Low Complexity Algorithms for Relay Selection and Power Control in Interference-Limited Environments

Lazaros Gkatzikis, Iordanis Koutsopoulos

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Lazaros Gkatzikis Iordanis Koutsopoulos
Department of Computer and Communications Engineering
University of Thessaly, Volos, Greece

***Abstract*—We consider an interference-limited wireless network, where multiple source-destination pairs compete for the same pool of relay nodes. In an attempt to maximize the sum rate of the system, we address the joint problem of relay assignment and power control. Initially, we study the autonomous scenario, where each source greedily selects the strategy (transmission power and relay) that maximizes its individual rate, leading to a simple one-shot algorithm of linear complexity. Then, we propose a more sophisticated algorithm of polynomial complexity that is amenable to distributed implementation through appropriate message passing. We evaluate the sum rate performance of the proposed algorithms and derive conditions for optimality. Our schemes incorporate two of the basic features of the LTE-Advanced broadband cellular system, namely *interference management* and *relaying*. We also provide guidelines on how our algorithms can be incorporated in such multichannel systems.**

Index Terms - 4G, LTE, power control, relay, SINR, sum rate maximization

I. Introduction

Cooperative communications exploit the broadcast nature of the wireless medium by using intermediate nodes as relays. Thus a virtual multiple input multiple output (MIMO) system is formed, realizing the benefits of spatial diversity even when each node is equipped with a single transceiver (see [1]).

Especially, in infrastructure-based wireless networks, it has been shown that the relays can be used to extend coverage and enhance throughput with minimum deployment cost. Although multihop communications require additional radio resources (frequency channels or time slots), relaying reduces the path loss significantly, by shortening the propagation path. This gain is maximized whenever an initial non line-of-sight (NLOS) path from a transmitter to the intended receiver can be split, through an intermediate relay, into two line-of-sight (LOS) links. The relay nodes also create diverse paths that mitigate the effects of fading during the transmission of data from the source to destination. Finally, relaying may also increase capacity by enabling spatial reuse, allowing thus multiple transmissions to take place simultaneously in the same frequency/time slot throughout a cell, as shown in [2]. However, in such scenarios interference management is of crucial importance.

In this work we consider the interference-limited environment that arises whenever multiple unicast communications take place over the same physical channel. Our network consists of active sources and destinations and inactive nodes that may serve as relays. In this setting, we study relay selection and power control as the main mechanisms of achieving the maximum sum rate performance. We derive simple heuristic algorithms that require minimal information exchange. These algorithms are ideal for scenarios where the resource allocation decisions need to be made fast e.g due to rapidly changing channel conditions. We also derive algorithms of polynomial complexity, exhibiting near optimal performance.

A. Related work

Several relaying strategies have been proposed with *Amplify and Forward* (AaF) and *Decode and Forward* (DaF) being the most common ones (see [3]). In the former, the relay acts as a repeater, amplifying the received signal (noise included) in the analog domain, whereas in DaF the relay decodes the received signal, re-encodes it and forwards it to the destination. Regardless of the strategy applied, the performance of cooperative communications highly depends on the efficient allocation of the network resources, namely the proper relay assignment to the sources and the power control across the transmissions.

In this direction, the authors of [4], propose an iterative relay selection algorithm with a max-min fairness objective. Relays that may offer a benefit to the minimum capacity achieving source are marked as candidate nodes and finally the best one is selected. After some iterations where reassignments occur, the proposed algorithm converges to the optimal assignment. In [5] the problems of relay selection and power allocation are modeled as auctions, where each user makes best response bids to maximize its utility and the relay allocates its transmission power according to the bids. This leads to a distributed algorithm that converges to a Nash equilibrium point. This resource allocation problem is modeled as a Stackelberg game in [6], with the sources being the buyers and the relays the sellers. A similar problem, that of scheduling users over multiple OFDM carriers in relay-enabled networks, is addressed in [7]. Our analysis in the sections that follow clarifies the similarities of these two problems.

The importance of the relay selection problem is indicated by the interest shown lately by the research community in the incorporation of relays in the next generation wireless systems.

A practical system that benefits from the introduction of relay nodes is the 802.16j that was recently finalized in [8]. Besides, LTE-Advanced and IEEE 802.16m (under development [9], [10]) consider the use of relays as a means to meet the requirements of the 4G mobile wireless communication system IMT-Advanced, especially for users located close to the cell edge. All these systems are based on orthogonal frequency division multiple access (OFDMA) schemes for the downlink, mitigating thus the effects of intersymbol interference (ISI) and providing robustness to frequency selective fading.

A single 802.16 cell consisting of a Base Station (BS), several infrastructure Relay Stations (RS) and Mobile Stations (MS) uniformly distributed within a cell is considered in [11]. The authors of this paper show that in the downlink, when the relays operate in transparent mode, i.e. just forward data but take no synchronization or control decisions, significant throughput improvement appears only for the half of the cell coverage area. In a similar setting [12] quantifies the tradeoff between coverage extension and capacity increase, that relays may offer through spatial reuse.

Most of the works in the field of cooperative communications, assume that the transmissions take place over orthogonal channels. In TDMA, OFDMA and CDMA for example, interference is caused only by transmitters using the same time slot, frequency channel and spreading code respectively. However, due to the scarcity of channel resources, in an attempt to improve spectrum efficiency, frequency reuse is very common in practice. For example, in cellular systems neighboring cells tend to use the same channels. Most existing works either consider this interference negligible or handle it as noise. Strong interference may appear though, whenever adjacent cells transmit over the same time/frequency to the same location. The impact of interference becomes even more significant when the coverage areas of neighboring BSs overlap. In such cases proper interference management is of crucial importance, requires though extensive cooperation among the BSs for the relay selection and frequency allocation. Furthermore, transmission power control can be used to tackle the problem of interference.

