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# On the Optimal Design of MAC Protocols in Multi-Hop Ad Hoc Networks

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**Abstract**—In this paper, we present our results on the performance of MAC protocols in multi-hop wireless ad hoc networks in terms of the newly proposed metric “aggregate multi-hop information efficiency”. This metric captures the impact of the traffic conditions, the quality of service requirements for rate and correct packet reception, the number of hops and distance between a sender and its destination, and the outage probability for packet transmissions. Our network model considers a wireless network where nodes are distributed according to a homogeneous 2-D Poisson point process. Packets are generated following a Poisson distribution, and are forwarded to their destinations through a variable number of hops. Approximate expressions are derived for the outage probability of the ALOHA and CSMA MAC protocols, and validated with simulations. Various modifications of these protocols are considered, and their performances are compared. Moreover, an analytical procedure is presented to optimally design the communication so the multi-hop information efficiency performance of the network can be maximized.

## I. INTRODUCTION

As the need for supporting many simultaneous point-to-point communication links in ad hoc networks is ever increasing, the more crucial it becomes to fully understand the behavior of such systems. Wireless ad hoc networks (such as military battle fields, emergency operations, and sensor networks) are particularly of interest, as they are self-configuring and can be rapidly formed and deployed from whatever nodes available. One of the main challenges in the design of ad hoc networks is how to manage the interference. A popular approach is the use of medium access control (MAC) protocols, where two of most commonly-used protocols are ALOHA and Carrier Sensing Multiple Access (CSMA). In this paper, we consider both of these protocols and investigate their performance in terms of *aggregate multi-hop information efficiency (AMIE)*.

AMIE was introduced in [1] to capture the impact of the spatial progress of information, as well as the quality of the communication links and the number of hops between a source and its destination. This metric is closely related to the *transmission capacity*, proposed by Weber *et al.* in [2] and defined as the product of the density of successful active links and their communication rate. The main difference between such metrics is that the latter evaluates the spatial spectral efficiency of single-hop ad hoc networks, while the former also evaluates the progress of the information through multi-hop links.

In all the above-mentioned works, only the slotted ALOHA protocol is assumed. This brings us to the natural extension to consider unslotted ALOHA and furthermore the CSMA protocol. In the recent works of [3]–[5], the authors analyze the performance of the ALOHA and CSMA MAC protocols in single-hop ad hoc networks in terms of *outage probability*. Outage probability is defined as the probability that a receiver is not able to decode a transmitted packet correctly after a given number of backoffs and retransmissions. The system model considered in [4], [5] resembles our model, with the difference that we also allow for multi-hop communication. As a consequence, our performance metric has been revised to track changes in more system parameters than the ones considered in the aforementioned works.

This paper serves as an extension to prior analysis performed on the performance of the ALOHA and CSMA protocols. The novelty of this work lies in three aspects: 1) The model is extended to involve multi-hop networks, allowing for arbitrary number of hops; 2) The metric used for analysis, namely AMIE, is an enhanced version of other metrics, in order to take into account traffic conditions, quality of service requirements, and the routing strategy of the multi-hop environment; and 3) Performance improvement is provided by optimal design the communication rate used by the links.

The remaining of this paper is organized as follows. In Section II, we present the system model used for the system analysis. Section III introduces and explains the AMIE performance metric. In Section IV, we explain the technique used for analysis and we derive approximate expressions for the outage probability of the various MAC protocols. Section V presents an analytical procedure to design the communication rate between the links in order to maximize the AMIE. The numerical results and related discussions are provided in Section VI, while Section VII concludes this paper.

## II. SYSTEM MODEL

Our system model considers an ad hoc network where transmitter nodes are located on an infinite 2-D plane according to a homogeneous PPP with spatial density  $\lambda_s$  [nodes/m<sup>2</sup>]. Each transmitter has packets, of constant length  $T$ , arriving in time according to an independent 1-D PPP with density  $\lambda_{\text{pkt}}$  [packets/sec/node].

Applying the same approach described in [3]–[5], we find spatial density  $\lambda$  [packets/m<sup>2</sup>] of generated packets in a given time period  $T$  as:

$$\lambda = \lambda_s \times \lambda_{\text{pkt}} \times T. \quad (1)$$

Upon the formation of each packet, it is transmitted with a constant power  $\rho$  to its receiver in the next hop, which is assumed to be positioned a distance  $d_{\text{sh}}$  away. The multi-hop distance, i.e., the distance from each transmitter to its final destination, is denoted by  $d_{\text{mh}}$ . We assume that  $d_{\text{mh}}$  is fixed and that we have a perfect routing protocol (i.e. packets travel in a straight line from their sources to their destinations). Each packet goes through  $h$  hops to reach its destination.

