Bounded-Hop Strong Connectivity for Flocking Swarms

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***Abstract*—In this paper we consider a set of n mobile wireless nodes, which have no information about each other. The only information a single node holds is its current location and future mobility plan. We develop a two-phase distributed self-stabilizing scheme for producing a bounded hop-diameter communication graph.**

The first phase is dedicated to the construction of an underlying topology for the dissemination of data needed for the second phase. In the second phase the required topology is constructed by means of an asymmetric power assignment under two modes — static and dynamic. The former aims to provide a steady topology for some time interval, while the latter uses the constant node locations changes to produce a constantly changing topology, which succeeds to preserve the required property of the bounded hop-diameter.

For the static mode we provide an $O(\lambda, \lambda^2)$-bicriteria approximation algorithm so that given a parameter $1 \le \lambda \le n-1$, we construct a power assignment which induces a static h-bounded hop communication graph, $h = n/\lambda + \log\lambda$, with a cost of at most λ times the optimum and network lifetime of at least $1/\lambda^2$ times the optimum. For the dynamic mode, given a parameter $1 \le h \le n-1$ we construct an optimal power assignment (in terms of network lifetime) which induces a dynamic h-bounded hop communication graph.

I. Introduction and related work

Consider a swarm of unmanned aerial vehicles (UAVs) that flock in the sky and would like to communicate efficiently. The members of the swarm may discover each other by exchanging messages; periodically collecting information concerning the position, speed and direction of other members. Given the collected information about the current layout and near future changes, the members form an ad-hoc network by deciding on individual transmission ranges. The resulting network has important topological properties such as strong connectivity and small diameter. The obtained network that is computed to last in spite of movements serves as the communication backbone for the swarm for a while. An efficient communication backbone is essential to any network as it may reduce the total network traffic. A desirable property of such a network is that it has a small number of hops between each pair of members; which yield small delays and efficient routing complexity. We propose schemes for the construction of such dynamic communication backbones in the presence of moving participants.

The required communication backbone might have different topological requirements. For example in a battle situation, military UAVs might be required to coordinate the movement of troops, and therefore the communication backbone must have the all-to-all topological structure. While scientific UAVs performing an environmental research might require an all-to-one communication possibilities. In wireless networks, producing a communication backbone which matches some topological requirements is called *topology control* [7].

The topology of the network depends on the current distribution of the wireless nodes as well as the transmission range assignment of each node. The transmission range r_v of node v is determined by the power assigned to v, $p(v)$. It is customary to assume that the minimal transmission power required to transmit to distance d is d^α, where the *distance-power gradient* α is usually between 2 and 4, [26]. Thus, node v receives transmissions from node s iff $p(u) \ge d(u,v)^\alpha$, where $d(u,v)$ is the Euclidean distance between u and v. For example, in Figure 1(a) node v can receive transmissions from u, while w cannot. There are two possible models: symmetric and asymmetric. In the symmetric settings, also referred to as the undirected model, there is an undirected communication link between two nodes, u and v, if $p(u) \ge d(u,v)^\alpha$ and $p(v) \ge d(v,u)^\alpha$, that is u can reach v and v can reach u. The asymmetric variant allows directed (one-way) communication links between two nodes. In Figures 1(b) and 1(c) there is an exposition of communication links under the symmetric and asymmetric models, respectively, following the range assignment in Figure 1(a). This paper addresses the asymmetric model, i.e. we allow directed (one-way) communication links between two nodes.

Our focus is on the *bounded-hop strong connectivity* topology property. A graph $H = (V, E_H)$ is *h-bounded hop strongly connected* (in short, h-bounded), if for any pair of nodes $u, v \in V$, there exists a path from u to v in H with at most h edges. The hop-diameter of H, denoted $\Delta(H)$, is the minimum value of h for which H is h-bounded. Naturally, the strong connectivity (all-to-all) property is one of the most basic topology requirements; by limiting the hop-diameter of the network, we allow faster data transmission and easier routing.

As wireless nodes typically do not have a constant power supply, but rather have to rely on some limited energy reserves, energy efficiency becomes a critical factor in wireless networks design. Each node has some initial battery charge, which is sufficient for a limited amount of time, reliant on the power assigned to that node. The network lifetime is defined as the time it takes the first node to run out of its battery charge. In this paper we are interested in two optimization objectives, *minimizing the total energy consumption* and *maximizing the network lifetime* through topology control.

Topology control in mobile networks. In theory it is impossible to devise a range assignment that will satisfy the topology

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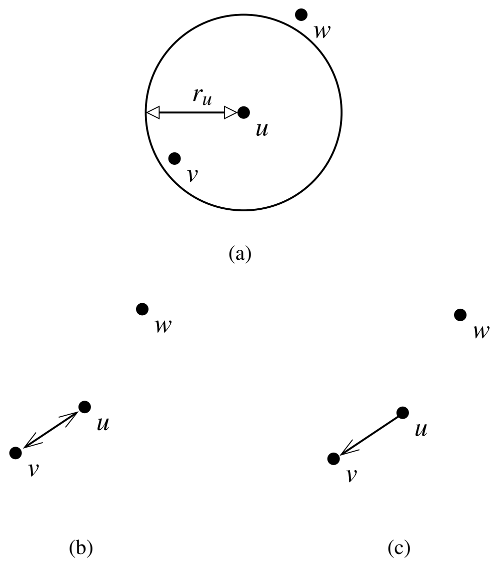


Fig. 1. Communication links under symmetric and asymmetric models: (a) Power assignment, $p(u) = (r_u)^2$; (b) Links under the symmetric model; (c) Links under the asymmetric model

requirement for a given period of time without knowing the future location changes. Each node has its own *mobility plan*, which is composed of direction vectors, velocity, acceleration, and so on. Basch et al. [2], [3] proposed an elegant method to handle topology updates for mobile nodes. They proposed a framework to maintain an invariant of a set of moving objects in a discrete manner, called the *kinetic data structure* (KDS in short). They introduce the idea of keeping certificates as triggers for updates. When an object moves and a certificate fails, the consistency of the kinetic data structure is invalidated and an update is mandatory. Each failure of a certificate incurs a setup of up to a constant number of new certificates. Hence we are allowed to monitor the dynamics of a set of objects discretely and efficiently. The kinetic data structure requires that we know the mobility plan (a specification of the future motion) of all nodes, and that the trajectory of each disk can be described by some low-degree algebraic curve. These structures are extremely efficient for topology maintenance, but do not address the issue of energy efficiency or the construction of initial topology. The approach taken in this paper resembles the spirit of KDS. Additional results for topology control in mobile networks may be found in [19], [22], [23], [25]. However, none of these works addresses the bounded-hop strong connectivity property.