It was only recently that the standardization committees recognized inter-cell interference as one of the primary limiting factors of the performance of current cellular systems and set interference management as one of the major research directions for the next generation communications. In this direction, the authors of [13] propose a heuristic power allocation scheme, where each relay selects its power so as to achieve a minimum bit error requirement and minimize the interference caused. [14] investigates the performance of several emerging half-duplex relay strategies in interference-limited cellular systems. In [15] a message passing based algorithm for distributed power control and scheduling in a line network is proposed. It is shown to be optimal for the K-hop interference model.

B. Our contribution

To the best of our knowledge, this is the first work that addresses the joint problem of relay assignment and power control towards maximizing the sum rate of the system in interference-limited two-hop networks. The difficulty of this problem lies on its high complexity, caused by the following:

i. Interference makes the relay selection and power control strongly coupled
ii. Due to interference, one's transmission power affects all the others
iii. The first and second hop transmission rates are coupled, since the achievable rate of the bottleneck link determines the maximum end-to-end rate.

Thus, initially, we assume that each source transmits without knowing whether a relay will be used to assist its transmission. Since rate is an increasing function of ones transmission power, everyone is expected to transmit at maximum power to achieve the maximum individual rate. However, later we relax this assumption, by introducing a protocol that allows sources and relays to coordinate their actions in an attempt to improve the total rate of the system.

The contributions of this paper are the following:

1) We develop lightweight resource allocation algorithms (of at most polynomial complexity), amenable to distributed implementation and applicable to any relay assisted network (from ad-hoc to infrastructure-based ones) and any relaying strategy.
2) We derive conditions for the optimality of our proposed algorithms and characterize their impact on the sum rate performance of the system.
3) We present a case study for the LTE-Advanced system, indicating the applicability of our proposed algorithms and the performance benefits derived.

This paper is organized as follows. In section II we present our system model and the assumptions made. In section III we describe our relay selection and power control algorithms and derive conditions for optimality. Section IV describes how our proposed algorithms can be applied in the LTE-Advanced system. Numerical results quantifying the performance of our proposed schemes are presented in section V for an interference-limited ad-hoc network and an LTE-Advanced scenario. Section VI concludes our study.

II. System Model

Consider a wireless network of N sources, N destinations and K intermediate nodes, which serve as relays for the transmitted signals, all arbitrarily located in a plane. We denote with $\mathcal{S} = \{S_1, S_2, \dots S_N\}$ the set of the sources, $\mathcal{D} = \{D_1, D_2, \dots D_N\}$ the set of the destination nodes and $\mathcal{R} = \{R_1, R_2, \dots R_K\}$ the set of the potential relays. Sets $\mathcal{S}$,$\mathcal{R}$ and $\mathcal{D}$ are disjoint sets. We consider only point-to-point (unicast) communications with $\{S_i, D_i\}$ defining communication pair i. Each source has always packets to transmit in each queue for the corresponding receiver and may transmit them either directly to the destination or through a relay. All the

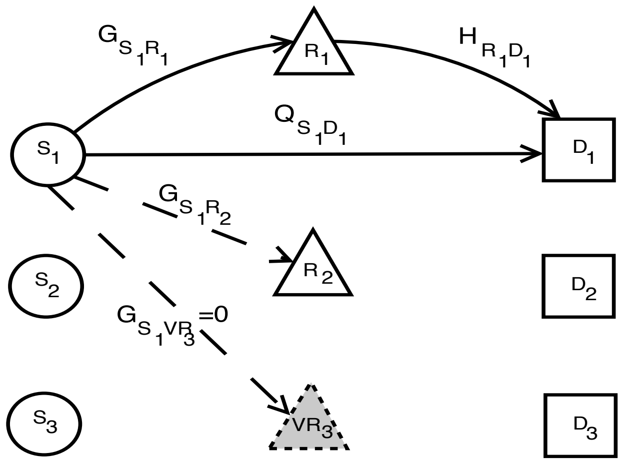


Fig. 1. A network of $N = 3$ communication pairs and $K = 2$ relays

transmissions take place in the same frequency channel and thus interference has to be taken into consideration.

Each node has a single transceiver and consequently, simultaneous transmission and reception is not feasible. Thus, transmissions are performed in a frame by frame basis, with each frame consisting of two timeslots of fixed duration $T/2$ time units each. During the first one, the sources transmit and the relays overhear the transmission. During the second one, each relay forwards it to the proper destination. Here for simplicity weassume that the two timeslots are of equal duration. Another option would be to set the relative duration of the timeslots, such that for each Source-Relay-Destination link, equal amount of information is transferred through the two hops. However, such a scheme would require extensive coordination among the transmitters in order to synchronize their transmissions. An analysis of the scenario of unequal timeslots is presented in [16]), but for the case of orthogonal channels, where no interference exists.

We use $\boldsymbol{G}$, $\boldsymbol{H}$ and $\boldsymbol{Q}$ to denote the source-relay, relay-destination and source-destination channel gain matrices respectively. For example, element $G_{S_iR_k}$ captures fading, path loss and antenna gains of the link between nodes S_i and R_k. We assume that the transmission frame length is small compared to the channel coherence time and as a result all channel gains can be considered fixed during the time of interest. Such a network is shown in Fig. 1.

