimal algorithms for this problem were proposed in [5], [6].

Since the problem of interest is rather involved, the aim of this paper is an algorithmic strategy offering a good performance/complexity tradeoff. Our contributions can be briefly summarized as follows:

- For the SINR model (1), we exploit the fact that the sum rate maximization problem is a difference of convex functions (DC) problem. This decomposition is, of course, a very general one. However, we show that in the studied problem it is derived with no additional computational cost, which is normally not the case.
- The considered decomposition enables the application of certain results from the DC programming theory. Combined with the theory of interference functions, we

utilize an iterative algorithm based on the linearization of the "non-convex part" of the objective function.

- It is shown how all necessary ingredients (subgradients/gradients, convex majorants, etc.) for the algorithm mentioned above can be obtained. This is done by efficiently exploiting the special structure which interference functions at hand possess.
- We prove that in the case of differentiable interference functions, the employed iterative algorithm always returns a stationary point.
- The computational efficiency of the proposed method is demonstrated by numerical simulations and comparisons with the related approaches from the literature. For this purpose, two problems are studied in more detail: (a) precoder optimization for the flat-fading, single-cell, downlink, multiple-input, single-output (MISO) system; (b) dynamic resource allocation in an orthogonal-frequency-division-multiplex (OFDM) based, multicellular, uplink, single-input, multiple-output (SIMO) system.

Finally, we remark that the framework of interference functions is very powerful not only in the sense that it includes many concrete application scenarios, but also in the sense that it enables a better mathematical analysis of the problems. It will be shown in the sequel that due to the structural model of the interference function, the joint optimization of powers and adaptive receive strategies ends up with the optimization solely with respect to powers. This facilitates the DC decomposition of the problems considered in this paper.

B. Notation

The following notation is used in the rest of the paper. Small and large bold fonts denote vectors and matrices, respectively. The dimensions, if not explicitely stated, will be clear from the context. Inequalities between vectors are component-wise inequalities. $\|(\cdot)\|_1$ and $\|(\cdot)\|_2$ are the l_1 and l_2 (Euclidean) vector norms, respectively [14]. The transpose and the conjugate (Hermitian) transpose operations are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. The set of subgradients of $f(\boldsymbol{x})$ at a point $\boldsymbol{x}$ is denoted by $\partial f(\boldsymbol{x})$. For a differentiable $f(\boldsymbol{x})$, $\nabla f(\boldsymbol{x})$ is the gradient at $\boldsymbol{x}$.

II. System Model and Problem Statement

Consider a system where K users from the set $\mathcal{K} = \{1, 2, \ldots, K\}$ can possibly transmit over N resources $\mathcal{N} = \{1, 2, \ldots, N\}$. Typical resources are frequency carriers, time slots, or spatial beams. Orthogonality is not required, so mutual interference can occur. We will assume that the resources can be accessed freely. In other words, each user can use multiple resources simultaneously, and each resource can be shared by multiple users.

Let $P_{k,n} \geq 0$ be the transmission power of user k on resource n. The case $P_{k,n} = 0$ corresponds to an unused (inactive) resource. By denoting with the link index l the user-resource pair (k, n), all powers $P_{k,n}$, $k = 1, \ldots, K$, $n = 1, \ldots, N$, are stacked into the power link vector $\boldsymbol{p}$, defined in (1). The dimension of $\boldsymbol{p}$ is $L = KN$. We further assume that $\boldsymbol{p}$ is from a compact, convex set $\mathcal{P}$, which also means that the power is finite.

In this paper, we will focus on a common strategy of maximizing the weighted sum rate

$$\begin{aligned} R &= \max_{\boldsymbol{p} \in \mathcal{P}} \sum_{l=1}^{L} \alpha_l \log\left(1 + \mathrm{SINR}_l(\boldsymbol{p})\right) \\ &= \max_{\boldsymbol{p} \in \mathcal{P}} \sum_{l=1}^{L} \alpha_l \log\left(1 + \frac{p_l}{Y_l(\boldsymbol{p})}\right) \end{aligned} \tag{2}$$

where the vector $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_L]^T \geq \mathbf{0}$ contains the corresponding link weighting factors. Note that in the general case, the optimum of (2) does not have to be equal to the sum capacity of the system (e.g., in the case of linear processing).