Bounded hop-diameter. The only results for h-bounded strong connectivity were obtained for stationary networks. For the linear case of node disposition, Kirousis et al. [24] develop an optimal power assignment algorithm in $O(n^4)$ time. In the Euclidean case, [10] obtains constant ratio approximation algorithms for the bounded-hop vertex connectivity for well spread instances. Beier et al. [4] discuss the problem of finding a bounded-hop path between pairs of nodes with minimized power consumption. They find an optimal path in $O(hn\log n)$ time. In [5] the authors obtain $(O(\log n), O(\log n))$ bicriteria approximation algorithms for the bounded-hop broadcast, bounded-hop connectivity and bounded-hop symmetric connectivity problems. In their output there are at most $h\log n$ hops and the cost is at most $\log n$ times the optimal. In [1] the authors present an exact algorithm for solving the 2-hop broadcast problem with a running time of $O(n^7)$ as well as a PTAS with a running time of $O(n^\mu)$ where $\mu = O((h^2/\varepsilon)^{2^h})$. Funke and Laue [18] provide a PTAS for the h-broadcast algorithm in $O(n)$ time. Shpungin and Segal [29] give approximation algorithms for k-fault resistant bounded-hop broadcast for the linear and planar layout of nodes. They develop power assignments with a total cost of $O(k)$ and $O(k^2)$ times the optimum for the linear and planar cases, respectively. More on bounded range assignments can be found in [8], [11]. These results apply to stationary networks only and do not address the network lifetime optimization objective, with or without the total cost, like we do in this paper.

There are several challenges a network designer faces when developing a topology control algorithm in a mobile wireless network, such as: *How do the nodes discover each other? How do the nodes share the current layout information? How do the nodes share their mobility plans? How to discover new nodes? How to discover a node failure?* – All these questions must be answered before the construction of the communication backbone with the desirable property can begin.

Due to the unreliable nature of wireless networks coupled with mobility opportunities, it seems reasonable to divide the problem of topology control into two main phases: *discovery* and *construction* [12]. In the first phase (described in Section IV), the nodes execute a very basic distributed algorithm, for the discovery of other nodes and disseminating the mobility and current layout information. The general idea is to execute this algorithm at constant time intervals. The second phase (described in Section V) takes place between two consecutive executions of the first phase; let the time interval in between be $[t_s, t_f]$. Having acquired the mobility plans and current layout, each node now has all the required information to carry out the topology control algorithm and decide on its own power assignment. The topology constructed in the second phase is valid for the time interval $[t_s, t_f]$. Note that at the beginning of the first phase, each node is only aware of its current location and its own mobility plan. By adopting this scheme we are able to handle node failures and corruptions and thus to operate in an hostile environment. We also react to changes in the initial network settings, such as nodes arrival and departure. Some work was done on data dissemination and topology discovery for mobile networks [6], [28], and for stationary networks [13], [14], [17], [20], [21]. All these papers assume some underlying infrastructure for message passing, which is not the case we consider. Data dissemination algorithms might be used in the first phase after some basic underlying communication backbone is obtained.

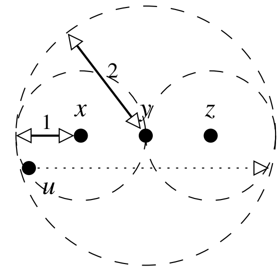
While in stationary wireless networks, the topology induced by a range assignment is fixed, in mobile wireless networks the topology changes frequently as a result of the nodes movement. It makes sense then to indicate the time interval, for which the induced topology is valid. In this paper we describe range assignments that induce a topology which is valid for some time interval $[t_s, t_f]$, where t_s and t_f are the start and finish times, respectively; we consider two possible modes for the topology construction – static and dynamic.

The **static mode**, preserves all the relevant communication links (those that are used for inducing the required topology) for the whole time interval $[t_s, t_f]$. Note that some other links might appear and disappear during the time interval, however the important links, which define the required topology remain unchanged. In other words, the communication graph, which is variant in time, always includes a subgraph which is unchanged for the whole time interval.

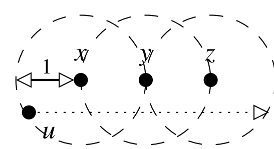
The **dynamic mode** is different in that there is no constant subgraph which holds the topology property. However, as communication links are added and removed, depending on the movement of the nodes, the topology property requirement (e.g. connected dominating set) is satisfied during the entire period $[t_s, t_f]$.

For example, in Figure 2, there are 4 nodes; x, y, z are stationary, while u moves along the dotted arrow. The topology requirement is to induce a connected dominating set. In Figure 2(a) we show the static mode. The power assignment $p(u) = 0$, $p(x) = p(z) = 1$, and $p(y) = 4$ ensures that the following edges are always present: $(x,y),(y,x),(y,z),(y,z),(y,u)$. The dynamic mode (Figure 2(b)) is different. The power assignment $p(u) = 0,\ p(x) = p(y) = p(z) = 1$ ensures the existence of the following edges: $(x,y),(y,x),(y,z),(y,z)$. However, there is always an edge to u from one of the nodes, x, y or z, therefore the connected dominating set property is always maintained.

This paper is organized as follows. In Section II we present our system settings, followed by Section III which states our contribution. Then, in Sections IV and V we describe the two phases of our scheme. Finally, in Section VI we conclude and discuss possible future work.