In this general setting, we would like to find the assignment $\boldsymbol{a} = [a_1, a_2, \dots, a_N]^T$ of the relays to the sources and the transmission power control in the sources $\boldsymbol{p}_S = [p_{S_1}, p_{S_2}, \dots, p_{S_N}]^T$ and the relays $\boldsymbol{p}_R = [p_{R_1}, p_{R_2}, \dots, p_{R_K}]^T$ that maximize the end-to-end rate of the system, under a maximum transmission power constraint for each transmitter. Without loss of generality, we assume that all nodes are characterized by the same maximum transmission power p_{max} and the assignments are described by:

$$a_i = \begin{cases} R_k, & \text{if relay } R_k \text{ is assigned to source } S_i \\ 0, & \text{no relay assigned (direct transmission).} \end{cases} \quad (1)$$

We mention here that we do not consider the scenario where multiple relays assist a single source. Nevertheless, we allow the same relay to be assigned to more than one sources. In this case some form of scheduling is required.

Our ultimate objective can be formally written as

$$\begin{array}{lll} \underset{\boldsymbol{p_S}, \boldsymbol{p_R}, \boldsymbol{a}}{\text{maximize}} & \displaystyle\sum_{i \in \mathcal{S}} r_{S_i}^{a_i} & \\ \text{s.t.} & 0 \leq p_{S_i} \leq p_{max} & \forall S_i \\ & 0 \leq p_{R_j} \leq p_{max} & \forall R_j. \end{array} \quad (2)$$

The expression of $r_{S_i}^{a_i}$, the end-to-end rate of source node S_i assisted by relay a_i, depends also on the strategy that the relay applies, with *Decode and Forward* (DaF), *Amplify and Forward* (AaF) and *Compress and Forward* (CaF) being the most common ones. Regardless of the strategy used, the rate is an expression of the following form:

$$r_{S_i}^{a_i} = f(\text{SINR}_{S_iD_i}, \text{SINR}_{S_ia_i}, \text{SINR}_{a_iD_i}). \quad (3)$$

We use the notation K_{ab} to denote a parameter K referring to link $a \to b$, where a is the transmitter and b the receiver. Thus, the signal to interference ratio (SINR) at the receiver of relay a_i, when it decodes the transmission of source S_i is denoted as $\text{SINR}_{S_ia_i}$ and is given by:

$$\text{SINR}_{S_ia_i} = \frac{G_{S_ia_i} p_{S_i}}{\displaystyle\sum_{l \in \mathcal{S} \setminus S_i} G_{la_i} p_l + \sigma_{a_i}^2}, \quad (4)$$

where $\sigma_{a_i}^2$ is the variance of the zero mean noise in the receiver of the relay. Obviously, the achievable rate of a transmission, where no relay is used, depends only on the SINR of the direct link, i.e. the first term.

In this work, we mainly focus on the DaF scenario, with only the sources transmitting during the first timeslot. Each relay decodes the signal and re-encodes it. During the second timeslot only the relays transmit, forwarding the signal to the respective destination. Finally, each destination has to retrieve the original signal out of these transmissions. We assume that a destination decodes either the signal received from the respective relay or the direct signal, if no relay is used, leading to the following end-to-end rate expression:

$$\begin{aligned} r_{S_i}^{a_i} &= \max\left\{ r_{S_i}^0, \min\left\{ r_{S_ia_i}, r_{a_iD_i} \right\} \right\} \\ &= \frac{W}{2} \log_2 \Bigg(1 + \max\Big\{ \text{SINR}_{S_iD_i}, \\ &\qquad \min\left\{ \text{SINR}_{S_ia_i}, \text{SINR}_{a_iD_i} \right\} \Big\} \Bigg), \end{aligned} \quad (5)$$

where W is the channel bandwidth. Here we have assumed that the sources transmit only during the first timeslot, leaving hence the second one for the relay transmissions. This is where the $1/2$ factor comes from. However, this assumption may be easily relaxed by removing the $1/2$ factor from the direct transmission term. In order to improve the achievable decoding rate at the destination, but at the cost of increased complexity, maximal ratio combining can be applied at the receiver (see [3], [4]). If no relay is used, the destination will decode the signal coming directly from the source, or the combination of signals received from the respective source and relay otherwise. Then, the achievable rate would be:

$$r_{S_i}^{a_i} = \frac{W}{2}\log_2\Bigg(1+\max\Big\{\text{SINR}_{S_iD_i}, \min\{\text{SINR}_{S_ia_i}, \text{SINR}_{S_iD_i}+\text{SINR}_{a_iD_i}\}\Big\}\Bigg). \quad (6)$$

III. Relay assignment and power control in interference-limited environments

In order to solve the optimization problem defined in (2), we need to solve the problems of finding the transmission powers at the sources and the relays, and the relay assignment that give the system-wide optimal sum rate performance. These two problems are strongly coupled, since the optimality of a relay assignment depends on the selected transmission powers and vice versa. Consequently, solving it even in a centralized way is extremely difficult. Initially, we will try to decouple these two operations, by solving the two problems in an iterative way. That is, given an initial transmission power allocation we will attempt to find the optimal relay assignment and then given this assignment, we will try to find the optimal power allocation.

Given the others' powers, the rate of a node is an increasing function of its transmission power. Hence, a natural starting point is to assume that all sources transmit at maximum power in an attempt to maximize their individual rate. We also consider fully cooperative relays, that transmit at maximum power in order to forward the signals to the destinations. Since we do not consider any power budget constraints, which would restrict the willingness of the relays to forward messages in order to save energy, this is also a logical assumption. Thus, the initial power allocation is $\boldsymbol{p}_S^0 = \boldsymbol{p}_R^0 = p_{max}$.

A. The relay selection problem

If we assume full Channel State Information (CSI) at the transmitter, the problem of finding the optimal relay assignment is of exponential in the number of sources complexity, namely $O\Big((K+1)^N\Big)$, since each source has $K+1$ choices, either to transmit through one of the K relays or directly to the destination. However, any schedule, where a relay is assigned to more than one sources cannot be sum rate optimal.

