Grooming in Unidirectional Rings: $K_4 - e$ Designs

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Abstract

In wavelength division multiplexing for unidirectional rings, traffic grooming is used to pack low rate signals into higher rate streams to share a wavelength. The grooming chosen determines the number of add-drop multiplexers used for the optical-to-electronic conversion. The determination of groomings to use the fewest multiplexers is equivalent to a graph design problem, which has been solved when up to four signals can be packed into a stream. We completely settle the case here when five signals pack into one stream, using old and new results on $(K_4 - e)$-designs.

1 Introduction

Traffic grooming is the process of packing low rate signals into higher rate streams which share a wavelength. In optical networks, particularly in SONET ring networks, grooming has received much attention; surveys are given in [5, 10, 16, 17]. The setting is a wavelength-division multiplexed (WDM) network; each wavelength is an optical communication medium which connects all nodes in a circle, and may be unidirectional or bidirectional. An *add-drop multiplexer (ADM)* is required on each wavelength at each node at which traffic is added or dropped. In general, there are two main goals, given a set of traffic requirements between nodes. The first is to minimize the number of wavelengths employed, while the second is to minimize the total number of ADMs (the *drop cost*). As we shall see, these goals can be in conflict.

A case of substantial interest (see [2, 3] and references therein) arises with symmetric uniform traffic requirements on a unidirectional ring. In this scenario, for every source node i and every target node j, the traffic requirement is for the fixed fraction $\frac{1}{C}$ of a wavelength. The quantity C is the *grooming ratio*, because we can "groom" C circles onto the same wavelength. A similar problem arises for bidirectional rings [7, 9], but we focus on the unidirectional case here.

Bermond and Coudert [2] establish that minimizing drop cost of a grooming on n nodes with grooming ratio C can be expressed as an optimization problem on graphs, as follows. Partition the edges of K_n into subgraphs $G_1, \ldots, G_w$ so that each G_i contains at most C edges, and the sum of the numbers of vertices of nonzero degree in the $\{G_i\}$ is minimized. Such a partition of K_n is a *C-grooming*; see also [13]. As with the work of Wan [19] (see also [7, 9]) for bidirectional rings, this recasts the problem for unidirectional rings as a graph decomposition problem.

When $C = 1$, each class forms a single edge and no savings in drop cost is possible. When $C = 2$, classes contain at most two edges, since the line graph of K_n is eulerian, by taking edges corresponding to consecutive vertices on an eulerian cycle yields a drop cost of $\lceil 3\frac{n(n-1)}{4} \rceil$. Bermond and Ceroi [1] settled the case when $C = 3$. When $C = 4$, the problem is somewhat easier again (see [3, 15]) and the drop cost is $\binom{n}{2}$ except when $n \in \{2, 4\}$, in which case it is one larger.

In each of these cases, the minimum drop cost is realized by a grooming in which the number of wavelengths is minimum. This does not occur in general. To see this, we use a simple economic model. Imagine that we can purchase graphs on at most C edges. The price we pay for a graph is the number of vertices of nonzero degree in the graph; this is precisely the number of ADMs required for this wavelength. We must "purchase" all edges of a complete graph among the graphs chosen, at the lowest possible total cost. In this context, it is natural to calculate the cost per edge obtained, which is the ratio of the number of vertices to the number of edges in the graphs chosen. The smallest ratio for a small number m of edges are 2 for $m = 1$, $\frac{3}{2}$ for $m = 2$, 1 for $m = 3$, 1 for $m = 4$, $\frac{4}{5}$ for $m = 5$, $\frac{2}{3}$ for $m = 6$, and $\frac{5}{7}$ for $m = 7$. Since the ratio for $C = 6$ is lower than that for $C = 7$, we prefer to take graphs in the partition with six edges rather than seven. As a result, it is possible for the minimum drop cost to be realized only by groomings with more than the minimum number of wavelengths (in fact, a K_4-design on 13 points has 13 graphs and drop cost 52; yet only 12 wavelengths are needed, but the lowest drop cost is 54). Indeed suppose that for some integer k, we have that $\binom{k}{2} < C \leq \lfloor \frac{k^2}{2} \rfloor$. Then the best C-grooming in terms of drop cost is actually a $\binom{k}{2}$-grooming for some values of n, in particular when a balanced incomplete block design of order n, block size k, and index one exists (see [6] for existence results on these).