(a)



(b)

Fig. 2. Power assignment modes in mobile settings. Nodes x, y, z are stationary, while u moves along the dotted arrow and $p(u) = 0$. The topology requirement is *connected dominating set*. (a) Static mode: $p(x) = p(z) = 1$, $p(y) = 4$; u is always reachable by y; (b) Dynamic mode: $p(x) = p(y) = p(z) = 1$; u is always reachable by some node

II. System settings and contributions

The topology control phase takes place after successful propagation of the mobility plans, and each node is aware of its own and other nodes mobility plans for some fixed time interval $[t_s, t_f]$, where t_s and t_f are the start and finish times, respectively. By $t \in [t_s, t_f]$ we indicate that $t_s \le t \le t_f$.

Let V be the set of n mobile wireless nodes. As the distance between any two nodes $u, v \in V$ may vary in time, we define $d_{u,v}(t)$ to be the Euclidean distance between u and v at time $t \in [t_s, t_f]$. A power assignment is a function $p : V \to \mathbb{R}^+$, which assigns each node $v \in V$ a transmission range $r_v = \sqrt[\alpha]{p(v)}$ (in this work we assume $\alpha = 2$ for simplicity, although our results can be easily extended to any constant α). The transmission possibilities resulting from a power assignment vary in time. Let $H_p(t) = (V, E_p(t))$, with $E_p(t) = \{(u,v) : r_u \ge d_{u,v}(t)\}$, be the induced directed communication graph at time $t \in [t_s, t_f]$. The cost of the power assignment is defined as $c(p) = \sum_{v \in V} p(v)$.

The lifetime of the network is defined as the time it takes the first node to run out of its battery charge. Each node $v \in V$ has some initial battery charge b_v, which is sufficient for some limited time, depending on the power assignment $p(v)$. It is common to take the lifetime of v to be $l_v = b_v / p(v)$, that is after a time interval of length $b_v / p(v)$ the battery is completely depleted. The lifetime of the whole network for a given power assignment p is $l(p) = \min_{v \in V} l_v$. This paper assumes that all the nodes have the same initial battery charge b.

Definition II.1. *A power assignment p induces a* static *h-bounded communication graph if there exists a directed h-bounded graph H_p, so that H_p is a subgraph of $H_p(t)$ for every $t \in [t_s, t_f]$.*

Definition II.2. *A power assignment p induces a* dynamic *h-bounded communication graph if for every $t \in [t_s, t_f]$, $H_p(t)$ is h-bounded.*

In this paper we consider the problem of energy efficient power assignment which induces a communication graph with a bounded hop-diameter for the two modes, static and dynamic. In particular we solve the following two problems.

Problem II.3 (Static Bounded Hop (SBH))**.** *Given the graph $G = (V, E_V)$, find a power assignment p which induces a static h-bounded communication graph so that $c(p)$ and h are minimized and $l(p)$ is maximized.*

Problem II.4 (Dynamic Bounded Hop (DBH))**.** *Given the graph $G = (V, E_V)$ and parameter $h > 0$, find a power assignment p which induces a dynamic h-bounded communication graph so that $l(p)$ is maximized.*

Note that the SBH problem aims to minimize the hop-diameter as one of its optimization objectives. This is due to the natural trade off between the cost of a power assignment and the hop-diameter of the induced communication graph. In Section V-A we propose an approximation algorithm which balances this trade off – the shorter the paths, the greater the ranges assigned. The definition of the DBH problem is somewhat simpler, as it is lacking the cost optimization constraint, and the desired hop-diameter is given as a parameter. As a result, in Section V-B we are able to optimally solve the problem.

III. Our contribution

In this paper we consider a set of n mobile wireless nodes, which have no information in advance about each other. The only information a single node holds is its current location and future mobility plan. Each node is capable of adjusting its transmission power to cover any range. We choose to divide the problem of topology control in this type of network into two main phases: (1) Dissemination of current layout and mobility plans; (2) Topology construction. These phases are periodically executed, which allows us to quickly react to outdated and corrupted data. The above self-stabilizing scheme [15] allows us to operate even in a hostile environment.

We design a simple, distributed algorithm for the first phase; after which, each node holds the mobility plans and locations of every node in the network. Then, for the second phase we:

- Propose an $O(\lambda, \lambda^2)$-bicriteria approximation algorithm for the SBH problem, so that given a parameter $1 \le \lambda \le n-1$, we construct a power assignment which induces a static h-bounded communication graph, $h = n/\lambda + \log\lambda$, with a cost of at most λ times the optimum and network lifetime of at least $1/\lambda^2$ times the optimum.
- Show an optimal polynomial time algorithm for the DBH problem under the assumption that every node moves in a single direction along a straight line with constant speed. Given a parameter $1 \le h \le n-1$, we construct a power assignment which induces a dynamic h-bounded communication graph with an optimal network lifetime.

Note that the SBH problem is NP-Hard since the problem of minimum power strong connectivity is NP-Hard even in stationary networks without the hop-diameter limitation [9].

IV. Layout and mobility plans dissemination

One of the main challenges of every ad-hoc wireless network is topology discovery. We propose a simple distributed algorithm to form a temporary underlying topology, which can be used for the dissemination of current location and mobility plans.

We assume that all participants have synchronized clocks (say by a common input from a GPS, or by using a self-stabilizing clock synchronization algorithm). Thus, they all repeatedly and simultaneously start the two phase algorithm, which in turn implies convergence and stabilization following the first restart.

Another assumption is that the first phase is happening very fast, so the location of nodes remains unchanged during its execution. To simplify things, we allow the nodes to transmit at the maximum possible transmission range $R_{\max}$ which can be adjusted. Now, suppose the algorithm is executed at some time t' - so the underlying topology to be used for the data dissemination is actually $H_{p_{\max}}(t')$, where $p_{\max}(u) = (R_{\max})^2$, for every $u \in V$. Without loss of generality, let $H_{p_{\max}}(t')$ be strongly connected (otherwise the algorithm is valid for every strongly connected component).