We explain now the interference modeling in (1). It is assumed that the interference functions $Y_l(\boldsymbol{p})$, $l = 1, \ldots, L$, in (1) satisfy the axioms of positivity, monotonicity, and scalability [9]. Additionally, it will be supposed that these functions are matrix-based concave:

$$Y_l(\boldsymbol{p}) = \min_{z_l \in \mathcal{Z}_l} \left(\boldsymbol{p}^T \boldsymbol{v}_l(z_l) + n_l(z_l)\right), \quad l = 1, \ldots, L \tag{3}$$

where $z = (z_1, \ldots, z_L)$ contains the receive strategies for the L links, chosen from compact sets $\mathcal{Z}_1, \ldots, \mathcal{Z}_L$, respectively [15]. In order to model the coupling between the non-orthogonal links in a compact manner, we define the coupling matrix $\boldsymbol{\Psi}(z)$ as

$$\boldsymbol{\Psi}(z) = \left[\begin{array}{ccc} \boldsymbol{v}_1(z_1) & \cdots & \boldsymbol{v}_L(z_L) \end{array}\right]^T. \tag{4}$$

It can be seen that decoupled receive strategies w.r.t. the link index l are implicitly assumed. In other words, the lth row of $\boldsymbol{\Psi}(z)$ depends only on z_l. These receive strategies can represent not only beamforming, which is our main concern, but also base station selection, shaping constraints, etc. The vector $\boldsymbol{n}(z) \triangleq [n_1(z_1), \ldots, n_L(z_L)]^T$ is associated with the effective noise powers, which are also influenced by the chosen receive strategies. Notice that the concavity of $Y_l(\boldsymbol{p})$ in (3) follows from the fact that $Y_l(\boldsymbol{p})$ is a pointwise minimum of affine functions [16].

As an example, one can consider the case of receive beamforming with L single-antenna transmitters, where the interference functions in (1) are readily given in the form (3)

$$Y_l(\boldsymbol{p}) = \min_{\|\boldsymbol{u}_l\|_2 = 1} \frac{\sum_{j=1, j\neq l}^{L} p_j \boldsymbol{u}_l^H \boldsymbol{R}_j^l \boldsymbol{u}_l + \sigma_l^2 \|\boldsymbol{u}_l\|_2^2}{\boldsymbol{u}_l^H \boldsymbol{R}_l^l \boldsymbol{u}_l} \tag{5}$$

for $l = 1, \ldots, L$. In (5), $\boldsymbol{u}_l$ is the beamformer of link l, and $\boldsymbol{R}_j^l = \boldsymbol{h}_j^l {\boldsymbol{h}_j^l}^H$, with $\boldsymbol{h}_j^l$ being the vector channel between the transmitter on link j and the (multiantenna) receiver on link l. The noise variances of all receive antennas on link l are assumed to have the same value σ_l^2, w.l.o.g.

Clearly, for fixed receive strategies, the interference functions (3) are linear. Since the main problem of interest (2) is NP-hard for linear interference functions [11], we conclude that the matrix-based concave case, studied in this paper, is NP-hard, as well. For this reason, suboptimal solutions are of interest. In the sequel, we will derive one such solution using the DC programming theory.

III. DC Programming Problems

DC programming problems in global (non-convex) optimization have attracted a lot of attention in recent years. It is an exceptionally large group of optimization problems, which admits efficient numerical methods that can often approach local or even global optima [17], [18], [19], [20]. Motivated by the special structure of the formulated problem of interest (2) regarding the power domain and the interference functions, the following definition for a DC programming problem is adopted in this paper:

$$\min_{\boldsymbol{x}\in\mathcal{X}} h(\boldsymbol{x}) \triangleq f(\boldsymbol{x}) - g(\boldsymbol{x}) \tag{6}$$

where $f(\boldsymbol{x}) : \mathbb{R}^L \to \mathbb{R}$ and $g(\boldsymbol{x}) : \mathbb{R}^L \to \mathbb{R}$ are continuous, convex functions on a compact, convex set $\mathcal{X} \in \mathbb{R}^L$. Notice that we start the analysis with no assumption regarding the differentiability of $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$.

The problem (6) is not convex in general. However, Algorithm 1 can be applied to solve this problem suboptimally.

Algorithm 1 Iterative, suboptimal solution for DC problems.