For the channel model, only path loss attenuation effects (with exponent  $\alpha > 2$ ) are considered, i.e., additional channel effects such as shadowing and fast fading are ignored. Each receiver potentially sees interference from all transmitters, and these independent interference powers are added to the channel noise power  $\sigma^2$ , resulting in a signal to interference plus noise ratio (SINR) of:

$$\text{SINR} = \frac{\rho d_{\text{sh}}^{-\alpha}}{\sigma^2 + \sum_{i \in \mathcal{I}} \rho d_i^{-\alpha}}, \quad (2)$$

where  $\mathcal{I}$  is the random set of interferers on the network area,  $d_i$  is the distance between the  $i$ -th interferer and the reference receptor.

Assuming that packets are transmitted with a required rate  $\eta$ ,<sup>1</sup> and neglecting the thermal noise, the signal-to-interference ratio (SIR) threshold required for a packet to be received successfully,  $\beta_{\text{req}}$ , can be obtained by applying the Shannon's capacity formula [6], which yields

$$\beta_{\text{req}} = 2^\eta - 1. \quad (3)$$

Now, if the received SIR at each single hop falls below this required threshold  $\beta_{\text{req}}$  at any time during a packet transmission, the packet is received in error and must be retransmitted. The probability of this is denoted by  $P_{\text{error}}$ ;

$$P_{\text{error}} = \Pr \left\{ \text{SIR} = \frac{d_{\text{sh}}^{-\alpha}}{\sum_{i \in \mathcal{I}} d_i^{-\alpha}} \leq \beta_{\text{req}} \right\}. \quad (4)$$

The packet transmissions occur according to either ALOHA or CSMA. In ALOHA, all packets are transmitted regardless of the channel conditions. In CSMA, on the other hand, the decision-making node (this would be the transmitter in CSMA<sub>TX</sub> and the receiver in CSMA<sub>RX</sub>) senses the channel prior to transmission. If the measured SINR is above the sensing threshold  $\beta_{\text{sens}}$ , the packet transmission is initiated; otherwise, it is backed off. Due to space constraint, in this work we assume that  $\beta_{\text{sens}} = \beta_{\text{req}} = \beta$ . Each packet is given a maximum of  $M$  backoffs, before it is dropped. Since the objective of this work is not to evaluate the backoff scheme,

<sup>1</sup>Note that the rates  $\eta$  and  $\eta_{\text{sh}}$  are different. Their mathematical relation will be stated in the sequel of this section.

but rather to analyze qualitatively what its impact is on the outage probability, we simply assume that the backoff times are random, uncorrelated, and exponentially distributed.

Once the transmission is initiated, but the packet is received in error, the receiver informs its transmitter of this, and the packet is retransmitted. Each packet has a maximum of  $N$  retransmission attempts in order to be received correctly. If this is not achieved after  $N$  retransmissions, the packet is dropped and counted to be in outage, contributing to the total outage probability  $P_{\text{out}}$ . The communication between the transmitter and its receiver is assumed to occur over an orthogonal control channel, and the delay introduced by the feedback is assumed to be insignificant compared to the packet length.

### III. AGGREGATE MULTI-HOP INFORMATION EFFICIENCY

Aggregate multi-hop information efficiency (AMIE) was proposed in [1] and is our metric for evaluating the efficiency of packet progress in space, time, and frequency. The mathematical definition of AMIE, which is denoted by  $\epsilon$  and has units (bits·m)/(s·Hz·m<sup>2</sup>), is given by

$$\epsilon = d_{\text{mh}} \times \eta_{\text{mh}} \times \lambda_{\text{suc}}, \quad (5)$$

where  $d_{\text{mh}}$  is the multi-hop distance, i.e., distance between the source and its destination node,  $\eta_{\text{mh}}$  is the multi-hop spectral efficiency, and  $\lambda_{\text{suc}}$  is the density of successful transmissions.

Intuitively, when an  $h$ -hop link is considered, where

$$h = \frac{d_{\text{mh}}}{d_{\text{sh}}}, \quad (6)$$

a packet requires  $h$  times as many channel uses as a single-hop link to reach its destination. This means that the overall spectral efficiency  $\eta_{\text{mh}}$  of a multi-hop link can be computed as

$$\eta_{\text{mh}} = \frac{\eta_{\text{sh}}}{h}, \quad (7)$$

where  $\eta_{\text{sh}}$  is the single-hop spectral efficiency, defined as the required rate  $\eta$  divided by the average number of transmissions that attempt to access the channel. To simplify the analysis, we assume independence between the hops so the density of successful multi-hop links  $\lambda_{\text{suc}}$  can be evaluated as

$$\lambda_{\text{suc}} = \lambda \times (1 - P_{\text{out}})^h. \quad (8)$$

Now, inserting Eqs. (6), (7), and (8) into Eq. (5), the AMIE formulation can be rewritten as

$$\epsilon = d_{\text{sh}} \times \eta_{\text{sh}} \times \lambda \times (1 - P_{\text{out}})^h. \quad (9)$$

As the only unknown parameter in our AMIE metric is  $P_{\text{out}}$ , we devote the following section to explaining the method used for the analysis and the derived results, as given in [5].