Construct Underlying Topology

```
   // Neighbors Discovery
1  N(u) ← ∅
2  transmit hello⟨u⟩ in range R_max
3  while not timeout do
4      if received hello⟨v⟩ then
5          add v to N(u)
   // Construct network underlying topology
6  transmit neighbors⟨u,N(u)⟩ in range R_max
7  initialize G_u to be an empty graph
8  add edges {(u,v) : v ∈ N(u)} to G_u (create new nodes if required)
9  Forwarded ← {u}
10 Unknown ← N(u)
11 while Unknown ≠ ∅ do
12     if received neighbors⟨v,N(v)⟩ then
13         if v ∉ Forwarded then
14             add edges {(v,w) : w ∈ N(v)} to G_u (create new
               nodes if required)
15             foreach w ∈ N(v) do
16                 if w ∉ Forwarded and w ∉ Unknown then
17                     add w to Unknown
18             remove v from Unknown
19             add v to Forwarded
20             transmit neighbors⟨v,N(v)⟩ in range R_max
```

We present a distributed algorithm, Construct Underlying Topology, which enables the nodes to acquire the knowledge about the topology of $H_{p_{\max}}(t')$. The algorithm is executed at every node at time t' and can be roughly divided into two steps. First (lines 1-5), the nodes discover their immediate neighbors in $H_{p_{\max}}(t')$. Then (lines 6-20) this information is flooded so that each node could locally construct $H_{p_{\max}}(t')$, which can later be used for layout and mobility plans dissemination.

The first step is carried out by each node transmitting its own unique id at range $R_{\max}$ (line 2). Since a node can receive a message only if it is within the transmission range from the sender, then node u can safely add every node v to its neighbor list $N(u)$ once it receives v's `hello` message.

In the second step, each node obtains the knowledge of the underlying topology $H_{p_{\max}}(t')$ through the flooding of the neighbor lists. Every node u transmits its own neighbor list (line 6) and then constructs the underlying graph G with the aid of two lists, *Forwarded* and *Unknown*. If $v \in Forwarded$, then u has received the `neighbors`$\langle v, N(v)\rangle$ message and graph G_u contains all the edges adjacent to v in $H_{p_{\max}}(t')$. If $v \in Unknown$, then u has not received the `neighbors`$\langle v, N(v)\rangle$ message.

For every new `neighbors`$\langle v, N(v)\rangle$ message received at u, the edges from v to every $w \in N(v)$ are added to G_u (line 13) (if some edge contains an endpoint which does not appear in G_u, it is added to the graph). Then, each node $w \in N(v)$, which is neither in *Forwarded* nor in *Unknown*, is added to *Unknown* (line 17). Finally, `neighbors`$\langle v, N(v)\rangle$ is forwarded (line 20).

Note that during the second step nodes need not transmit at $R_{\max}$ as they only need to reach the farthest node in their neighbor list. That is, node u can forward `neighbors` messages using $\max_{v\in N(u)} d_{u,v}(t')$ as the transmission range. The next theorem shows the correctness, time and message complexity of the CONSTRUCT UNDERLYING TOPOLOGY algorithm.

Theorem IV.1. *The message complexity of* CONSTRUCT UNDERLYING TOPOLOGY *is $O(|E_{p_{\max}}(t')|\cdot n)$, and after at most $\Delta(H_{p_{\max}}(t'))$ rounds, for every $u \in V$, $G_u = H_{p_{\max}}(t')$.*

Proof: Note that all the messages actually passed along the edges of $H_{p_{\max}}(t')$ since all the nodes transmit at $R_{\max}$. There are $2n$ distinct messages (`hello` and `neighbors`). Only the `neighbors` messages are forwarded (once at each node).

Therefore there are at most $O(|E_{p_{\max}}(t')|\cdot n)$ messages received. Clearly, after $\Delta(H_{p_{\max}}(t'))$ rounds, no messages are forwarded and each node u has constructed some image of the underlying topology G_u. We argue that $G_u = H_{p_{\max}}(t')$. Since the graph $H_{p_{\max}}(t')$ is strongly connected, then G eventually receives all the `neighbors` messages. So we only need to show that all these messages will be eventually handled. In other words, if $Unknown = \emptyset$ then $Forwarded = V$. Suppose by contradiction that at some point $Unknown = \emptyset$ and $Forwarded \subset V$.

Since $H_{p_{\max}}(t')$ is strongly connected, there exists $(v,w) \in E_{p_{\max}}(t')$so that $v \in Forwarded$ and $w \notin Forwarded$. Therefore, w was added to $Unknown$ at some point, either line 10 or 17. Note that a node is removed from $Unknown$ (line 18) only if it is added to $Forwarded$. Therefore w is never removed from $Unknown$, and as a result $Unknown \neq \emptyset$. A contradiction. ■

Once every node holds an image of the underlying topology it can disseminate its current location and mobility plan through $H_{p_{\max}}(t')$ by using one of the existing algorithms [6], [28].

The first phase described in this section is executed periodically with an interval of $t_f - t_s$ time units between two consecutive executions; every time starting from scratch, where each node is only aware of its own current location and future mobility plan. Once the data is disseminated, each node has all the required information to carry out the second phase, which is the actual topology construction algorithm, and decide on its own power assignment. Each nodes carries out the topology control algorithm and decides on its own power assignment. The topology constructed in the second phase is valid for the time interval $[t_s,t_f]$.

V. BOUNDED HOP STRONG CONNECTIVITY

Once every node $u \in V$ has acquired the mobility plans of all the other nodes (as described in Section IV), it is able to compute the value of $d_{u,v}(t)$ for every $v \in V$ and $t \in [t_s,t_f]$. We assume that the computation time is negligible compared to the message transmission time. After that each node executes the algorithms proposed in this section, which in turn defines its transmission power. In what follows we first propose an approximation algorithm for Problem II.3 (SBH), and then show a polynomial time optimal solution for Problem II.4 (DBH).

A. Static bounded hop communication graph

In this section we propose an approximation algorithm for the SBH problem. We wish to find a power assignment p', which induces a low cost static h-bounded communication graph, with high network lifetime and low hop-diameter.

We need some definitions. Let $G_V = (V,E_V)$ be an undirected complete graph. For any $t \in [t_s,t_f]$, let $w_t(u,v) = (d_{u,v}(t))^2$, for every $(u,v) \in E_V$, a weight function over the edge set E_V. Note that $w_t(u,v)$ matches the amount of energy required to transmit from u to v, at time t. For any weight function w, defined on a weight set E_V, the weight of a graph $H = (V,E_H)$, $E_H \subseteq E_V$, is $w(H) = \sum_{(u,v)\in E_H} w(u,v)$. Let $MST(w)$ be a minimum weight spanning tree of G_V based on a weight function w.