1: Initialize $\boldsymbol{x}^{(0)}$, set $n = 0$ (iteration number).
2: **repeat**
3: Define an auxiliary function $\hat{h}^{(n)}(\boldsymbol{x})$ as

$$\hat{h}^{(n)}(\boldsymbol{x}) \triangleq f(\boldsymbol{x}) - g(\boldsymbol{x}^{(n)}) - \boldsymbol{y}^T_{\boldsymbol{x}^{(n)}}(\boldsymbol{x} - \boldsymbol{x}^{(n)}) \tag{7}$$

where $\boldsymbol{y}^T_{\boldsymbol{x}^{(n)}} \in \partial g(\boldsymbol{x}^{(n)})$.
4: Solve the optimization problem

$$\boldsymbol{x}^{(n+1)} = \arg\min_{\boldsymbol{x}\in\mathcal{X}} \hat{h}^{(n)}(\boldsymbol{x}). \tag{8}$$

5: $n \leftarrow n + 1$.
6: **until** the sequence $\{h(\boldsymbol{x}^{(n)})\}$ converges.

The algorithm relies on the fact that the term $-g(\boldsymbol{x})$ in the objective function of (6) is replaced by its convex majorant $-g(\boldsymbol{x}^{(n)}) - \boldsymbol{y}^T_{\boldsymbol{x}^{(n)}}(\boldsymbol{x} - \boldsymbol{x}^{(n)})$ in (7).[1] Since $f(\boldsymbol{x})$ and the domain $\mathcal{X}$ are convex, (8) is a convex optimization problem, and it can be solved using algorithms from convex optimization theory [21], [22], [16].

The convergence of the algorithm can be easily proved. At the nth iteration step, both functions $\hat{h}^{(n)}(\boldsymbol{x})$, given by (7), and $h(\boldsymbol{x})$ have the same value at $\boldsymbol{x}^{(n)}$. Therefore, one concludes that

$$h(\boldsymbol{x}^{(n)}) = \hat{h}^{(n)}(\boldsymbol{x}^{(n)}) \geq \hat{h}^{(n)}(\boldsymbol{x}^{(n+1)}) \geq h(\boldsymbol{x}^{(n+1)}). \tag{9}$$

The first inequality in (9) is the consequence of $\boldsymbol{x}^{(n+1)}$ being the optimal point of the problem (8), and the second follows from $\hat{h}^{(n)}(\boldsymbol{x}) \geq h(\boldsymbol{x})$, $\forall \boldsymbol{x}$. Therefore, the sequence $\{h(\boldsymbol{x}^{(n)})\}$ monotonically decreases as n increases. The compactness and continuity assumptions on the problem domain and the objective function, respectively, confirm the claimed convergence. An illustration of the iterative procedure described by Algorithm 1 is given for a one-dimensional problem in Fig. 1.

We analyze now the problem (6) under an additional assumption that $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are (continuously) differentiable. In this case, it can be shown that Algorithm 1 returns a stationary point of the objective function $h(\boldsymbol{x})$.

Theorem 1: For differentiable $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$, Algorithm 1 returns a stationary point of $h(\boldsymbol{x})$ in $\mathcal{X}$.

Proof: Remember that a stationary point of a function $h(\boldsymbol{x})$ in an arbitrary problem domain $\mathcal{X}$ is a point $\bar{\boldsymbol{x}}$ which fulfills the following condition (see, e.g., Chapter 2 in [21])

$$\nabla h(\bar{\boldsymbol{x}})^T(\boldsymbol{x} - \bar{\boldsymbol{x}}) \geq 0, \quad \forall \boldsymbol{x} \in \mathcal{X}. \tag{10}$$

Consider the proved convergence of the objective function $h(\boldsymbol{x})$ in problem (6), when $n \to \infty$ in Algorithm 1. In the limit, all inequalities in (9) become equalities. In other words, both $\boldsymbol{x}^{(n)}$ and $\boldsymbol{x}^{(n+1)}$ are optimal points of $\hat{h}^{(n)}(\boldsymbol{x})$ on $\mathcal{X}$. Since $\hat{h}^{(n)}(\boldsymbol{x})$ is convex, these points have to be stationary points of $\hat{h}^{(n)}(\boldsymbol{x})$, as well [21]. By exploiting the relation (10), the latter claim can be equivalently rewritten for $\boldsymbol{x}^{(n)}$ as

$$\left(\nabla f(\boldsymbol{x}^{(n)}) - \nabla g(\boldsymbol{x}^{(n)})\right)^T (\boldsymbol{x} - \boldsymbol{x}^{(n)}) \geq 0, \quad \forall \boldsymbol{x} \in \mathcal{X}. \tag{11}$$

The fulfillment of the condition (11) means that $\boldsymbol{x}^{(n)}$ is also a stationary point of $h(\boldsymbol{x})$ in $\mathcal{X}$ when $n \to \infty$, which concludes the proof. ■

We remark that in the case of continuously differentiable functions, in general, there exist more efficient solutions for solving the inner problem (8), compared with the non-differentiable case [21], [22], [16].