The ratios calculated determine the drop cost only if we can indeed partition K_n into copies of the relevant graphs. So, although the ratios provide a lower bound, it remains an open question whether this phenomenon occurs for any $C < 7$, or for any C not satisfying $\binom{k}{2} < C \leq \lfloor \frac{k^2}{2} \rfloor$. For $C \leq 4$, we have seen that lowest drop cost is realized among groomings with the fewest frequencies (see [1]). Our main objective here is to establish that the same holds true for $C = 5$; indeed we prove:

Theorem 1.1 [Main Theorem] *The minimum drop cost for a 5-grooming of K_n is*

$$4 \cdot \lfloor \frac{1}{5}\binom{n}{2} \rfloor + \begin{cases} 0 & if \quad n \equiv 0,1 \pmod 5 \ \textit{and} \ n \neq 5 \\ 1 & if \quad n = 5 \\ 2 & if \quad n \equiv 2,4 \pmod 5 \ \textit{and} \ n \neq 7 \\ 3 & if \quad n = 7 \\ 3 & if \quad n \equiv 3 \pmod 5 \ \textit{and} \ n \neq 8 \\ 4 & if \quad n = 8. \end{cases}$$

and this drop cost is realized by a 5-grooming using the minimum number $\lceil \frac{n(n-1)}{10} \rceil$ of wavelengths.

We shall see that this meets the elementary lower bound except when $n \in \{5, 7, 8\}$.

2 Lower Bounds

Given any 5-grooming, we first determine the structure under the single assumption that the drop cost is minimum. In a putative 5-grooming, let a_i be the number of classes containing i edges, for $i = 1, 2, 3, 4, 5$. We establish some inequalities on i subject to the constraints that $\sum_{i=1}^{5} i \cdot a_i = \binom{n}{2}$, and that $2a_1 + 3a_2 + 3a_3 + 4a_4 + 4a_5$ is minimized, where each a_i is a nonnegative integer. Considering such selections of $(a_1, a_2, a_3, a_4, a_5)$ which realize the minimum does not, of course, guarantee that there is such a 5-grooming, but it tells us what is possible in theory as the minimum.

We tabulate some cases here in which the weighted sum is not minimum, showing a selection leading to a lower value:

$a_1 \geq 2$	$(a_1 - 2, a_2 + 1, a_3, a_4, a_5)$
$a_2 \geq 2$	$(a_1, a_2 - 2, a_3, a_4 + 1, a_5)$
$a_3 \geq 3$	$(a_1, a_2, a_3 - 3, a_4 + 1, a_5 + 1)$
$a_4 \geq 2$	$(a_1, a_2, a_3 + 1, a_4 - 2, a_5 + 1)$
$(a_1, a_2) = (1, 1)$	$(a_1 - 1, a_2 - 1, a_3 + 1, a_4, a_5)$
$(a_1, a_3) = (1, 2)$	$(a_1 - 1, a_2 + 1, a_3 - 2, a_4, a_5 + 1)$
$(a_1, a_4) = (1, 1)$	$(a_1 - 1, a_2, a_3, a_4 - 1, a_5 + 1)$
$a_2 = 1,\ a_3 \geq 1$	$(a_1, a_2 - 1, a_3 - 1, a_4, a_5 + 1)$
$(a_2, a_4) = (1, 1)$	$(a_1 + 1, a_2 - 1, a_3, a_4 - 1, a_5 + 1)$
$(a_3, a_4) = (2, 1)$	$(a_1, a_2, a_3 - 2, a_4 - 1, a_5 + 2)$

In plain terms, this states that at most one of a_1, a_2, or a_4 is nonzero, and if so then it is one. In the first and third cases, a_3 may then be 0 or 1; in the second, it must be zero. Now observing that $\binom{n}{2}$ is always $0, 1, 3 \pmod 5$, we find that (a_1, a_2, a_3, a_4) is one of (0,0,0,0), (1,0,0,0), (0,0,1,0), or (0,0,2,0).