For any two nodes $u,v \in V$, in order that an edge (u,v) would exist in every $H_p(t)$, $t \in [t_s,t_f]$, the power assigned to u should be at least the square of the maximum distance between u and v during $[t_s,t_f]$.We define a weight function w' which reflects this amount of energy for any pair of nodes,

$$w'(u,v) = \max_{t\in[t_s,t_f]} (d_{u,v}(t))^2, \text{ for every } u,v \in V.$$

The following lemma shows that w' holds the weak triangle inequality.[1]

Lemma V.1. *For any $u,v,w \in V$, it holds*

$$w'(u,v) \le 2(w'(u,w) + w'(w,v)).$$

Proof: Let $t_0 \in [t_s,t_f]$ be the moment so that $w'(u,v) = (d_{u,v}(t_0))^2$. Due to the triangle inequality in the Euclidean space, for any $u,v,w \in V$ and $t \in [t_s,t_f]$, it holds $d_{u,v}(t) \le d_{u,w}(t) + d_{w,v}(t)$. Therefore,

$$\begin{aligned} w'(u,v) = (d_{u,v}(t_0))^2 &\le (d_{u,w}(t_0) + d_{w,v}(t_0))^2 \\ &\le 2((d_{u,w}(t_0))^2 + (d_{w,v}(t_0))^2) \\ &\le 2(w'(u,w) + w'(w,v)). \end{aligned}$$

The last inequality follows from the definition of w'. ■

In [16] the authors show that given a complete graph $G = (U,E)$ with n nodes, a weight function w that holds a weak triangle inequality, and a parameter λ, $1 \le \lambda \le n-1$, it is possible to construct in polynomial time a spanning tree T of G, so that $\Delta(T) \le n/\lambda + \log\lambda$, and the weight of T is at most $O(\lambda)$ times the weight of the minimum weight spanning tree of G. In addition the weight of an edge in T is at most λ^2 times the maximum weight of an edge in the minimum weight spanning tree of G.

We use this construction to obtain a spanning tree $T' = (V,E')$ of G_V with a weight function w', which has similar properties; this is possible since w' holds the weak triangle inequality (Lemma V.1). Let e' and e_{MST} be the maximum weight edges in T' and $MST(w')$, respectively. The next theorem summarizes the properties of T'.

Theorem V.2 ([16]). *For any λ, $1 \le \lambda \le n-1$, and a weight function w' which holds the triangle inequality, it is possible to*

[1] A weight function w holds a weak triangle inequality, if there exists some constant value $\mu > 1$, so that for any three nodes x,y,z, $w(x,y) \le \mu(w(x,z) + w(z,y))$.

construct a spanning tree T' of G_V so that $w'(e') \leq \lambda^2 w'(e_{MST})$, $\Delta(T') \leq n/\lambda + \log\lambda$, and $w'(T') \leq O(\lambda)w'(MST(w'))$.

We are now ready to define the power assignment p'. Let $p'(u) = \max_{(u,v)\in E'} w'(u,v)$, for every $u \in V$. The hop-diameter of the induced communication graph and the cost of p' are derived in the following lemma.

Lemma V.3. *The power assignment p' induces a static h-bounded communication graph for $h = n/\lambda + \log\lambda$ and $c(p') \leq 2w'(T')$.*

Proof: Let T'_D be the directed version of T' (each undirected edge appears as two directed edges). If an undirected edge (u,v) is in T', then the two directed edges, (u,v) and (v,u), appear in $H_{p'}(t)$ for every $t \in [t_s,t_f]$, since $p(u) \geq w'(u,v)$ and $p(v) \geq w'(v,u)$ (which ensures that u and v are within the transmission range of each other for the whole time interval $[t_s,t_f]$). Therefore, for every $t \in [t_s,t_f]$, T'_D is a subgraph of $H_{p'}(t)$. Clearly, p' induces a static h-bounded communication graph for $h = n/\lambda + \log\lambda$ as the hop-diameter of T'_D is at most $n/\lambda + \log\lambda$. Finally,

$$c(p') = \sum_{u\in V} \max_{(u,v)\in E'} w'(u,v) \leq \sum_{u\in V} \sum_{(u,v)\in E'} w'(u,v) \leq 2 \sum_{(u,v)\in E'} w'(u,v) = 2w'(T').$$

This completes our proof. ■

In the next two lemmas we derive the lower and upper bounds for the cost and network lifetime, respectively, of a power assignment which induces a static h-bounded communication graph, for any $h \geq 1$.

Lemma V.4. *Let p_h^C be the minimum cost power assignment which induces a static h-bounded communication graph for some parameter $h \geq 1$. Then, it holds $c(p_h^C) \geq w'(MST(w'))$.*

Proof: It is easy to see that $c(p_h^C) \geq c(p_{n-1}^C)$, as every graph which is h-bounded, $h \geq 1$, is also $(n-1)$-bounded. We prove $c(p_{n-1}^C) \geq w'(MST(w'))$.

Let p be some power assignment which induces a static $(n-1)$-bounded communication graph. Therefore, there exists a directed graph $H_p = (V,E_p)$, so that H_p is strongly connected and for every $t \in [t_s,t_f]$, H_p is a subgraph of $H_p(t)$. From the definition of H_p, if $(u,v) \in E_p$, then $p(u) \geq w'(u,v)$.

Choose an arbitrary node $r \in V$ as the root. For $u \in V$, $u \neq r$, let P_u be a simple directed path from u to r in H_p. Denote by $E(P_u)$ the set of directed edges in P_u. The union of the edges in all the paths, $E = \bigcup_{u\in V,u\neq r} E(P_u)$ forms a directed tree $T = (V,E)$, rooted at r, where all the edges are directed toward the root. For every edge (u,v) in T, the power assigned to u is at least $w'(u,v)$ (since $E \subseteq E_p$). As there is only one outgoing edge from every node (toward the root), we obtain $w'(T) = \sum_{(u,v)\in E} w'(u,v) \leq \sum_{u\in V,u\neq r} p(u) \leq c(p)$. Let T_U be the undirected version of T obtained by omitting the edge directions. Clearly T_U is a spanning tree of G_V, and $w'(T) = w'(T_U) \geq MST(w')$.