Irrespectively of the differentiability assumption, it should be noted that there is no guarantee that Algorithm 1 will return a global optimum of a DC problem. However, there exist related branch-and-bound approaches based on outer approximations, which are indeed globally optimal [23]. The provable convergence to a global optimum of these methods is accompanied though with an increase in computational complexity.

IV. Application to Sum Rate Maximization

We start by rewriting the problem of sum rate minimization (2) as

$$\min_{\boldsymbol{p}\in\mathcal{P}} \sum_{l=1}^{L} \left(-\alpha_l \log(p_l + Y_l(\boldsymbol{p})) + \alpha_l \log(Y_l(\boldsymbol{p}))\right). \tag{12}$$

Thanks to concavity of the interference function $Y_l(\boldsymbol{p})$, given by (3), and the general property of positivity, it can be concluded that the functions $\log(p_l + Y_l(\boldsymbol{p}))$ and $\log(Y_l(\boldsymbol{p}))$ are concave, as well (see, e.g., Section 3.2.4 in [16]). Scaling with non-negative α_l clearly does not change these concavity properties. Therefore, (12) is a DC problem according to the

[1] In fact, the approximate majorant is affine. This clearly presents a reasonable choice for most performance measures of interest.

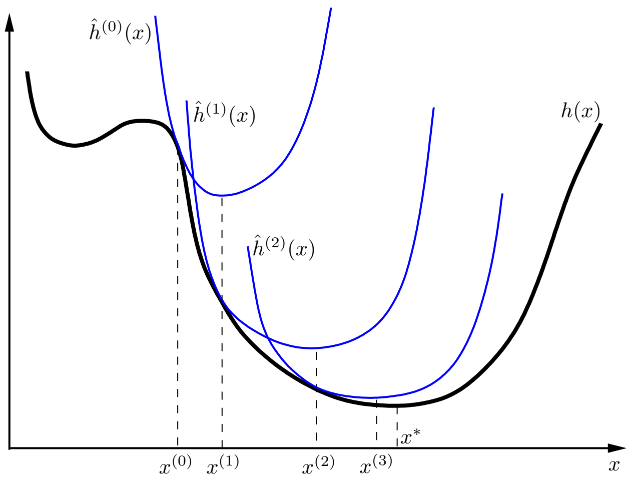

Fig. 1. An illustration of the monotonic convergence in Algorithm 1 for a DC problem with one scalar variable. x^* is the optimal point (minimum) of $h(x)$.

definition (6), with the variable $\boldsymbol{p}$, and

$$\begin{aligned} f(\boldsymbol{p}) &= -\sum_{l=1}^{L} \alpha_l \log(p_l + Y_l(\boldsymbol{p})) \\ g(\boldsymbol{p}) &= -\sum_{l=1}^{L} \alpha_l \log(Y_l(\boldsymbol{p})). \end{aligned} \tag{13}$$

We remark that a DC decomposition exists for a very large class of (non-convex) functions [17]. However, there are no general practical procedures for constructing such decompositions, and their calculation requires often a significant effort. In our problem formulation, thanks to the noticed concavity of the interference functions with arbitrary receive strategies, this decomposition is immediately obtained.

The last requirement for a practical application of Algorithm 1 is the calculation of the subgradients of $f(\boldsymbol{p})$ and $g(\boldsymbol{p})$ in order to formulate and solve (8). This issue is resolved by using the following lemma.

Lemma 1: Let $z^* = (z_1^*, \dots, z_L^*)$ be an optimal set of the receive strategies when calculating $Y_1(\bar{\boldsymbol{p}}), \dots, Y_L(\bar{\boldsymbol{p}})$ in (3). The transposed rows of the system coupling matrix $\boldsymbol{\Psi}(z^*)$, defined in (4), present a set of subgradients of the interference functions $Y_1(\boldsymbol{p}), \dots, Y_L(\boldsymbol{p})$ at $\bar{\boldsymbol{p}}$.