Remark 1: The sum rate optimal assignment is a matching from the set of sources $\mathcal{S}$ to the set of relays $\mathcal{R}$.

Proof: We will prove this by contradiction. Assume that the sum rate $\tilde{r}$ achieved by an assignment where more than one sources, say $\{S_1, S_2, \ldots, S_l\}$ use the relay R_k is optimal. Since each relay has a single transceiver and operates in a specific frequency, a time sharing schedule has to be applied. Given the randomness of the channel gains the probability that any two of these sources achieve equal rates, i.e. $r_{S_i}^{R_k} = r_{S_j}^{R_k}$ is zero. As a result the rates are ordered, with say $r_{S_i}^{R_k}$ having the largest value. Thus, if instead of having the relay being shared by all these sources, we assign it to user S_i we get a $r^* > \tilde{r}$. Consequently no assignment, where a relay is assigned to more than one sources, can be sum rate optimal . ■

This way the complexity of the problem is reduced, but remains exponential. In this paper we aim to develop distributed protocols that can guarantee near optimal performance in at most polynomial time.

From the rate expressions above, we notice that under the full CSI assumption at the transmitter, the sources are ignorant of the CSI of the second hop, and consequently they are not able to identify whether using a relay is beneficial. However, they can exclude some relays, which compared to the direct transmission cannot offer any rate improvement. Generally, each sourceS_i is able to categorize the relays in two sets, namely the bad ones $\mathcal{B}_{S_i} = \{R_k : \text{SINR}_{S_iD_i} \geq \text{SINR}_{S_iR_k}\}$ and the unknown ones $\mathcal{U}_{S_i} = \{R_k : \text{SINR}_{S_iD_i} < \text{SINR}_{S_iR_k}\}$. The first one includes all the relays, which cannot improve the performance of source S_i, whereas no decision can be made a priori for the relays in the latter one, since the achievable rate depends also on second hop parameters. As a result, each source has to make a guess on which relay may offer the maximum rate. This could be done in the way that follows.

One-shot greedy algorithm

Each source greedily selects the relay that is expected to maximize its own rate. That is, assuming full CSI at each source for its links to the relays, the source selects out of $\mathcal{U}_{S_i}$ the relay that has the best first hop performance, or none if $\mathcal{U}_{S_i} = \emptyset$. This relay selection can be formally described as:

$$\begin{aligned} a_i &= \arg\max_{k\in\{\mathcal{R}\cup 0\}} \hat{r}_{S_i}^k \\ &= \arg\max_{k\in\mathcal{R}}\{\text{SINR}_{S_iD_i}, \text{SINR}_{S_ik}\} \end{aligned} \quad (7)$$

We use the hat symbol ˆ to denote that this is an estimation of the actual achievable rate, based only on the first hop. Whenever a relay k is selected by two or more sources, since a relay within one timeslot cannot decode and forward more than one signal (see previous remark), it will forward the one that achieves the best rate, i.e. the strongest signal.

This myopic relay selection approach is of linear complexity and is expected to yield suboptimal assignments, whenever the performance of the second hop is the limiting factor. It may also yield suboptimal assignments even when this is not the case, i.e. even if all first hop channels are worse than the corresponding second hop ones. Since each source S_i either uses its best candidate relay out of $\mathcal{U}_{S_i}$ or none (direct transmission to the destination), whenever a relay is selected by two or more sources, useful relays may remain unassigned and the diversity gain is not fully exploited.

Remark 2: Whenever i) the achievable rates in the first hop links are smaller than the corresponding second hop ones (i.e. the first hop is the bottleneck) for all the communication pairs, and ii) the greedy algorithm returns an assignment where no relay is selected by more than one source, this is the optimal assignment. In this case the optimal assignment is a maximum matching of size $|S|$ from the set of sources $\mathcal{S}$ to the set $\mathcal{R} \cup \mathcal{D}$.

Other possible strategies that we do not consider in this work are either the unassigned sources not to transmit at all or the unassigned sources by using a marking mechanism to select in a subsequent round the best out of the available relays. Alternatively, we could allow the relays to decide on their own which signal to forward.

In this approach each source acts for itself and no coordination of actions exists. In the following paragraph we propose an alternative algorithm, which enforces the cooperation of the nodes through appropriate message exchanges.

Bipartite Maximum Weighted Matching (MWM) approach

The relay selection problem can be mapped into the problem of finding the maximum weighted matching in a properly constructed complete bipartite graph $\mathcal{G} = \{\mathcal{S}, \mathcal{R}, \mathcal{E}\}$, as the one shown in Fig. 2, where the weights of the edges are given by $w_{S_i R_k} = r_{S_i}^{R_k}$. If the interference in the receiver of each link is known, then these weights can be easily calculated.

As mentioned earlier, we assume that each node transmits whenever it has something to transmit, without considering the interference it causes. Then, interference in the first hop can be easily estimated through appropriate pilot transmissions. However, the interference in the second hop cannot be known a priori, since it depends on which relays will be selected to forward data. Thus, before selecting an assignment we can not estimate the interference in the receiver of each destination, and without the interference estimation we cannot calculate the achievable rate in the second hop. To overcome this, we will take a conservative approach and assume that in the optimal assignment all the relays are used, hence probably making an overestimation of the interference.

