The weak limit of Boltzmann random matchings on diluted graphs

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Session : Optimisation combinatoire

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par Charles Bordenave, Marc Lelarge et **Justin Salez**

A matching on a finite graph $G = (V, E)$ is a collection of pairwise non-adjacent edges $M \subseteq E$. The Boltzmann random matching at temperature $z > 0$ on G is distributed as follows : for any matching M on G,

$$\mathbb{P}(\mathcal{M}_G^z = M) = \frac{z^{|V|-2|M|}}{P_G(z)}, \text{ with } P_G(z) = \sum_M z^{|V|-2|M|}.$$

We are interested in the asymptotic behavior of $\mathcal{M}_G^z$ as $|G| \to \infty$. Specifically, we establish that for any graph sequence $(G_n)_{n\geq 1}$ converging to an infinite tree $\mathcal{T}$ with finite Hausdorff dimension, $\mathcal{M}_{G_n}^z$ converges in distribution to a properly defined random matching $\mathcal{M}_{\mathcal{T}}^z$ on $\mathcal{T}$ with determinantal marginals. Moreover, the zero-temperature limit $\mathcal{M}_{\mathcal{T}}^0 = \lim_{z\to 0} \mathcal{M}_{\mathcal{T}}^z$ exists in some sense, and under an extra condition on $\mathcal{T}$ it is precisely the weak limit of a the uniform maximum matching on G_n. When the $(G_n)_{n\geq 1}$ are random and converge weakly to a Galton-Watson tree, the limit turns out to be characterized by a recursive distributional equation, which we solve. We thus obtain an explicit formula for the asymptotic size of a maximum matching on G_n, generalizing that of Karp and Sipser for Erdös-Rényi graphs.

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Journées MAS 2010, Bordeaux

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