Proof: Since $Y_l(\boldsymbol{p})$ is concave, its subgradient at $\boldsymbol{p} = \bar{\boldsymbol{p}}$ is defined as a vector $\boldsymbol{y}_l$, which satisfies the following condition (cf. [21], p. 731)

$$Y_l(\boldsymbol{p}) - Y_l(\bar{\boldsymbol{p}}) \le \boldsymbol{y}_l^T(\boldsymbol{p} - \bar{\boldsymbol{p}}), \quad \forall \boldsymbol{p}. \tag{14}$$

With z^* being a minimizer in (3) for $\boldsymbol{p} = \bar{\boldsymbol{p}}$, we have

$$Y_l(\bar{\boldsymbol{p}}) = \bar{\boldsymbol{p}}^T \boldsymbol{v}_l(z_l^*) + n_l(z_l^*). \tag{15}$$

Therefore, for an arbitrary power vector $\boldsymbol{p}$ we obtain

$$\begin{aligned} & Y_l(\boldsymbol{p}) - Y_l(\bar{\boldsymbol{p}}) && (16) \\ &= \min_{z_l \in \mathcal{Z}_l} \left(\boldsymbol{p}^T \boldsymbol{v}_l(z_l) + n_l(z_l)\right) - \bar{\boldsymbol{p}}^T \boldsymbol{v}_l(z_l^*) - n_l(z_l^*) && (17) \\ &\le \boldsymbol{p}^T \boldsymbol{v}_l(z_l^*) + n_l(z_l^*) - \bar{\boldsymbol{p}}^T \boldsymbol{v}_l(z_l^*) - n_l(z_l^*) && (18) \\ &= \boldsymbol{v}_l(z_l^*)^T (\boldsymbol{p} - \bar{\boldsymbol{p}}) && (19) \end{aligned}$$

where the inequality is based on the fact that $\boldsymbol{z}^*$ might not be an optimal receive strategy for $\boldsymbol{p}$. This concludes the proof of the lemma. ■

While a subgradient $\boldsymbol{y}_l$ of $Y_l(\boldsymbol{p})$ at $\bar{\boldsymbol{p}}$ is directly obtained from Lemma 1, a subgradient of $\log(Y_l(\boldsymbol{p}))$ at $\bar{\boldsymbol{p}}$ can be calculated using the chain rule as $\boldsymbol{y}_l / Y_l(\bar{\boldsymbol{p}})$. The latter claim can be easily verified by noticing that

$$\begin{aligned} \log(Y_l(\boldsymbol{p})) &\le \log\left(Y_l(\bar{\boldsymbol{p}}) + \boldsymbol{y}_l^T(\boldsymbol{p} - \bar{\boldsymbol{p}})\right) && (20) \\ &\le \log\left(Y_l(\bar{\boldsymbol{p}})\right) + \frac{1}{Y_l(\bar{\boldsymbol{p}})} \boldsymbol{y}_l^T(\boldsymbol{p} - \bar{\boldsymbol{p}}) && (21) \end{aligned}$$

holds for any $\boldsymbol{p}$. The inequality in (20) follows from the definition of the subgradient (14) and the monotonicity of the logarithmic function. The concavity and the differentiability of the logarithmic function imply then (21). A subgradient of $\log(p_l + Y_l(\boldsymbol{p}))$ can be calculated a similar way.

The resulting non-differentiable, convex optimization problem (8) can be solved then using some of the methods from the convex optimization theory which require the construction of *a* subgradient. A classical example is the ellipsoid method [22], and some more advanced approaches can be found in [20], [21].

We focus now on an important case when the interference functions at hand are continuously differentiable. This property of the functions (3) is ensured if the optimal receive strategies are unique [24]. We remark that there are interesting practical examples, such as the beamforming example (5), where there are more choices for the optimal receive strategies (beamformers) that minimize the interference. However, if the channels are deterministic, as assumed for (5), all optimal (maximum SINR) beamformers differ only by the scaling factors. The coupling matrix $\boldsymbol{\Psi}(z)$ remains unique, and the corresponding interference functions are still continuously differentiable.