Under this assumption, the distributed algorithm of [17] can be applied to find the maximum weighted matching. The relays and the sources exchange messages and finally after $O\Big(\max(N, K)\Big)$ iterations we converge to the optimal assignment. However, this algorithm has some limitations that should be taken into account. First of all it works only on balanced bipartite graphs. Thus, in order to turn our graph into a balanced one, we need to introduce $|N - K|$ virtual nodes, either on the left side (virtual sources) with zero weights, if $N < K$, or on the right side (virtual relays) of the graph with $w_{S_i R_k} = r_{S_i}^0$ otherwise. Secondly, it converges only when the maximum weighted matching is unique. To guarantee this, we have to add infinitesimal random values ε_i (disturbances) into the weight of each link, making thus the previously equally weighted matchings, ordered.

In the following remark we derive conditions for the optimality of the MWM algorithm.

Remark 3: With high probability, i.e. with probability going to 1 as the number of sources over the number of relays goes to infinity ($\frac{N}{K} \to \infty$), our MWM algorithm finds the optimal assignment.

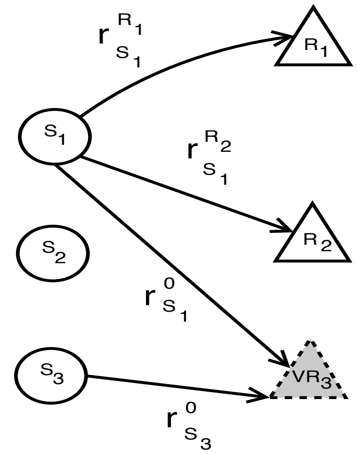


Fig. 2. The bipartite graph that we use to model the relay assignment as a MWM problem

Whenever in the optimal assignment every relay is assigned to a source, all the relays will eventually transmit. Consequently, the interference estimation that we made for the second hop will be accurate and the MWM will return the optimal assignment. As $\frac{N}{K} \to \infty$ every source can find a relay to improve its rate performance leading thus to a perfect matching.

Starting from the initial power allocation $\boldsymbol{p}_S^0$, $\boldsymbol{p}_R^0$, these algorithms can find an assignment of the available relays to the sources. Given this, we may then modify the transmission powers of the sources and the relays in an attempt to maximize the sum rate. In the following section we propose two alternatives for the power control part.

B. The power control problem

Given the relay assignment $\boldsymbol{a}$, we have to find the optimal transmission powers for the sources and the relays. In this direction, we propose the following heuristic power control that attempts to equalize the rates of the first and the second hop.

Rate equalization algorithm (Req)

In our setting, we have two apparently independent steps of power control, one in the first and one in the second hop. Nevertheless, they are coupled, since the rate of neither hop can be greater than the other. If $r_{S_i a_i} > r_{a_i D_i}$, the relay cannot forward the data to the destination at the rate they are transmitted by the source. Thus, we say that for this transmission pair the 1st hop is the bottleneck link. On the other hand, if $r_{S_i a_i} < r_{a_i D_i}$, the transmission rate of the relay cannot be fully utilized, leading to a bottleneck in the second hop.

As a result, for the given relay assignment and starting from initial power vectors $\boldsymbol{p}_S^0$, $\boldsymbol{p}_R^0$, the sources and the relays should iteratively update their powers to match the rate of the other hop. However, since increasing the transmission power, also increases interference, something that may lead us to worse sum rate performance, the rate equalization process should be applied only in the non-bottleneck links, leading the transmitters to reduce their transmission power. That is, for each source-relay assignment where $r_{S_i a_i} > r_{a_i D_i}$ the source S_i will reduce its transmission power to match the rate of the second hop. Otherwise the corresponding relay will have to reduce its transmission power.

The proposed heuristic for each communication pair can be formally written as:

Algorithm 1 Rate equalization step for communication pair i

if $r_{S_i a_i} \geq r_{a_i D_i}$ **then**
  $p_{S_i}^{t+1} = y$ such that $r_{S_i a_i}(y) = r_{a_i D_i}(\boldsymbol{p}_R^t)$
else
  $p_{a_i}^{t+1} = z$ such that $r_{a_i D_i}(z) = r_{S_i a_i}(\boldsymbol{p}_S^t)$
end if

If we apply this iteratively and since the power modification is always a power reduction we get a contraction mapping and convergence to a stationary point is guaranteed.

Joint source and relay power control algorithm (JsrPC)

The problem of sum rate maximization through power control has been extensively studied for single hop networks. Although it has not been solved yet due to its nonconvex nature, several approximations have been proposed. The authors of [18] proposed the approximation $\log(1+\text{SINR}) \approx \text{SINR}$ for the low SINR regime. On the other hand, in [19] a distributed algorithm that converges geometrically fast was proposed for the power control in the high SINR regime, based on the approximation $\log(1+\text{SINR}) \approx \log \text{SINR}$.

In our two-hop scenario the rates of the first and the second hop have to be equal. Thus, we may modify the aforementioned algorithms by incorporating the additional constraint $r_{S_i a_i} = r_{a_i D_i}$ for each communication pair i. To achieve this we may apply unconstrained power control on either hop, say the second one, and then a constrained one on the other. If we apply the unconstrained power control in the relay hop in timeslot t, the transmission powers of the sources for the high SINR regime will be given by:

$$p_{S_i}^{t+1} = min\left\{ \left(\sum_{k \in \mathcal{S} \setminus S_i} \frac{G_{S_i a_k}}{\sum_{l \in \mathcal{S} \setminus S_k} G_{S_l a_k} p_{S_l}^t + \sigma_{a_k}^2} \right)^{-1}, \; y \text{ such that } r_{S_i a_i}(y) = r_{a_i D_i}(\boldsymbol{p}_R^t), \; p_{max} \right\} \quad (8)$$

Here, the first term is the actual power control as described in [19]. The second term corresponds to the constraint introduced by the fact that increasing the rate of the source-relay link beyond the rate of the relay-destination link is meaningless. This is actually an incorporation of the Req algorithm(see Algorithm 1). The third term corresponds to the physical limitation of the maximum transmission power p_{max} that we assumed for every node of the system.