### IV. OUTAGE PROBABILITY ANALYSIS

In order to derive the outage probability of ALOHA, we apply the concept of *guard zones* [7]. Define  $s$  to be the distance between the receiver under observation, RX<sub>0</sub>, and its closest interfering transmitter that causes the SINR to fall

just below the threshold  $\beta$ . By manipulation of the SINR expression,  $s$  is derived to be:

$$s = \left( \frac{d_{\text{sh}}^{-\alpha}}{\beta} - \frac{\sigma^2}{\rho} \right)^{-\frac{1}{\alpha}}. \quad (10)$$

Consider a circle of radius  $s$  around  $\text{RX}_0$ , and denote this by  $B(\text{RX}_0, s)$ . There are two events that can cause error in the received packet of  $\text{RX}_0$ ; (1) If the accumulation of powers from all the interferers *outside*  $B(\text{RX}_0, s)$  results in the SINR at  $\text{RX}_0$  to fall below the threshold  $\beta$ , and (2) if at least one active transmitter, other than  $\text{RX}_0$ 's own transmitter,  $\text{TX}_0$ , falls inside  $B(\text{RX}_0, s)$  at any time during the packet transmission. The latter event yields a *lower bound* to the outage probability [2], which is shown to be tight around the actual outage probability. Hence, we only focus on this bound in our analysis.

#### A. ALOHA Protocol

In order to find the outage probability of ALOHA, we first need to find the density of interferers at each time instant. Allowing for retransmissions is equivalent to increasing the number of packets that attempt to access the channel. Since the waiting times are random and uncorrelated, we can neglect the correlation between the amount of interference detected in each retransmission attempt. Given that the probability of a packet being retransmitted in ALOHA is  $P_{\text{rt}}$ , we have that the density of active interferers is

$$\lambda_{\text{aloha}}(P_{\text{rt}}) = \lambda(1 + P_{\text{rt}} + P_{\text{rt}}^2 + \dots + P_{\text{rt}}^N) = \lambda \frac{1 - P_{\text{rt}}^{N+1}}{1 - P_{\text{rt}}}. \quad (11)$$

1) *Slotted ALOHA*: In *slotted* ALOHA, transmitters can only start their transmissions at the beginning of the next time slot after each packet has been formed. Such slotting of time improves the performance, as it removes the problem of partial overlap of packets. However, this is achieved at the expense of a need for synchronization. Using the expression for outage probability in a PPP environment,  $P_{\text{out}} = 1 - e^{-\mathbb{E}[\# \text{ of interferers}]}$ , we reach the following theorem.

*Theorem 1*: The outage probability of slotted ALOHA can be approximated by  $\tilde{P}_{\text{out}}(\text{Slotted ALOHA}) \approx P_{\text{rt}}^{N+1}$ , where  $P_{\text{rt}}$  is the solution to:

$$P_{\text{rt}} = 1 - e^{-\lambda \frac{1 - P_{\text{rt}}^{N+1}}{1 - P_{\text{rt}}} \pi s^2}. \quad (12)$$

The proof of this theorem, as well as the following ones, can be found in [5].

2) *Unslotted ALOHA*: In *unslotted* ALOHA, communication is continuous in time, meaning that packets are transmitted as soon as they are formed. Continuous-time protocols are particularly of interest in systems that have no synchronization abilities. The outage probability of unslotted ALOHA is stated in the following theorem.