Since we chose p to be any power assignment which induces a static $(n-1)$-bounded communication graph, we therefore conclude $c(p_{n-1}^C) \geq w'(T) \geq w'(MST(w'))$. ■

Recall that the network lifetime is defined as the time it takes the first node to run out of its battery charge. For equal initial battery charges b and a power assignment p, the network lifetime $l(p)$ is defined as $l(p) = \min_{v\in V} b/p(v)$.

Lemma V.5. *Let p_h^L be the maximum network lifetime power assignment which induces a static h-bounded communication graph for some parameter $h \geq 1$. Then, $l(p_h^L) \leq b/w'(e_{MST})$.*

Proof: Since p_h^L induces a static h-bounded communication graph, there exists a directed h-bounded graph $H_{p_h^L}$. It is a well known fact that for any spanning tree $ST = (V,E_{ST})$ of G_V, $\max_{e\in E_{ST}} w'(e) \geq w'(e_{MST})$. Let $e = (u,v)$ be the maximum weight edge in $H_{p_h^L}$. As in the proof of Lemma V.4, $p(u) \geq w'(e)$. From the definition of network lifetime, $l(p_h^L) \leq b/p(u)$. Therefore, since $H_{p_h^L}$ is strongly connected, $w'(e) \geq w'(e_{MST})$. We conclude $l(p_h^L) \leq b/w'(e) \leq b/w'(e_{MST})$. ■

Note that the bounds shown in Lemmas V.4 and V.5 do not depend on the value of h. We can now state the main result of this section based on Theorem V.2, and Lemmas V.3, V.4, and V.5.

Theorem V.6. *Given n mobile wireless nodes V, and a parameter λ, $1 \leq \lambda \leq n-1$, it is possible to construct in polynomial time a power assignment p_h that induces a static h-bounded communication graph, $h = n/\lambda + \log\lambda$, so that $c(p_h) = \lambda c(p_h^C)$ and $l(p_h) \geq l(p_h^L)/\lambda^2$, where p_h^C and p_h^L are optimal (in terms of cost and network lifetime, respectively) power assignments that induce a static h-bounded communication graph.*

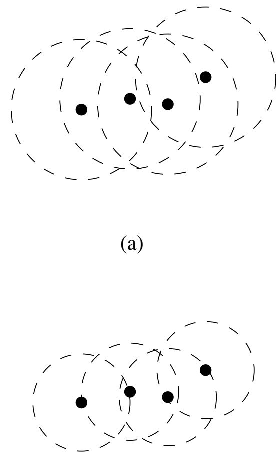
B. Dynamic bounded hop communication graph

In the case that the wireless nodes share the same initial battery charge b, maximizing the network lifetime is equivalent to minimizing the maximum power assigned to any node. The authors in [27] noted that if the required optimization is to minimize the maximum power assigned, it is possible to assign the same power level to all nodes. Hence, all we need to do is to choose the minimum power level which induces an h-bounded communication graph, for any given $h \geq 1$.

For non-mobile nodes, given a power level x it is easy to test whether the induced communication graph is h-bounded in polynomial time. Furthermore, it makes sense to test only those power levels x, for which there exists a pair of nodes at a distance exactly $\sqrt{x}$ of each other, otherwise the power level can be decreased. Thus, there are at most $\binom{n}{2}$ possible power levels. In Figure 3(b) the same topology as in Figure 3(a) is induced with a lower power level.

Adopting the above scheme to mobile nodes is challenging for two reasons. First, given a specific power level, it is difficult to test if the induced communication graph is dynamically h-bounded in the whole time interval $[t_s,t_f]$. Second, it is unclear what power levels should be considered, as the distance between any pair of nodes might changes constantly.

In the rest of this section we assume that every node moves with constant speed in a single direction along a straight line during $[t_s,t_f]$. We show that given a power level x, it is possible to test in polynomial time whether the power assignment induces a dynamic h-bounded communication graph, and also



(a)

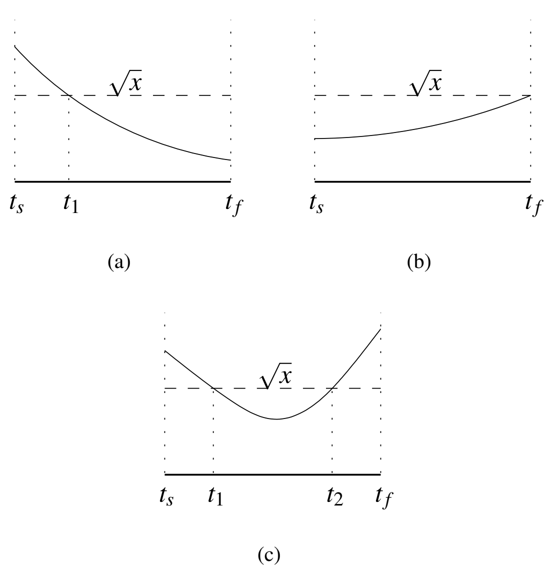
(b)

Fig. 3. Power efficiency: (a) an inefficient power level; (b) an efficient power level, same topology.

show that the number of possible power levels is at most $O(n^4)$. Let p_x be a power assignment, where for every $u \in V$, $p_x(u) = x$.

The distance between any two nodes, each moving with constant speed in a single direction along a line, can be either constant or first decrease to some minimum value and then constantly increase (see Figure 4 for the exposition of different types), as summarized in the following observation.

Observation V.7. *If during the time interval $[t_s, t_f]$ every node moves in a single direction along a straight line with constant speed then there exists $t' \in [t_s, t_f]$ for any pair of nodes $u, v \in V$ so that the distance function $d_{u,v}(t)$ is monotone non-increasing in $[t_s, t']$ and monotone non-decreasing in $[t', t_f]$.*



(a) (b)

(c)

Fig. 4. Distance function types and different cases of the $C_x^{u,v}$ set: (a) $C^{u,v}(x) = \{t_1\}$; (b) $C_x^{u,v} = \emptyset$; (c) $C_x^{u,v} = \{t_1, t_2\}$.