In the following remark we derive conditions for the optimality of the JsrPC algorithm.

Remark 4: In the high SINR regime (i.e. SINR>> 1) the JsrPC algorithm yields the optimal power allocation whenever the bottleneck is in the same hop for all the communication pairs.

Any combination of the aforementioned algorithms along with some control messages can lead to the design of distributed protocols of at most polynomial complexity.

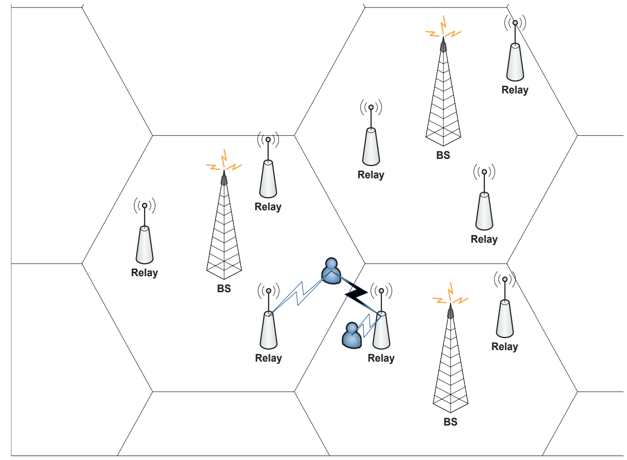


Fig. 3. The cellular structure of an LTE-Advanced system

IV. Our Algorithms in the Context of LTE-Advanced

The cellular downlink (uplink) communication scenario, where all the logical transmitters (receivers) reside in the same physical entity, namely the Base Station, comes as a special case of the previously described system model. In this section we deal with the downlink of the upcoming LTE-Advanced system. Such a system is shown in Fig. 3.

In these systems the communication network can be thought of as a multilevel hierarchical tree structure. The BS lies at the highest level, being the root, the RSs are organized in the intermediate levels and the MSs are the leaves. Thus, each downlink communication can be represented as a top-down path starting from the root, whereas the uplink takes place the reverse way. Here, we consider a two-hop downlink scenario consisting only of a single level of relays, which is also the case for LTE-Advanced. Nevertheless, the analysis performed here can be easily extended to the uplink.

In contrast to the generic scenario described earlier, here the relays are infrastructure nodes, strategically placed, either relatively close to the cell edge or in places where coverage holes appear, and much fewer in number than the MSs. Besides, since communications within a cell take place over orthogonal subcarriers, only inter-cell interference is apparent. Thus, now each BS has to decide how to assign the relays to the MSs, and how to allocate the subcarriers to the communication pairs.

From the point of view of the BS, it is easily deduced that in order to maximize the downlink throughput of the cellit will allocate all the available subcarriers. Thus, for the first timeslot the BS has to assign every single subcarrier to a receiver, which may be either an RS or an MS. If we assume that the subcarrier allocation is performed in an end-to-end path basis, i.e. the subcarriers used in the first timeslot for communication with relay R_k, are used in the second timeslot only by this same relay for transmission to the corresponding MS, the problem of relay selection and power control for each subcarrier maps back to our original problem, as a special case.

If we focus on a single subcarrier, the sources of interference are the transmitters of the adjacent cells that use the same subcarrier, namely BSs during the first hop and either BSs and/or relays during the second hop. Besides, in this setting no contention for the available relays exists among the BSs, since each one can only use the relays within its cell. In other words for each BS_i there exists a set of relays $\mathcal{R}_i$ that can be used, but all these are disjoint sets. A representation of this structure for a single subcarrier, the SC1, is shown in Fig. 4. From this figure it is obvious that for each subcarrier and each BS a proper bipartite graph like the one depicted in 2 can be created . Then, the relay selection algorithms of the previous section can be directly applied in this graph. The proposed power control algorithms are also directly applicable to this new setting, with the only difference being the sources of interference. However, in the cellular networks all the resource allocation decisions are made by the BS. Thus, the distributed versions developed earlier are needless. Instead centralized approaches should be applied like the well known Hungarian algorithm for MWM [20].

We saw earlier that in order to maximize the benefits of cooperative communications, the source (the BS in our case) has to be aware of the physical layer conditions for the BS-RS, BS-MS and RS-MS links. In LTE-Advanced such information can be acquired through the CSI-reference signals (CSI-RS), which estimate the condition of a channel and assist the beamforming and scheduling decisions. CSI-RSs are transmitted in every kth subframe, with k being configurable. In our scenario the BS needs also to know the interference caused by the adjacent cells. This can be realized through the Relative Narrowband TX Power (RNTP) indicator, which is transmitted by each BS to its neighbors through the X2 interface.

V. Numerical Results

In this section we present some simulation results for the generic scenario of an interference-limited ad-hoc network. We also simulate a cellular network, similar to the upcoming LTE-Advanced, in order to get some insight on the relative performance of the proposed schemes.