*Theorem 2*: The outage probability of unslotted ALOHA can be approximated by  $\tilde{P}_{\text{out}}(\text{Unslotted ALOHA}) \approx P_{\text{rt}}^{N+1}$ ,

where  $P_{\text{rt}}$  is the solution to:

$$P_{\text{rt}} = 1 - e^{-2\lambda \frac{1 - P_{\text{rt}}^{N+1}}{1 - P_{\text{rt}}} \pi s^2}. \quad (13)$$

#### B. CSMA Protocol

The CSMA protocol improves the performance of ALOHA as it allows the nodes to sense their channel prior to each transmission. If the measured or estimated received SINR at the start of the packet is greater than  $\beta$  as given by Eq. (3), the transmission is initiated immediately. Otherwise, it is backed off a random time. The probability of this is denoted by  $P_{\text{b}}$ . If the packet transmission is initiated, it has a probability  $P_{\text{rt}}$  to be received in error and thus be retransmitted. Based on this and also assuming independence between the backed off packets, we have that the density of packets attempting to access the channel is

$$\begin{aligned} \lambda_{\text{csma}}(P_{\text{b}}, P_{\text{during}}) &= \begin{cases} \lambda \sum_{m=0}^{M-1} P_{\text{b}}^m & ; \text{for } N = 0 \\ \lambda \left[ \sum_{m=0}^{M-1} P_{\text{b}}^m + (1 - P_{\text{b}}^M) P_{\text{during}} \sum_{n=0}^{N-1} P_{\text{rt}}^n \right] & ; \text{for } N > 0 \end{cases} \\ &= \lambda \left[ \frac{1 - P_{\text{b}}^M}{1 - P_{\text{b}}} + (1 - P_{\text{b}}^M) P_{\text{during}} \frac{1 - P_{\text{rt}}^N}{1 - P_{\text{rt}}} \right], \end{aligned} \quad (14)$$

where  $P_{\text{during}}$  is the probability that the packet goes in outage during its transmission, and  $P_{\text{rt}}$  is given by:  $P_{\text{rt}} = P_{\text{rx}} + (1 - P_{\text{rx}})P_{\text{during}}$ , with  $P_{\text{rx}}$  being the probability that the packet is in outage at the start of the packet once it decides to initiate transmission.

The total outage probability of CSMA can be expressed as

$$P_{\text{out}}(\text{CSMA}) = P_{\text{b}}^M + (1 - P_{\text{b}}^M) [P_{\text{b}} + (1 - P_{\text{b}})P_{\text{during}}]^N \left[ P_{\text{rx}|\text{transmit}} + (1 - P_{\text{rx}|\text{transmit}})P_{\text{during}} \right], \quad (15)$$

where  $P_{\text{rx}|\text{transmit}}$  is the probability that the receiver is in outage at its first transmission attempt.

1) *CSMA with Transmitter Sensing*: In  $\text{CSMA}_{\text{TX}}$ , the *transmitter* senses the channel upon arrival, estimates the SINR at its receiver, and thereby makes the backoff decision.  $\text{CSMA}_{\text{TX}}$  is the conventional CSMA protocol, and its outage probability is stated in the following theorem.

*Theorem 3*: The outage probability of  $\text{CSMA}_{\text{TX}}$  is given by Eq. (15), where

- $P_{\text{b}} \approx \tilde{P}_{\text{b}}$  is the approximate backoff probability, and is found as the solution to

$$\tilde{P}_{\text{b}} = 1 - e^{-\lambda \left( 1 - \tilde{P}_{\text{b}}^M + (1 - \tilde{P}_{\text{b}}^M) \tilde{P}_{\text{during}} \frac{1 - \tilde{P}_{\text{rt}}^N}{1 - \tilde{P}_{\text{rt}}} \right) \pi s^2}, \quad (16)$$

where  $P_{\text{rt}} = P_{\text{b}} + (1 - P_{\text{b}})P_{\text{during}}^{\text{TX}}$ .

- $P_{\text{during}} \approx \tilde{P}_{\text{during}}^{\text{TX}}$  is the approximate probability that an activated packet is received erroneously some time during its transmission and must thus be retransmitted;

$$\tilde{P}_{\text{during}}^{\text{TX}} = 1 - e^{-\int_{s-d_{\text{sh}}}^s \lambda_{\text{csma}} \left[ 2\pi - 2 \cos^{-1} \left( \frac{r^2 + d_{\text{sh}}^2 - s^2}{2d_{\text{sh}}r} \right) \right] r dr},$$

- $\tilde{P}_{\text{rx}|\text{transmit}}$  is the approximate probability that the receiver

is in outage at the start of the packet at the first transmission attempt, given its transmitter has decided to transmit;

$$\tilde{P}_{\text{rx}|\text{transmit}} = \tilde{P}_b \left[ 1 - \frac{1}{\pi s^2} \left( 2s^2 \cos^{-1} \left( \frac{d_{\text{sh}}}{2s} \right) - d_{\text{sh}} s \sqrt{1 - \frac{d_{\text{sh}}^2}{4s^2}} \right) \right].$$

2) *CSMA with Receiver Sensing*: CSMA<sub>RX</sub> is the modified version of the conventional CSMA protocol, and was proposed in [3]. In this protocol, the *receiver* senses the channel at the start of each packet and informs its transmitter over an orthogonal control channel whether or not to back off. The simple feedback channel added in CSMA<sub>RX</sub> was shown to provide considerable performance gain. The outage probability of CSMA<sub>RX</sub> is given by the following theorem.