In the case of differentiable interference functions, the subgradients from Lemma 1 are singletons, and they define the corresponding gradients. The transposed coupling matrix is then the Jacobian matrix of the vector of interference functions (cf. also [24]). The gradients of $\log(Y_l(\boldsymbol{p}))$ are readily obtained as

$$\nabla \log Y_l(\boldsymbol{p}) = \frac{1}{Y_l(\boldsymbol{p})} \boldsymbol{v}_l, \quad l = 1, \dots, L. \tag{22}$$

Therefore, standard methods from convex optimization theory for solving the inner problem in Algorithm 1 can be applied [21], [16].

A. Necessary Condition for Global Optimality Under the Sum Power Constraint

We show in this subsection an important necessary condition if the power domain $\mathcal{P}$ is defined by a sum power constraint.

Besides obvious applications in examining an eventual global maximum, and in serving as a useful initialization, this condition will be of interest for numerical examples in Section V.

Lemma 2: Any globally optimal solution of (12) under a maximum sum power constraint $P_{\max}$ satisfies this constraint with equality.

Proof: Suppose that for an optimal point $\boldsymbol{p}^*$ of (12) the relation $\|\boldsymbol{p}^*\|_1 < P_{\max}$ holds. Then, there exists $\mu > 1$, such that $\|\mu\boldsymbol{p}^*\|_1 = P_{\max}$. Since for any $\mu > 1$, and any axiomatic interference function, $Y_l(\mu\boldsymbol{p}) < \mu Y_l(\boldsymbol{p})$ [9], it can be concluded that

$$\frac{p_l^*}{Y_l(\boldsymbol{p}^*)} = \frac{\mu p_l^*}{\mu Y_l(\boldsymbol{p}^*)} < \frac{\mu p_l^*}{Y_l(\mu\boldsymbol{p}^*)}. \tag{23}$$

Therefore, we have

$$\sum_{l=1}^{L} \alpha_l \log\left(1 + \frac{p_l^*}{Y_l(\boldsymbol{p}^*)}\right) < \sum_{l=1}^{L} \alpha_l \log\left(1 + \frac{\mu p_l^*}{Y_l(\mu\boldsymbol{p}^*)}\right). \tag{24}$$

In other words, a higher sum rate is achieved with the power vector $\mu\boldsymbol{p}^*$. This is a contradiction, so the starting claim that a global optimum does not satisfy the sum power constraint with equality is not true. ■

Finally, we remark that related necessary conditions can be derived for several other practically relevant power constrains. If, e.g., per-user power constraints are imposed in the system, it can be shown in a similar way that in the optimum, at least one of these constraints has to be active.

V. Numerical Examples

In this section, we apply the proposed algorithm on two problems. The first one considers linear precoder design in a single-cell, flat-fading scenario. The second problem assumes a multicellular, uplink transmission using OFDM.

A. Linear Downlink Precoding in a Single-Cell, Flat-Fading, Multiuser MISO System

In this problem, we consider the problem of sum rate maximization for a single-cell, flat-fading, downlink MISO system. Notice that this is just one special case of the general framework (3), for which, however, there exist results from the literature that we can compare with. According to the uplink/downlink duality, we have an equivalent uplink formulation for the sum rate maximization. We solve namely the equivalent uplink problem by the proposed algorithm and then transfer the solution to the downlink via duality [13]. The interference functions in the dual uplink problem have the decoupled beamformers, as given by (5). For the simulation, we assume that the base station has $N_T = 3$ transmit antennas and communicates with $L = K = 3$ single-antenna users (in this scenario, the terms users and links can be used interchangeably). The flat fading, normalized, complex Gaussian channel is assumed. The achieved sum rates are averaged over 1000 channels. The power region $\mathcal{P}$ in (12) is defined assuming that the total transmit power is 1 (or less or equal to 1, see Lemma 2).