Now it is easily seen that if this minimum can be achieved, it can be done (indeed, *must* be done) with the fewest wavelengths possible. Hence our goal is to partition K_n, or K_n minus an edge, a triangle, or two triangles, into copies of $K_4 - e$. We pursue this in the remainder of the paper.

3 Graph Designs and Constructions

Let G and H be (finite, simple, undirected) graphs. A G-decomposition of H is a partition of the edges of H into classes so that the edges within each class form a graph isomorphic to G. When H is a complete graph of *order* n, the graphs in a G-decomposition of H form a *G-design* of order n (and *index* one, since each edge of H appears in exactly one of the graphs chosen). A K_k-design of order n is a *Steiner system* $S(2, k, n)$.

We shall be interested not only in taking H to be complete, but also taking H to be "nearly" complete. To this end, define a complete multipartite graph to be of *type* $g_1^{u_1} \cdots g_s^{u_s}$ if it has exactly $\sum_{i=1}^{s} u_i$ classes in the multipartition, and there are u_i parts of size g_i for $i = 1, \ldots, s$. A G-decomposition of the complete multipartite graph of type $g_1^{u_1} \cdots g_s^{u_s}$ is termed a *G-group divisible design* of *type* $g_1^{u_1} \cdots g_s^{u_s}$, and is often called a G-GDD for short. The special case when a G-GDD has type $1^r h^1$ is an *incomplete G-design* of *order* $r + h$ with a *hole* of size h; in graph-theoretic vernacular this is a partition of the edges of $K_{r+h} - K_h$ into copies of G.

In view of the application to grooming, we are primarily concerned in this paper with the case that $G = K_4 - e$, the unique graph on four vertices and five edges. We recall some existence results:

Lemma 3.1 [4] *There exists a $(K_4 - e)$-design of order n if and only if $n \equiv 0, 1 \pmod 5$ and $n \neq 5$.*

Lemma 3.2 [8] *There exists a $(K_4 - e)$-GDD of type g^u if and only if $u \geq 3$, $g^2\binom{u}{2} \equiv 0 \pmod 5$ (equivalently, $g \equiv 0 \pmod 5$ or $u \equiv 0, 1 \pmod 5$), and $(g, u) \neq (1, 5)$.*

We prove another simple lemma.

Lemma 3.3 *Let $n \geq 1$. Then there is a $(K_4 - e)$-GDD of type $n^2(2n)^1$.*

Proof: Let L be a latin square of side n. The GDD to be formed has $4n$ elements, $\{r_1, \ldots, r_n\}$, $\{c_1, \ldots, c_n\}$, and $\{s_{1i}, \ldots, s_{ni} : i \in \{0, 1\}\}$. When $L(i, j) = k$, form the $K_4 - e$ containing edges $\{r_i, c_j\}$, $\{r_i, s_{k0}\}$, $\{r_i, s_{k1}\}$, $\{c_j, s_{k0}\}$, and $\{c_j, s_{k1}\}$. This forms n^2 $(K_4 - e)$s which are edge disjoint, and hence forms the required GDD. □

Now we give the main construction.

Theorem 3.4 *Let n be a positive integer, $n \notin \{5, 7, 8, 9\}$, and write $n = 5t + h$ for $0 \leq h \leq 4$. Then there exists a partition of $K_n - K_h$ into copies of $K_4 - e$ (i.e., an incomplete $(K_4 - e)$-design of order n with a hole of size h).*

$n = 4$

abcd cd

$n = 5$

abcd {ae, be, ce, de, cd}

$n = 6$

feba bacd dcef

$n = 7$

abfg cdfg ceab efg {ca, cb, ce}

$n = 8$

hfac aceg gebd bdfh [abcd] [efgh]