*Theorem 4*: The outage probability of CSMA<sub>RX</sub> is given by Eq. (15), where

- $P_b \approx \tilde{P}_b$  is the approximate backoff probability, found as the solution to Eq. (16);  $P_{\text{rx}|\text{transmit}} = 0$ ; and
- $P_{\text{during}} \approx \tilde{P}_{\text{during}}^{\text{RX}}$  is the approximate probability that an activated packet is received erroneously some time during its transmission and must thus be retransmitted;

$$\tilde{P}_{\text{during}}^{\text{RX}} = 1 - e \left( - \int_{s-d_{\text{sh}}}^s \int_{\gamma(r)}^{2\pi-\gamma(r)} \lambda_{\text{csma}} P(\text{active}|r, \phi) r \, d\phi dr \right),$$

where  $\lambda_{\text{csma}}$  is given by Eq. (14), and

$$P(\text{active}|d, \phi) = 1 - \frac{1}{\pi} \cos^{-1} \left( \frac{r^2 + 2d_{\text{sh}}^2 - s^2 - 2d_{\text{sh}}r \cos \phi}{2d_{\text{sh}} \sqrt{r^2 + d_{\text{sh}}^2 - 2d_{\text{sh}}r \cos \phi}} \right)$$

$$\nu(r) = \cos^{-1} \left( \frac{r^2 + 2d_{\text{sh}}s - s^2}{2d_{\text{sh}}r} \right). \quad (17)$$

## V. OPTIMAL OPERATING POINT OF AMIE

The AMIE of the ALOHA and CSMA protocols, given by Eq. (9), is dependent on several system parameters, namely  $d_{\text{sh}}$ ,  $\eta_{\text{sh}}$ ,  $M$  and  $N$ .<sup>2</sup> Hence, in order to achieve optimal performance of a multi-hop ad hoc network, we must optimize AMIE with respect to all these system parameters. Due to the space limitation, we only present in this paper a simple analytical procedure to optimize the AMIE of the network in terms of the required rate  $\eta_{\text{sh}}$ .

Following the classical approach, the global maximum of a convex function can be obtained by first derivative test. In our interesting case,

$$\left. \frac{\partial \epsilon}{\partial \eta_{\text{sh}}} \right|_{\eta_{\text{sh}} = \eta^*} = 0, \quad (18)$$

where  $\eta^*$  is the required rate that optimizes  $\epsilon$ .

## VI. NUMERICAL RESULTS

In this section, we evaluate the AMIE, given by (9), for the ALOHA and CSMA protocols described in Section IV. The numerical results are obtained for the following configuration:

<sup>2</sup>It is worth to notice that  $\lambda$  is not a design parameter, but rather it is a network characteristic.

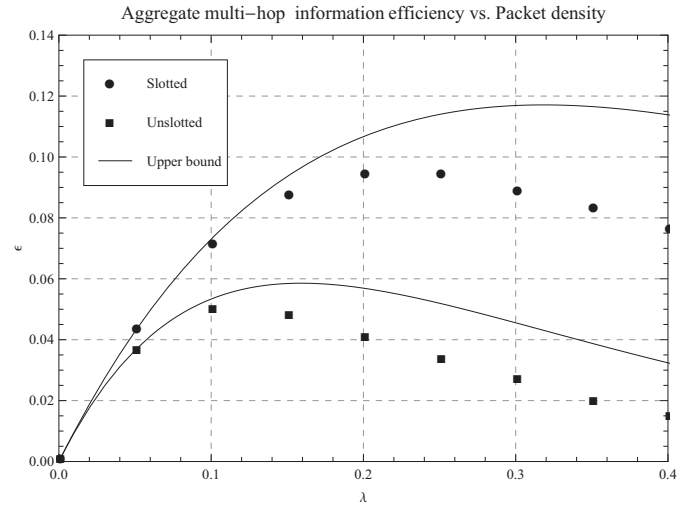


Fig. 1: Analytical and simulated AMIE ( $\epsilon$ ) versus the packet density  $\lambda$ , considering  $N = 0$ ,  $\eta = 1$  bit/s-Hz,  $\alpha = 4$ ,  $d_{\text{sh}} = 1$  m, and  $d_{\text{mh}} = 1$  m for slotted and unslotted ALOHA.

multi-hop distance  $d_{\text{mh}} = 1$  meter, packet duration  $T = 10$  seconds, and path-loss exponent  $\alpha = 4$ . In the following subsections, we first consider the slotted and unslotted ALOHA protocols, before we continue to evaluating CSMA<sub>TX</sub> and CSMA<sub>RX</sub>. In addition, due to the length limitation of the paper, we consider in this analysis  $N = 0$  and  $M = 1$ .