In Fig. 2, the performances of several schemes are compared. Since the global optimum is unknown for the case of linear beamformers, we approximate it (the line with markers "$*$") by the exhaustive search with resolution $0.01\times0.01\times0.01$ on the boundary of the optimization set. The decomposition of the complete transmit precoder into the power control and the beamforming part proves useful here. Namely, for each power allocation, we know the optimal (maximum SINR) receivers, so the search is done only on the sum power constraint boundary. We can see that the proposed algorithm has only a minor loss in performance compared with the exhaustive search. It gives approximately the same performance as the algorithms proposed in [5] and [6]. These three precoding approaches outperform the simple uplink MMSE reception with equal power allocation (the same sum rate is achievable in downlink thanks to duality), denoted by the line with markers "$\times$". As expected, all approaches with linear precoding yield smaller values of the sum rates compared with the iterative waterfilling approach, where the interference is partially precompensated by dirty-paper coding.[2]

We plot the required total numbers of iterations of the proposed algorithm and the schemes in [5] and [6] in Fig. 3. The algorithms had to achieve the accuracy $|R^{(n)} - R^{(n-1)}| < 10^{-6}$ to stop the iteration, where $R^{(n)}$ was the achieved sum rate in the nth iteration. For the proposed algorithm and the approach based on geometric programming (GP) from [6], we count the total number of the iterations (inner and outer) when the quasi-Newton method for solving the convex problems in these approaches is applied. We can see that the GP approach [6] has high complexity, especially when it converges to a point where some entries approach zero. This happens often in the low SNR region. The required number of iterations of the proposed algorithm does not vary much over $P_{\max}/\sigma_l^2$ (σ_l^2 is assumed to be equal for all users/links, $l = 1, \ldots, L$). Although the iterations of the algorithm in [5] have a somewhat simpler structure, it can be seen that this approach becomes definitely more complex for high $P_{\max}/\sigma_l^2$. It should be noticed that the problem formulation in our paper is also more general in comparison with [5].

B. Dynamic Resource Allocation in a Multicellular, OFDM-Based, SIMO Uplink System

In the second problem, we consider a multicellular system with 7 cells and 3 users per cell. We assume a fixed base station assignment, illustrated in Fig. 4. The users (uplink transmitters) are equipped with single-antennas, while each base station (receiver) has 4 antennas and performs linear receive beamforming. In uplink scenarios, for fixed powers, the beamformers can be calculated analytically (and, in fact, in a decentralized way), and no duality theory is needed. There are 3 orthogonal carriers (denoted by A, B, and C in Fig. 4) in the system, and it is assumed that the per-user power constraints of 22dBm are imposed. The channel model in this

[2]Notice that the exhaustive search approaches the global optimum for the case of *linear* beamforming.

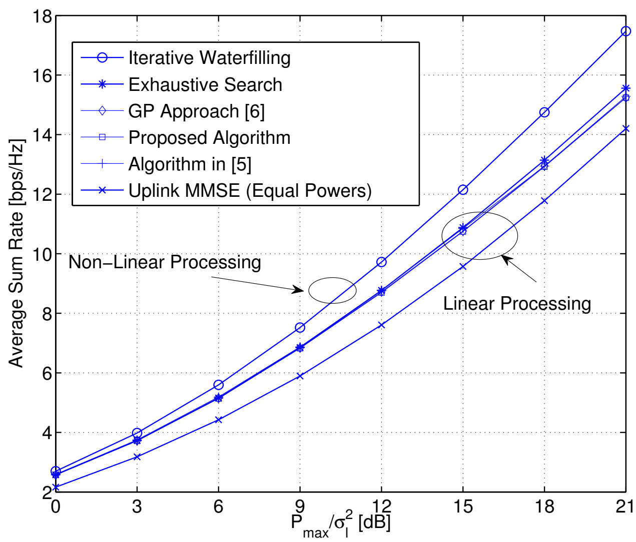

Fig. 2. Average sum rate vs. $P_{\max}/\sigma_l^2$. The noise variance σ_l^2 is equal for all users/links. $P_{\max} = 1$, $L = K = 3$, $N_T = 3$.