Verifying the hop-diameter. Given a power level x, we would like to verify that the communication graph induced by $p_x(u) = x$, for every $u \in V$, is h-bounded. The general idea is to verify the hop-diameter in a finite set of critical time points, and then based on these verifications to conclude for the whole time interval $[t_s, t_f]$.

For any $u, v \in V$, let $[t_s^{u,v}, t_f^{u,v}] \subseteq [t_s, t_f]$ be a non-empty time interval (if exists) so that $d_{u,v}(t) \leq \sqrt{x}$ for $t \in [t_s^{u,v}, t_f^{u,v}]$. If such an interval exists, we define the set of critical time points, $C_x^{u,v}$, to be

$$C_x^{u,v} = \{t_s^{u,v}, t_f^{u,v}\} \setminus \{t_s, t_f\}.$$

Otherwise, $C_x^{u,v} = \emptyset$. That is, $C_x^{u,v}$ is a set of time points, in the open time interval (t_s, t_f), when the nodes u and v change their connectivity status. Note that $C_x^{u,v} = C_x^{v,u}$. Let $C_x = \bigcup_{u,v \in V} C_x^{u,v}$. We claim that it is sufficient to verify the hop-diameter only for the time points in $C_x \cup \{t_s, t_f\}$.

Lemma V.8. *Given a power level x, the power assignment p_x induces a dynamic h-bounded communication graph if and only if for each $t \in C_x \cup \{t_s, t_f\}$, $H_{p_x}(t)$ is h-bounded.*

Proof: Let $t_s = t_0 \leq t_1 \leq \ldots \leq t_m = t_f$ be the time points in $C_x \cup \{t_s, t_f\}$ sorted in ascending order. Let us focus on a single arbitrary interval $[t_i, t_{i+1}]$, where $0 \leq i \leq m-1$. From the definition of C_x, for any pair of nodes $u, v \in V$ and any time point t', $t_i < t' < t_{i+1}$, it holds $d_{u,v}(t') \neq \sqrt{x}$. Thus, there are no edge changes inside the open time interval (t_i, t_{i+1}). Also, due to the definition of C_x, edge cannot exist only at t_i. Therefore, the graphs $H_{p_x}(t_i)$ and $H_{p_x}(t')$ are identical for every t', $t_i \leq t' < t_{i+1}$.

The *only if* case is trivial, so we concentrate on the *if* case. Suppose that for every i, $0 \leq i \leq m$, $H_p(t_i)$ is h-bounded. From the above, the graphs remain unchanged within each interval $[t_i, t_{i+1})$, $0 \leq i \leq m-1$. Therefore, $H_{p_x}(t)$ is h-bounded for every $t \in [t_s, t_f]$. ∎

Possible power levels. Although the nodes constantly change their location, there is a finite set of possible power levels which should be tested. The irrelevant power levels are either those which do not supply full coverage for a specific node, or those which supply excessive coverage and may be reduced.

There are three types of power levels which should be considered. Intuitively, the first type L_1 allows nodes to remain within the reach of each other for the whole time interval $[t_s, t_f]$. That is the power level matches the definition of w' in Section V-A,

$$L_1 = \left\{ x : \exists u, v \in V, \max_{t \in [t_s, t_f]} d_{u,v}(t) = \sqrt{x} \right\}.$$

The second type is due to the following simple logic; if something changes at one place, something has to change at some other place, otherwise the power level is either insufficient or exaggerated. In other words, a second type of relevant power levels consists of values which match the squared distance of a pair of nodes, shared at the same time by at least two pairs.

$$L_2 = \left\{ \begin{array}{ll} x : & \exists t \in [t_s, t_f], \exists u, v, x, y \in V, (u,v) \neq (x,y), \\ & d_{u,v}(t) = d_{x,y}(t) = \sqrt{x} \end{array} \right\}.$$

The third type covers all possible power levels at time points t_s and t_f, similar to the non-mobile case.

$$L_3 = \left\{x : \exists u,v \in V, d_{u,v}(t_s) = \sqrt{x} \text{ or } d_{u,v}(t_f) = \sqrt{x}\right\}.$$

Denote by $L = L_1 \cup L_2 \cup L_3$. We are ready to present the next lemma.

Lemma V.9. *Let x be the minimum possible (optimal) power level so that p_x induces a dynamic h-bounded communication graph, where for every $u \in V$, $p_x(u) = x$. Then, $x \in L$.*

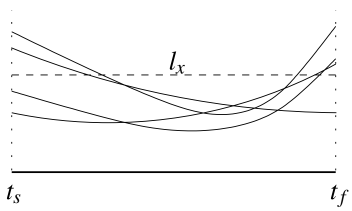
Proof: Suppose by contradiction that $x \notin L$. There are two possible cases to consider.

Case 1: The induced graph remains unchanged for the entire time interval $[t_s,t_f]$. that is for every $t_1,t_2 \in [t_s,t_f]$ the graphs $H_{p_x}(t_1)$ and $H_{p_x}(t_2)$ are identical. Then, from the definition of L for every pair of nodes $u,v \in V$ and $t \in [t_s,t_f]$, $d_{u,v}(t) \neq \sqrt{x}$. Therefore, if $d_{u,v}(t_s) < \sqrt{x}$, then it is also $d_{u,v}(t) < \sqrt{x}$ for any $t \in [t_s,t_f]$.

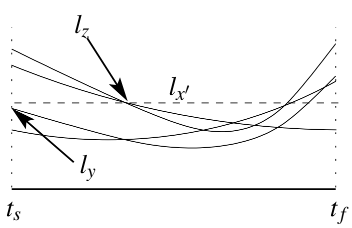
Let N_u be the set of nodes which are within the transmission range $\sqrt{x}$ from u in $[t_s,t_f]$ (note that this set does not change for the whole time interval). Let,

$$x' = \max_{u\in V, v\in N_u, t\in[t_s,t_g]} d_{u,v}(t).$$

It is easy to verify that $x' < x$ and that $p_{x'}$ induces a dynamic h-bounded communication graph. A contradiction that x is optimal.



(a)



(b)

Fig. 5. The Illustration of Lemma V.9, Case 2: (a) Distance functions of all pairs; (b) Lowering the l_x bar to obtain x'.