$n = 9$

agfe gbcd icda hcef habd bife def ghi

$n = 12$

cfbk lahb gfja gbeh kiab dgck kejh fdhl djba ceal chij ligj eidf kl

$n = 13$

lhbd egjc icdh hgmf cfbm lagc aekh iefl fjla ibgk kdgf bdaj mija medb jkch klm

$n = 17$

gleh hiob fklh qlib hemj mgkq aknb oagq mipd pcgj fica odje nchq kije ombl
paeh ngdi okpc fnmo dpbf ldca ebcn qfej nlpj bgfj dqhk amcj pq

$n = 18$

kdmo gnik rkih ojcm dgrp mcrn frle clkq fjdn pejn jqik mifh jbra cide mlpe
keba jglh hedo ahpc nhlq bfqh dbln qgeo fago bgcm fpkc pbio aqdm ilao nrao pqr

$n = 22$

dqBi Blgm maoh ckAd fqoA spfr inmo eotp aljc hBji egqi ratB cbip smtA pqam
edfs solh fbth gApo Areh jcem fklm litA Bpnt bnjA asig kipr rdmg gtch cron
fjri lhpd bmeg lrqb cBsf sqnb kjsg jtqA nelh bokB jdop qhkc tndk dabA ekaB
fnga AB

$n = 23$

pajm fdme kAjl jfgB Ahpe Bepq saCn shiq sced Aimq oaeg sblf bqak dpbl cCjn
tBdc lfan mkCt jhdo iBag opsc ikeo mBlh tnhq bnij tCep jsrm jlqe rdka irpc
obCt skBg tglr Cidl Atsa tifj qmrc bmge qofd chgl gqCp orAl onmB Adgn kahc
kpfn nerg brBh Cfhr Acbf ABC

$n = 27$

AGil FqAc ngiq nalb amjF ojfE eimb pcie Brta Apom jqkG ckab Dtpi Coar asGp
thjE Cslg lrdE Fdpb Bqmp aqef oehs Ebpq gkmo djnc smfb cDGg gAEB Fftn sirF
pghf daAD toqb Bnec dmCo dkie rGgb hCBF loic sdqE ljBp Emnc gtda bAhC CGep
Bfib fhdG jFDr jgeb aihE Cijq klfh Fleg skDB nGok nDCA DEef GBdE cCtf lDqb
EkFC oBDF nsth rfAe sAjc tAke tmGl rpnk mDhr rhcq FG

$n = 28$

secq aeAj AgcB Bsth Fbsn jfsh FaEq HEsB bpla lfGF hqnm jFAp oBjD hgDF ckFf
eHCm Hfbp ogls pqgB itGb etph mGdn gCdf jEGC GqkD ohdp ahrH rmfk aGsg qdHf
jgmi icHm AEqo diBe afBD GAbp tkjA GcBo Hgkt Canl kiha ADHd hCAG gEne cjqD
CFio CrDs Elhk elBn moab bchE rbBg iEfp nDps rdEp jHlr ilqD mlAs fnAt FmBD
crnl Cqbt bDek ksdp tEDm reGF kBCn noHi acdt oefk rotq iArs jdbn dtFl pCcm
FGH

Table 1: Solutions for Smaller Values of n

$n = 14$, incomplete
giad lchf ijkf befi bgmn egjl nhie aljn emad klbd kahf ckge mnkf jhbm hfdg
imcl jncd
$n = 19$, incomplete
mfkn nrsc cqfh rbmf gpnr mlgc jqms qipr ebqs nkej dljh qgko igad kicb jief
bplo jgbh nhib msih rahk dnqo ojra skdl efgh emoa erdl aspf cpej alqn dpmf
hkpo olfi scog
$n = 24$, incomplete
sait emct iwfh sflk vwmg egsx wdsk ioem nicu xavk mqlx dipl towr erkd hqgr
tvnd elvj bgor xjtw nefh htpb uprv xufd gtfc bivj usom wclu nsrq cxps uhlj
ohxa lpno dnjo rxil svhj vcrq arjw euab jpgf pqak mdgh hckf ovfk qtui joqc
lktb mfra agnl kgui ewpq mknj bnwx bpms fqdb
$n = 29$, incomplete
tCrc kyaA shCB Alch fBag ejik ljwC sjtv bmxk qugh Bizd wnCi otpv fbtu ogja
fkxn bzwC hoxd oqzn aenq loyr gbpr mrvd hzgc ampC zAmn wfdA eyuv xCup jAxa
qdtx pysi sefz qrkj jduz nsur lqpB dnyp iuaA mtyu wuor Bmjn lsdi ivfx Agdt
crie mose fzpr bqyi vpwu talz CvqA cjpn vkdz lukz kcsg sAbq Bxwt thin gimC
fmlh oBbA eCdB Aper fyCj axsr Bcvu elxg twek gnvx
yxcz kphB fcoq blnv havw bhje wmcq wgys ryhB okCi