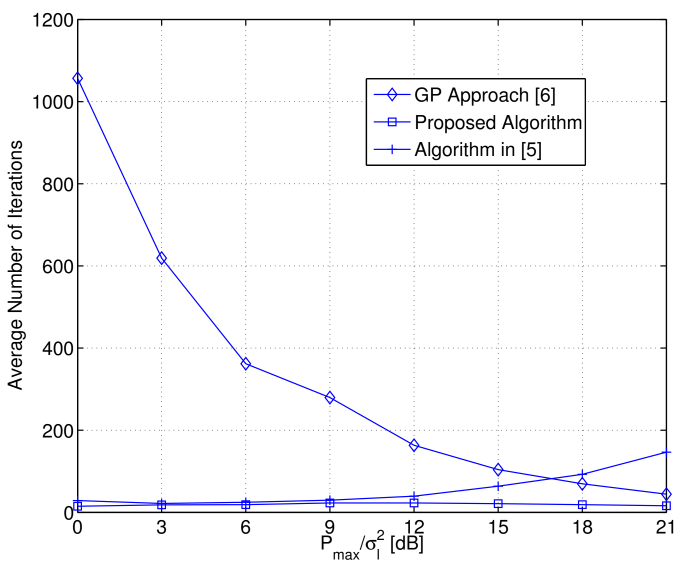

Fig. 3. Average number of iterations vs. $P_{\max}/\sigma_l^2$. $P_{\max} = 1$, $K = 3$, $N_T = 3$. The convergence condition is $|R^{(n)} - R^{(n-1)}| < 10^{-6}$. The required number of iterations of the proposed linearization algorithm does not vary much over $P_{\max}/\sigma_l^2$.

example is more realistic, and it resembles the 3GPP-SCME model (see, e.g, [25], [26]). The average sum-rate per cell is shown in Table I for the proposed method, based on Algorithm 1, and for three simpler strategies:

- Omnidirectional beamforming with uniform full power distribution, and no interference by performing a randomly chosen one-to-one assignment between subsets of users and subcarriers.
- Optimal power control and subcarrier allocation (using Algorithm 1) with omnidirectional beamforming.
- Optimal power control and subcarrier allocation (using Algorithm 1) with matched filtering as the adaptive receive strategy.

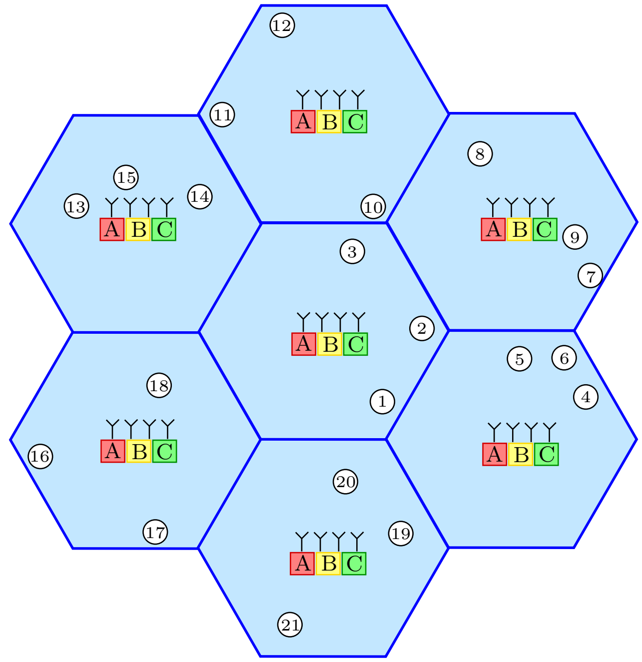

Fig. 4. A snapshot of the multicellular uplink system with 3 orthogonal subcarriers A, B, and C.

Beamforming	Power Control	Subc. Allocation	Spectral Effic.
Omnidirectional	Equal: 22 dBm	1 Subc./User	5
Omnidirectional	Joint Optimum		13
Matched Filter	Joint Optimum		21
Joint Optimum			35

TABLE I
AVERAGE SPECTRAL EFFICIENCY PER CELL FOR THE SYSTEM IN FIG. 4.

It can be seen that the joint optimization of all system resources yields significant gains.

Notice that in this example, a certain degree of cooperation between receivers for calculating the power control would be necessary (remember that the uplink beamformers are calculated in a distributed way). Reducing the amount of data that has to be exchanged in this cooperation would be an interesting topic for the future work.

VI. Conclusion

The paper exploits the DC structure in the problem of sum rate maximization in an interference limited system. Power allocation and interference filtering are optimized jointly using an iterative algorithm, which requires solving a sequence of convex problems. The generic algorithm is very general in terms of supporting a number of relevant system scenarios, with both differentiable and non-differentiable interference functions. In the differentiable case, it returns surely a stationary point of the problem, and admits very efficient solutions. Numerical simulations show its good practical performance and fast convergence. There are no heuristics involved in the algorithm, and its applicability does not depend on a particular channel realization. It also offers opportunities for extensions in order to perform a global DC optimization.

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