Case 2: The induced graph changes during the time interval $[t_s,t_f]$. Let us look at some distance function $d_{u,v}(t)$ in time interval $[t_s,t_f]$, and let $l_x = \sqrt{x}$ be a horizontal line corresponding to the transmission range threshold. Whenever $d_{u,v}(t)$ is not above l_x, the edges (u,v) and (v,u) both exist in the communication graph. Once $d_{u,v}(t)$ crosses l_x, a topology change occurs and nodes u,v either connect or disconnect. This resembles the definition of $C_x^{u,v}$.

We now draw all the distance functions (there are $\binom{n}{2}$ such functions) on the same coordinates system (see Figure 5(a)). The topology of the communication graph can be described as follows.

Top-1 Initially, at time t_s, an edge (u,v) exists in the graph $H_{p_x}(t_s)$ if $d_{u,v}(t_s)$ is not above l_x.

Top-2 Then, each time some distance function crosses l_x, a topology change occurs and either a new edge is added or an existing is removed.[2] This resembles the definition of C_x.

Top-3 Finally, at time t_f, an edge (u,v) exists in the graph $H_{p_x}(t_f)$ if $d_{u,v}(t_f)$ is not above l_x.

For some power level x', let $l_{x'} = \sqrt{x'}$. We say that a power level x' preserves the topology properties of a power level x if the following conditions hold

Cond-1 Graph $H_{p_x}(t_s)$ is identical to $H_{p_{x'}}(t_s)$, and graph $H_{p_x}(t_f)$ is identical to $H_{p_{x'}}(t_f)$ (Top 1 and Top 3)

Cond-2 The distance functions cross l_x and $l_{x'}$ at exactly the same order (Top 2).

Since p_x induces a dynamic h-bounded communication graph, it is easy to see that if a power level x' preserves the topology properties of power level x, then $p_{x'}$ also induces a dynamic h-bounded communication graph. Before we proceed, we make some observations.

Fact-1 Since $x \notin L_l$, then there does not exist a constant distance function $d_{u,v}(t) = \sqrt{x}$.

Fact-2 Since $x \notin L_2$, only one distance function can cross or tangent to l_x at any time point.

Fact-3 Since $x \notin L_3$, each distance function is either strictly above or strictly below l_x at times t_s and t_f.

We next use the monotonicity characteristics of the distance functions (Observation V.7) and the above facts to derive a power level $x' < x$ so that x' preserves the topology properties of x. From Facts 1 and 3, there exists a power level $y < x$ so that graph $H_{p_x}(t_s)$ is identical to $H_{p_y}(t_s)$, and graph $H_{p_x}(t_f)$ is identical to $H_{p_y}(t_f)$ (complying with Cond 1). From Fact 2 we conclude that there exists a power level $z < x$ so that the distance functions cross l_x and l_z at exactly the same order (complying with Cond 2).[3] Intuitively, this is achieved by lowering the l_x bar until it first encounters one of the following: (1) a distance function on one of the vertical lines t_s or t_f, (2) a constant distance function, (3) a intersection between two distance functions. In figure Figure 5(b) we can see the l_y power level, as an intersection between t_s and one of the distance functions, while $l_z = l_{x'}$ is an intersection between two distance functions.

Clearly x' preserves the topology properties of x, and therefore $p_{x'}$, induces a dynamic h-bounded communication graph. A contradiction to the optimality of x. ■

Running time analysis. To implement the scheme above we first need to compute L. Then for each value $x \in L$ we compute C_x and compute the hop-diameter of $H_{p_x}(t)$ for every $t \in C_x \cup$

[2]If a distance function does not cross, but only a tangent to l_x, then no topology change occurs, since the relevant edge does not appear in the communication graph for a continuous time period.

[3]Due to Observation V.7 we can also conclude that any two values a,b so that $y \leq a \leq x$ and $z \leq b \leq x$, comply with conditions Cond 1 and Cond 2, respectively.

$\{t_s, t_f\}$. We choose the minimum value of x for which all the graphs $H_{p_x}(t)$, $t \in C_x \cup \{t_s, t_f\}$ are h-bounded. We base our time analysis on the following lemma.

Lemma V.10. *In polynomial time it is possible to compute L, C_x, for any $x > 0$, and the hop-diameter of $H_{p_x}(t)$ for every $t \in C_x \cup \{t_s, t_f\}$. It also holds $|L| = O(n^2)$ and $\max_{x \in L} |C_x \cup \{t_s, t_f\}| = O(n^2)$.*

Proof: Clearly $|L_1|$ and $|L_3|$ can be computed in polynomial time, and $|L_1| = O(n^2)$, $|L_3| = O(n^2)$. Since all the nodes travel at constant speeds without direction changes, each pair of distance functions intersects at most 3 times. Therefore, we can compute L_2 in polynomial time and $|L_2| = O(n^2)$.

From Observation V.7 it is easy to see that for any x, $|C_x^{u,v}| \leq 2$ and therefore $\max_{x \in L} |C_x \cup \{t_s, t_f\}| = O(n^2)$.

To compute the hop-diameter of a graph we apply the BFS algorithm from an arbitrary node r and then run another BFS from the most distant node from r. This can be done in time linear to the number of edges, i.e. $O(n^2)$ time. ■

Clearly, Lemma V.10 implies the main theorem of this section.

Theorem V.11. *Given n mobile wireless nodes V, and a parameter h, $1 \leq h \leq n-1$, if during the time interval $[t_s, t_f]$ every node moves in a single direction along a straight line with constant speed then it is possible to construct in polynomial time a power assignment p_h that induces a dynamic h-bounded communication graph, and $l(p_h) = l(p_h^L)$, where p_h^L is a maximum network lifetime power assignment that induces a dynamic h-bounded communication graph.*

VI. Conclusions and Future Work

In this paper we consider a set of n mobile wireless nodes, which have no initial information about each other. The only information a single node holds is its current location and future mobility plan. We proposed a two-phase scheme to solve the stretchable topology control problem under two modes, static and dynamic. The scheme optimize the obtained network for an entire continuous time segment, rather than discrete time(s).

We believe that these realistic settings should be further studied. One of the most interesting future direction is to try and provide a power assignment for the DBH problem which is both cost and network lifetime efficient. It would be also intriguing to perform some simulation tests for the first phase of the scheme and calibrate the various parameters.

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