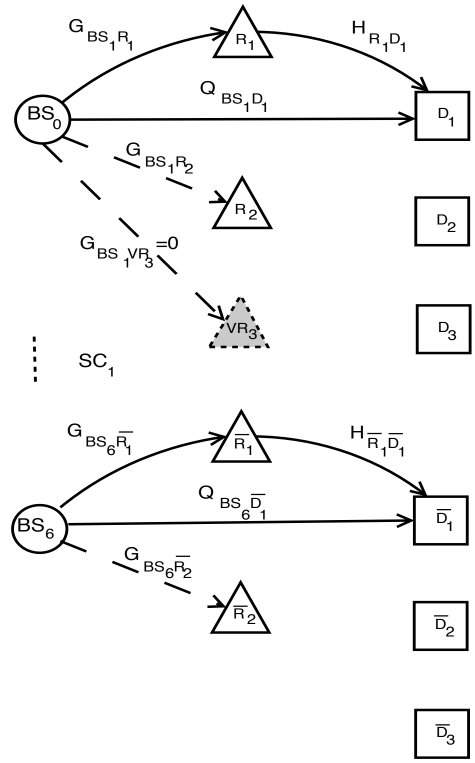


Fig. 4. A cellular system of 7 cells, with the central one having $N = 3$ MSs and $K = 2$ RSs

A. The generic scenario

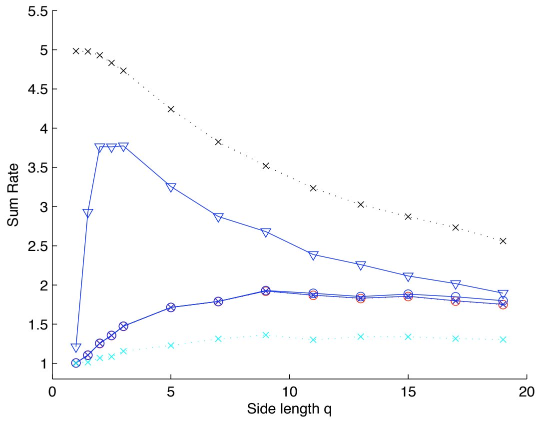
For the following simulations we assume a specific number of nodes, uniformly distributed over a given $q \times q$ square area. We use the channel gain function

$$G_{S_iR_k}(d_{S_iR_k}) = [\max(d_{S_iR_k}, d_0)]^{-\beta}, \tag{9}$$

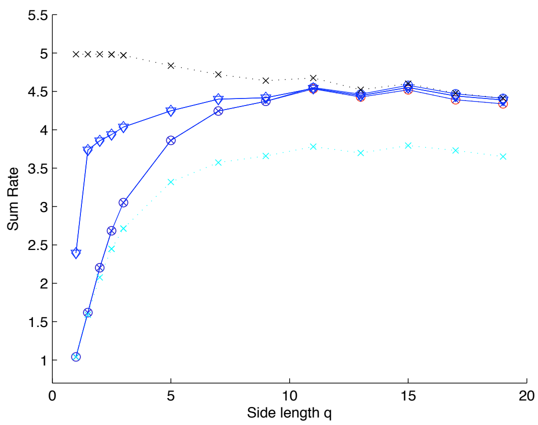
with $d_{S_iR_k}$ denoting the distance between S_i and R_k, and $d_0 = 1$ the radius around the transmitter where unit gain is assumed to hold. Besides, a path loss coefficient of $\beta = 2$, unit maximum power (p_{max}) and bandwidth (W) and white Gaussian noise (AWGN) of zero mean and variance 0.01 are also considered.

In an attempt to quantify the performance loss due to the decoupling of the relay selection and the power control, we begin with a simple motivating example of only two communication pairs and one relay, all randomly located within a square. In Fig. 5(a) we depict the impact of the side length q on the sum rate performance of the system. As the side length q increases, the mean distance of any two nodes increases(or equivalently the node density of the plane decreases). This causes both the actual transmissions and interference to experience higher path loss. We notice that up to a point, the gain from the proper relay selection increases and then remains stable. This is justified by the logarithmic nature of the rate expressions.

In such a simple network the relay assignment is quite trivial, leading thus to identical sum rate performance for the greedy and the MWM approach. Furthermore, applying the Req algorithm has no significant impact, since during

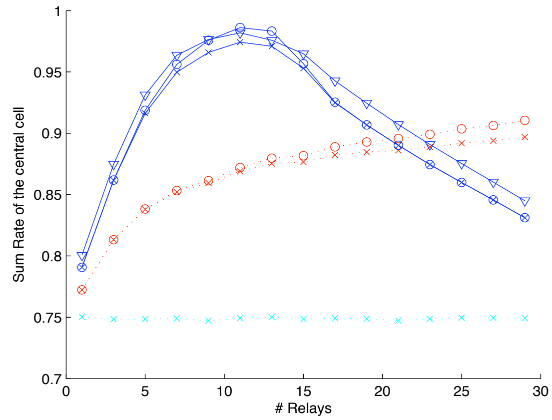


(a) Generic SINR

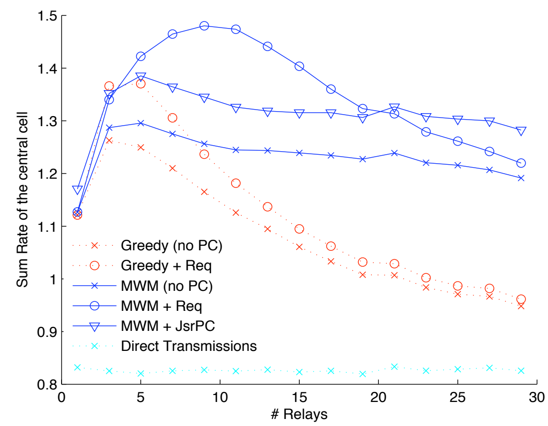


(b) High SINR regime

Fig. 5. Sum rate performance (for different SINR regimes) vs. Side length q of the square area



(a) Square dimension $q = 2$



(b) Square dimension $q = 10$

Fig. 6. Sum rate performance for 15 sources inside a square of side length q vs. Number of Relays K

the second hop there is no interference at all. On the other hand, applying the JsrPC algorithm, which combines the MWM relay selection with power control the performance is improved towards the optimum solution,especially for small distances where the gain from the interference mitigation is more significant.