Table 2: Incomplete $(K_4 - e)$-designs

Proof: Lemma 3.1 handles cases when $n \equiv 0, 1 \pmod 5$, $n \neq 5$. In Table 1 (see also [12, 14] for examples), packings are given for all values of n less than 30 when $n \equiv 2, 3 \pmod 5$, and also when $n \in \{4, 5, 6, 9\}$, writing abcd for the graph containing edges ab, ac, ad, bc, bd. In each case except for $n \in \{5, 7, 8, 9\}$, what is shown is a collection of $\lfloor \binom{n}{2}/5 \rfloor$ edge-disjoint copies of $K_4 - e$, and one K_2, or K_3 to account for the remaining 1 or 3 edges (when elements ab are listed, it is a K_2; abc denotes a K_3). Taking the vertices of the small complete graph as the hole yields the required solution in these cases except when $n \equiv 4 \pmod 5$, which omit a K_4. Table 2 presents solutions for $n \in \{14, 19, 24, 29\}$, in each case having a hole on the four symbols a,b,c,d. The "missing" K_4 can be partitioned into a $K_4 - e$ and a K_2 to complete the partition (see Table 1).

We prove the theorem inductively. When $n \geq 30$, write $n = 10s + 5m + h$ with $2 \leq s \leq m \leq 2s$ and $0 \leq h \leq 4$. Form a $TD(3, s)$ (equivalently, a latin square of side s, or a K_3-GDD of type s^3). Replace points in two of the groups by five points each, and in the third group by either five or ten points. Since $(K_4 - e)$-GDDs of type 5^3 and $5^2 10^1$ exist by Lemmas 3.2 and 3.3, we obtain a $(K_4 - e)$-GDD of type $(5s)^2(5m)^1$ whenever $2 \leq s \leq m \leq 2s$. Now adjoin h new vertices, and fill in groups using incomplete $(K_4 - e)$-designs of orders $5m + h$ and $5s + h$, each with a hole of size h to be aligned on the h new vertices. $\square$

Theorem 3.4 establishes that K_n can, except when $n \in \{5, 7, 8, 9\}$, be decomposed into $\lceil \binom{n}{2}/5 \rceil$ subgraphs, of which $\lfloor \binom{n}{2}/5 \rfloor$ are copies of $K_4 - e$. When $n \equiv 2, 3, 4 \pmod 5$, the remaining subgraph is a K_2 or a K_3. Table 1 gives decompositions of K_5, K_7, K_8, and K_9 as follows. K_5 is decomposed into $K_4 - e$ and a star plus an edge. K_7 is decomposed into three $(K_4 - e)$s, one K_3 and a 3-star. K_8 is decomposed into four copies of $K_4 - e$ and two 4-cycles. K_9 is decomposed into three copies of $K_4 - e$ and two copies of K_3; in this case, unlike the previous three, the drop cost does reach the theoretical minimum. To complete the discussion, it must be established that no 5-groomings exist with drop cost 8 for $n = 5$, 18 for $n = 7$, or 23 for $n = 8$; this verification is straightforward but overly tedious. Then the examples in Table 1 provide the best results, namely drop cost 9 for $n = 5$, 19 for $n = 7$, and 24 for $n = 8$.