### A. ALOHA protocol

Fig. 1 presents a comparison between the actual values of  $\epsilon$ , obtained by Monte-Carlo simulations, and their analytical upper bound for the ALOHA protocol. It is easy to see that such an upper bound can be used as a good approximation for lower densities, while it performs poorly as  $\lambda$  increases. By construction of the lower bound on the outage probability, the events where the aggregate power of the non-dominant interferers can lead to an erroneous packet reception are not considered. Thus, when the density of interferers increases, such events become more frequent and, consequently, the lower bound to the outage probability becomes looser. This explains the behavior of the upper bound on the aggregate multi-hop information efficiency. Nevertheless, it is worth to point out that, in real networks, the quality requirements normally impose low outage probabilities, corresponding to lower densities, where our derived bounds are tight.

Fig. 1 also shows that there exists a density  $\lambda$  that optimizes  $\epsilon$ . Intuitively, increasing the packet density, more information will be transmitted throughout the network, which increases  $\epsilon$ . On the other hand, this  $\lambda$  increase leads to more interference, increasing the probability of outage events. This optimal value of  $\epsilon$  reflects the best trade-off between these aspects.

Moreover, we observe that slotted ALOHA always outperforms its unslotted version. This fact indicates the advantage of having synchronous transmissions in ALOHA networks (if the system has synchronization abilities).

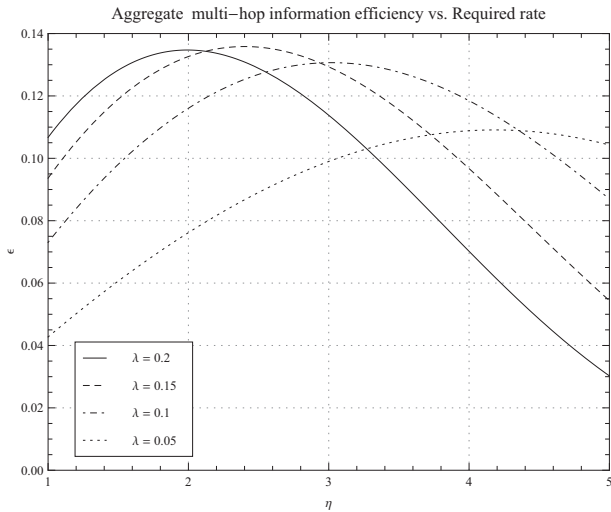


Fig. 2: AMIE ( $\epsilon$ ) of slotted ALOHA (upper bound) as a function of the required communication rate  $\eta$  considering  $N = 0$ ,  $\alpha = 4$ ,  $d_{sh} = 1$  m, and  $d_{mh} = 1$  m for different  $\lambda$ .

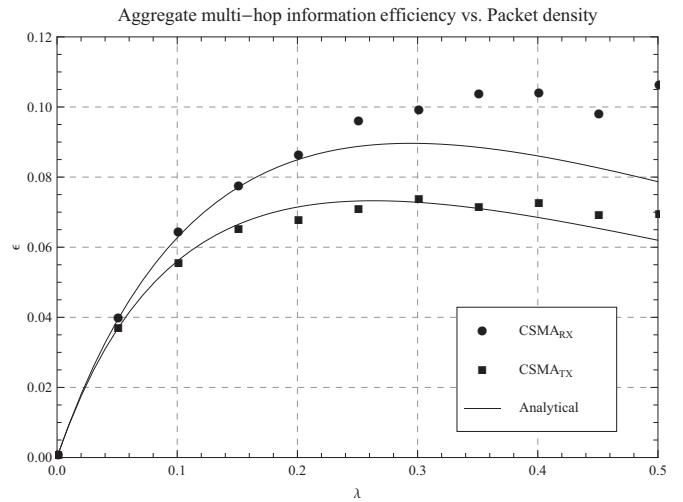


Fig. 4: Analytical and simulated AMIE ( $\epsilon$ ) versus the packet density  $\lambda$ , considering  $N = 0$ ,  $M = 1$ ,  $\beta = 1$ ,  $\eta = 1$  bit/s-Hz,  $\alpha = 4$ ,  $d_{sh} = 1$  m, and  $d_{mh} = 1$  m for CSMA<sub>RX</sub> and CSMA<sub>TX</sub>.