However, in the previous setting, at least for small distances the interference is significant, making the assumption of high SINR regime not valid. Thus, we consider a different scenario where the two communication pairs lie on the opposite sides of the square, on the vertices and the relay is randomly positioned within the square. In Fig. 5(b) we show the impact of the side length q, which is also the distance separating the two communication pairs, in the high SINR regime. As we expected here our proposed algorithms perform much better, exhibiting near optimal performance. We mention that all the values depicted are mean values of at least 1000 simulation runs.

In order to study the scalability and the impact of the number of relays K on our algorithms, we consider larger scale random topologies consisting of $N = 15$ sources. We expect that up to a point as the number of available relays increases, better sum rate performance can be achieved. However, this is not always the case for our algorithms.

The greedy approach makes a decision based only on the first hop, an obviously myopic choice. On the other hand, the MWM relay selection overestimates interference in the second hop, when the optimal assignment leaves some relays unused. Thus, we expect worse performance as K gets larger than N, something evident in Fig. 6(a). Here, the small side length q maximizes the impact of interference overestimation, because the redundant terms have large values (due to high channel gains). Nevertheless, by comparing with the direct–transmission–only strategy, it is evident that our relay selection algorithms exploit the benefits of cooperative diversity. Furthermore, the proposed power control schemes improve the achievable sum rate in general. However, since all transmissions take place in the same channel, and we are considering a dense network, we lie in the extremely low SINR regime, where the JsrPC performs poorly.

In Fig. 6(b) we increase the side length q and since this also causes the path-loss to increase, we notice that the benefit from using a relay becomes more significant. Besides, this leads us to an environment of less interference, where the power control algorithms perform much better. Through proper relay selection and power control the achievable sum rate gain is almost 100%. Finally here the increase of relays does not cause that important performance losses, since due to the increased pathloss most of the relays are beneficial.

B. The LTE-Advanced scenario

In this section we study the impact of our proposed algorithms on the 3GPP LTE-Advanced system, a candidate for the upcoming 4G mobile wireless communication system IMT-Advanced. We consider a system of 7 cells, one in the center and its six direct neighbors, as the one shown in Fig. 3. Each cell has a radius of 3km, the RSs are deterministically placed in a distance of 1.5km from the BS, so that cell edge users benefit the most and 20 MSs are randomly located within each cell.

Although OFDMA is used in the downlink of LTE-Advanced, interference from neighboring cells exists. Our system consists of 32 data subcarriers sharing a total bandwidth of 5 Mhz. Here we assume LOS paths for the BS-RS, and the RS-MS communications. This is a logical assumption, since usually infrastructure relays are placed on the rooftops of high buildings. On the other hand, we consider NLOS path loss for the BS-MS links. We also assume lognormal shadowing. The NLOS model that we use is described by $36.5 + 23.5\log_{10} d + \chi_{\text{NLOS}}$ with $\chi_{\text{NLOS}} \sim \mathcal{N}(0,8)$ and $\chi_{\text{LOS}} \sim \mathcal{N}(0,3.4)$ in dB, and d being the distance between the transmitter and the receiver in meters.

Under this model we study the sum rate performance of the central cell. The neighboring cells not only serve as sources of interference, but also participate in the power control updates. Fig. 7 shows the performance of our proposed algorithms as a function of the number of available relays in the cell. The first thing one may notice is that in the cellular setting the proposed algorithms perform much better. This is due to the fact that in the LTE-Advanced the interference experienced by the receivers is in general some orders of magnitude smaller than the actual signal. Besides, the path loss model used here is more realistic and captures the fact that the relays exploit the LOS benefits. Thus, the sum rate performance of the cell is improved significantly. Concluding we could say that all the remarks made for the interference limited scenario also hold here, but now the environment is less demanding (lower interference, higher transmission powers etc.).

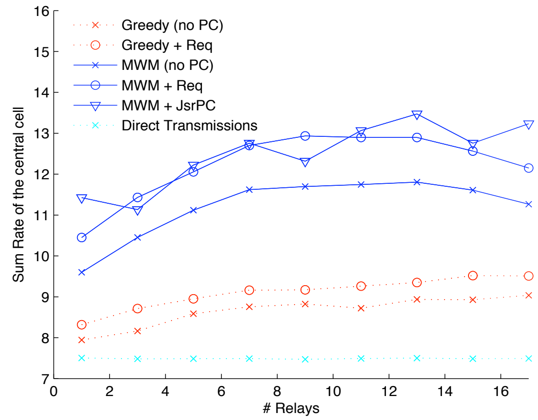


Fig. 7. The sum rate performance of 20 MSs inside a cell vs. the number of RSs K of the cell

VI. Conclusion

This work is a first step towards understanding the interaction between the relay selection and the power control process in interference-limited networks. We developed easy to implement distributed algorithms of at most polynomial complexity that are applicable to any type of relay assisted wireless systems and offer significant improvement in the sum rate performance. We also showed how our algorithms meet the objectives of interference management and relaying exploitation in the context of 4G wireless systems.

Throughout this work we assume full CSI at the transmitter. This is a logical assumption for contemporary cellular environments, which incorporate mechanisms for channel state estimation and where all the resource allocation decisions are made by the BS. However, in distributed scenarios, such as ad-hoc networks, where no central coordination entity exists, propagating CSI is not an easy task. Consequently, the impact of imperfect CSI or time varying channels on the performance and convergence of the proposed schemes is an interesting topic of future study, especially for the ad–hoc scenario.

Finally, it would be interesting to derive online versions of the proposed schemes that capture the dynamic scenario of nodes entering or leaving the system. In this case instead of running the algorithm from scratch, online matching schemes like the ones proposed in [21], [22] can be used.

VII. Acknowledgements

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