We therefore obtain the Main Theorem, Theorem 1.1.

4 A Hillclimbing Method

It must be admitted that Theorem 3.4 is not difficult to obtain, and that absent an application it would not be of terribly much interest. However, it does have an application to grooming. In fact, for all $k \geq 1$, it is not difficult to establish that $K_{6k+4} - K_{4k+2}$ admits a partition into copies of $K_4 - e$, and hence the solutions for $n \in \{22, 28\}$ in Table 1 are not needed. Nevertheless, in the process of proving the main theorem, we required a number of small examples, and devised an interesting heuristic method to find them. We report on this method here.

Consider K_n. Compute $s = \lfloor \binom{n}{2}/5 \rfloor$. This is the required number of $K_4 - e$ subgraphs in a decomposition of K_n (with 0, 1, or 3 edges remaining). So place at random a collection of s copies of $K_4 - e$ on n points. This fails in general to be a decomposition because some edges appear in more than one $K_4 - e$. How "bad" is it? Let us determine, for each edge of the K_n that appears in a $K_4 - e$, the *excess* number of times it appears in a $K_4 - e$, by setting the excess to be one less than the number of subgraphs in which it appears. This is extended to all edges by assigning those in no $K_4 - e$ an excess of zero. The *defect* of the collection is then the sum, over all edges of K_n, of the excess.

Now if the defect is 0, the collection is a decomposition; so the goal is to find simple transformations which reduce the defect. Actually, we settle for less – we consider transformations which *do not increase* the defect. We select at random some edge b. If b appears in at least one copy of $K_4 - e$, choose F to be one of the $(K_4 - e)$s in the collection which contains b. On the four points of F, there is exactly one copy of $K_4 - e$, F', which does not contain b. However, F' contains one edge, a, not contained in F. If replacing F by F' does not increase the defect (that is, if b has positive excess, or a appears in no $K_4 - e$, prior to the transformation) then replacement of F by F' in the collection is an *effective transformation*.

In general, this alone cannot produce edge-disjoint $(K_4 - e)$s because it never changes the points underlying the graphs; indeed if two $(K_4 - e)$s share three or more vertices, they cannot be made disjoint. So we define another transformation as follows. Let F be one of the graphs, and let v be a vertex of degree two having neighbours x and y. Select at random a vertex w not appearing in F, and let F' be the graph obtained from F by replacing v by w. This transformation is effective if the excess of $\{v, x\}$ plus the excess of $\{v, y\}$ is no smaller than the number of appearances of $\{w, x\}$ plus the number of appearances of $\{w, y\}$, prior to the transformation. This second transformation allows the set of vertices in one subgraph to change by replacement of a single vertex.

It is not clear *a priori* that these two transformations suffice to produce $(K_4 - e)$-designs. However, we proceeded by selecting at random an edge, choosing a $K_4 - e$ containing this edge, and carrying out one of the two transformations if and only if it is effective. Our computational experience with this method is very encouraging. Starting with random sets of the appropriate number of K_4-e graphs on 25 vertices, we found (K_4-e) designs in as few as 900 transformations, and rarely encountered the need for more than 25000. Nevertheless, the method is heuristic, and can cycle indefinitely. For this reason, we placed a limit on the number of transformations of 100000, and restarted with a new random collection if this iteration limit was exceeded.

The edge-disjoint (K_4-e)s in Table 1 are some of those produced by this hillclimbing method. We made a very simple adjustment to the method, by permitting the specification of certain edges not to appear in any K_4-e. This is handled by defining excess for such edges to be the number of appearances (hence excess zero corresponds to no appearance). By specifying edges of a K_h not to appear, the method then constructs incomplete (K_4-e)-designs; examples with $h=4$ appear in Table 2.

Hillclimbing techniques have been remarkably effective in a number of problems [11]. The one presented here differs from the one for Steiner triple systems [18]. In that case, triples are changed to permit the addition of a new triple; here we start with the correct number of K_4-e graphs, and employ hill-climbing to make them more and more edge-disjoint.

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