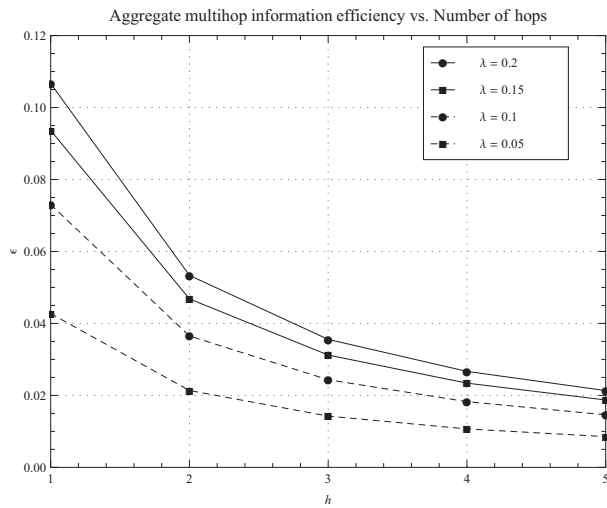


Fig. 3: AMIE ( $\epsilon$ ) of slotted ALOHA as a function of the number of hops  $h$  considering  $N = 0$ ,  $\beta = 1$ ,  $\alpha = 4$ , and  $d_{mh} = 1$  for different  $\lambda$ .

Fig. 2 illustrates how  $\epsilon$  varies as a function of the required rate  $\eta$  for slotted ALOHA<sup>3</sup>. The increase in  $\eta$  has a two-fold effect in the network. On one side, using a higher  $\eta$ , the efficiency of the links directly increases. On the other side, as  $\eta$  increases, so does the SIR threshold  $\beta$ , which implies a higher outage probability. The results reveals that, regardless the packet density considered, there is a rate  $\eta^*$  that maximizes  $\epsilon$ , reflecting the best tradeoff between interference robustness and the link spectral efficiency. This  $\eta^*$  can be obtained using the procedure described in the previous section.

<sup>3</sup>Similar behavior is seen for unslotted ALOHA, although with lower  $\epsilon$ .

Furthermore, Fig. 2 reveals that there is a relation between the best operational point of  $\eta$  and the packet density  $\lambda$ . As  $\lambda$  increases, the required rate  $\eta$  that maximizes the aggregate multi-hop information efficiency decreases. This fact indicates that in networks with higher interference levels (high values of  $\lambda$ ), the transmission rate  $\eta$  must be lower to guarantee a reliable communication. This result points out the dependence on the activity and the interference robustness to optimize the performance of the network.

Finally, Fig. 3 presents the aggregate multi-hop information efficiency as a function of the number of hops  $h$ . Considering a fixed multi-hop distance  $d_{mh}$ , we vary the fixed single-hop distance  $d_{sh}$  to control  $h$ . If a small  $d_{sh}$  is considered, the multi-hop link will be composed of several more robust single-hop links due to the distance-dependent path-loss. However, as the number of hops increases, the activity of the network also increases. Our results show that the degradation of the network performance due to the higher activity is the preponderant factor in the multi-hop aggregate information efficiency of the network regardless of the packet density  $\lambda$ .

### B. CSMA protocol

Fig. 4 shows the actual values and the analytical approximation of AMIE as a function of the packet density for the CSMA<sub>TX</sub> and CSMA<sub>RX</sub> protocols. We observe that the analytical formulas yield a good approximation to the simulation values of  $\epsilon$ , except for high densities. Clearly, increasing  $\lambda$  also increases the outage probability. However, due to the contention nature of the CSMA protocol, the network area will be composed of silent zones where some links can successfully transmit their packets. For this reason, the aggregate throughput and the AMIE of the network tends to a saturation value, i.e.,  $\lim_{\lambda \rightarrow \infty} \epsilon = c$ . This behavior can be seen for the simulation points, while the analytical approximations

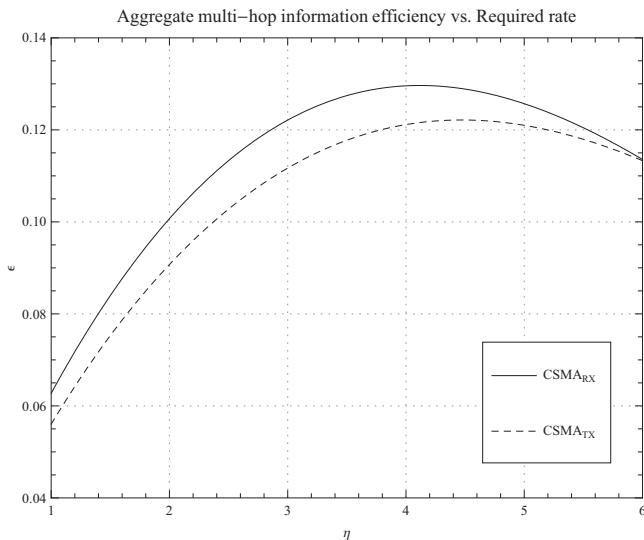


Fig. 5: AMIE ( $\epsilon$ ) as a function of the required communication rate  $\eta$  considering  $\lambda = 0.1$ ,  $N = 0$ ,  $M = 1$ ,  $\beta = 1$ ,  $\alpha = 4$ ,  $d_{sh} = 1$  m, and  $d_{mh} = 1$  m for both CSMA<sub>RX</sub> and CSMA<sub>TX</sub>.

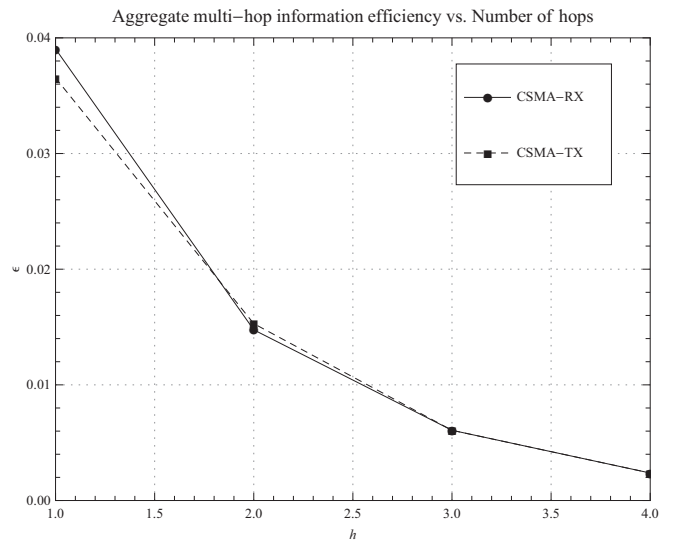


Fig. 6: AMIE ( $\epsilon$ ) as a function of the number of hops  $h$  considering  $\lambda = 0.005$ ,  $N = 0$ ,  $M = 1$ ,  $\beta = 1$ ,  $\alpha = 4$ , and  $d_{mh} = 1$  m for both CSMA<sub>RX</sub> and CSMA<sub>TX</sub>.

tend to decrease towards 0. In fact, the several assumptions on which our approximate outage probability expressions are based lead to a lower bound on  $\epsilon$ , where  $\lim_{\lambda \rightarrow \infty} \epsilon = 0$ .

Moreover, Fig. 4 also reveals that CSMA<sub>RX</sub> outperforms CSMA<sub>TX</sub>. This is due to the fact that the existence of a silent zone around the receiver is more effective in terms of avoiding collisions than the silent zone that the transmitter sensing provides. This yields a higher  $\epsilon$  for the CSMA<sub>RX</sub>.

Fig. 5 presents AMIE as a function of the required rate  $\eta$  for the CSMA protocols. Similar to Fig. 2, there exists a rate  $\eta$  that optimizes  $\epsilon$ , which reflects the best tradeoff between interference susceptibility and efficiency of the links.

Fig. 6 also presents AMIE versus  $h$  for the two different types of CSMA protocol studied in Section IV-B. Similar to Fig. 3, the increase in the number of hops degrades the network efficiency quantified in terms of AMIE, although the single-hop links are more robust against interference in CSMA as compared to ALOHA.

## VII. CONCLUSION

In this paper, we have investigated the performance of multi-hop ad hoc networks when various MAC protocols are applied for communication between the point-to-point links. The metric used for analysis is aggregate multi-hop information efficiency (AMIE), which captures the effect of traffic conditions, the quality of service requirements, and the routing strategy of the multi-hop network. Our ad hoc network consists of transmitter nodes distributed according to a 2-D Poisson point process, and each transmitter communicating with its own receiver located a fixed distance (through an arbitrary number of hops) away. Our derived analytical results are validated with Monte Carlo simulations, and the AMIE of

the various MAC protocols are compared for varying system parameters. It is shown that, in order to maximize AMIE, there exists an optimal tradeoff between the required transmission rate, the density of transmissions, and the SIR threshold for correct reception of packets as required by the system. Furthermore, we present an analytical procedure to find the required rate that reaches such optimal point.

As a natural extension of the work presented here, we plan to optimally design the several systemic parameters so as to achieve the global maximum AMIE for a given network scenario and subject to quality constraints.

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