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Abstract: Fair resource allocation is usually studied in a static context, in which a fixed amount of resources is to be shared. In dynamic resource allocation one usually tries to assign resources instantaneously so that the average share of each user is split fairly. The exact definition of the average share may depend on the application, as different applications may require averaging over different time periods or time scales. In this paper we address the question of defining and computing resource allocation in such situations. We examine how the constraints on the averaging durations impact the amount that each user gets.

Key-words: Resource allocation; Multiscale fairness; α-fairness; T-scale fairness; Wireless networks.

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Équité a plusieurs échelles de temps et ses applications dans l'allocation dynamique des ressources dans les réseaux sans fil

Résumé : L'allocation équitable des ressources est étudiée d'habitude dans un contexte statique, où une quantité fixe de ressource doit être partagée. Dans l'allocation dynamique des ressources, on a comme objectif de contrôler l'allocation instantanée de manière à ce que la quantité moyenne soit distribuée équitablement. La définition exacte de la notion de quantité moyenne pourrait dépendre de l'application, surtout que des application différentes pourraient avoir des contraintes différentes sur la durée des périodes sur lesquelles on va moyenner . Nous allons étudier quel est l'impact de ces contraintes (sur les périodes pour calculer les moyennes), sur la quantité de ressource que chaque usager obtient.

Mots-clés : allocation des ressources; équité multi-échelle; alpha-équité; équité T-échelle; réseaux sans-fil.

1 Introduction

Let us consider some set S of resource that we wish to distribute among I users by assigning user i a subset S_i of it. We shall be interested in allocating subsets of the resource fairly among the users. The set S may actually correspond to one or to several resources. We shall consider standard fairness criteria for sharing the resources among users. We shall see, however, that the definition of a resource will have a major impact on the fair assignment.

We associate with each user i a measurable function x_i that maps each point in S to some real number. Then, we associate with each i a utility u_i which maps all measurable subsets S_i to the set of real numbers. We shall say that S is a resource if $u_i(S_i)$ can be written as

$$u_i(S_i) = f\left(\int_{S_i} x_i(s)ds\right)$$

for each $S_i \subset S$.

As an example, consider I mobiles that wish to connect to a base station between 9h00 and 9h10 using a common channel. The time interval is divided into discrete time slots whose number is N. Assume that the utility for each mobile s of receiving a subsets $\mathcal{N}_i$ of slots depend only on the number of slots N_i it receives. Then the set of N slots is considered to be a resource.

Next assume that if mobile i receives the channel at time slot t then it can transmit at a throughput of X_t^i. Assume that the utility of user i is a function of the total throughput it has during this fraction of an hour. Then again the N slots are considered as a resource.

We adopt the idea that fair allocation should not be defined in terms of the object that is split but in terms of the utility that corresponds to the assignments. This is in line with the axiomatic approach for defining the Nash bargaining solution for example. With this in mind, we may discover that the set of N slots cannot always be considered as a resource to be assigned fairly. Indeed, a real time application may consider the N slots as a set of n resources, each containing $B = N/n$ consecutive slots. A resource may correspond to the number of time slots during a period of 100 msec. The utility of the application is defined as a function of the instantaneous rate, i.e. the number of slots it receives during each period of 100 msec. (With a playout buffer that can store 100 msec of voice packets, the utility of the mobile depends only on how many slots are assigned to it during 100 msec and not which slots are actually assigned to it.)

What is the impact on data transfer applications of splitting the resource of N slots into B smaller resources? We shall show that allocating fairly each of these B resources results in performance degradation for the data transfer applications. This raises the question of how to define fair assignment when the very notion of a resource varies from one user to another.

Another example where this question arises is frequency allocation. Assume that frequency bandwidth needs to be split between users, who bid for N carriers, each of bandwidth b. There may be users who need carriers of bandwidth mb. They can make use of a carrier only if they receive a set of m consecutive carriers. For these users, a resource may correspond to the set of N/m group of carriers, each of which containing m consecutive carriers.

Related work

Our work is based on the α-fairness notion introduced by Mo and Walrand [7]. This notion provides a continuum of fairness definitions through the real parameter α and it includes various known fairness concepts that are obtained for some specific values of the real parameter α (the max-min fairness, the proportional fairness and the harmonic fairness). This, as well as other fairness notions can be defined through a set of axioms, see [6]. This paper is inspired by several papers which already observed or derived fairness at different time-scales [1, 2, 3, 4, 8]. However, we would like to mention that the T-scale fairness (a unifying generalization of long- and short- term fairness) and multiscale fairness are new concepts introduced in the present work.

Structure of the paper

The paper is organized as follows: In the next Section 2 we introduce a resource sharing model which is particularly suitable for wireless network applications. In Section 2 we also define several fairness criteria, illustrate them by examples and prove theoretical properties of the introduced fairness criteria. In Section 3 we derive explicit formulae for instantaneous α-fairness in the case of linear resources. The case of linear resources corresponds to the frequency as a resource in wireless networks. In Sections 4 and 5 we apply different fairness criteria to spectrum allocation in fading channels and to indoor-outdoor scenario, respectively. Section 6 concludes the paper and provides avenues for future research.

2 Resource Sharing model and different fairness definitions

Consider n mobiles located at points $x_1, x_2, ..., x_n$, respectively. We assume that the utility U_i of mobile i depends on its location x_i and on the amount of resources s_i it gets.

Let $\mathbf{S}$ be the set of assignments; an assignment $s \in \mathbf{S}$ is a function from the vector x to a point in the n-dimensional simplex. Its ith component, $s_i(x)$ is the fraction of resource assigned to mobile i.

Definition 2.1 *(α-fair assignment) An assignment s is α-fair if it is a solution of:*

$$\begin{aligned} Z(x,\alpha) &:= \max_s \sum_i Z_i(s_i, x_i, \alpha) \\ s.t. \quad \sum_i s_i &= 1, \; s_i \geq 0 \forall i = 1, ..., n \end{aligned} \quad (1)$$

where,

$$Z_i(x_i, s_i, \alpha) := \frac{(U_i(x_i, s_i))^{1-\alpha}}{1-\alpha}$$

for $\alpha \neq 1$. *For* $\alpha = 1$ *we define*

$$Z_i(x_i, s_i, \alpha) := \log(U_i(x_i, s_i))$$

We shall assume throughout that U_i is non-negative, strictly increasing and is concave in s_i. Then for any $\alpha > 0$, $Z_i(x_i, s_i, \alpha)$ is strictly concave in s_i. We conclude that $Z(x_i, s_i, \alpha)$ is strictly concave in s for any $\alpha > 0$ and therefore there is a unique solution $s^*(\alpha)$ to (1).

Definition 2.2 *(Mo and Walrand [7]) We call* $Z_i(s_i, \cdot, \alpha)$ *the fairness utility of mobile* i *under* s_i, *and we call* $Z(s, \cdot, \alpha)$ *the instantaneous degree of* α*-fairness under* s.

In applications, the state X will be random, so that the instantaneous amount of resource assigned by an α-fair allocation will also be a random variable. Thus, in addition to instantaneous fairness we shall be interested in the expected amount assigned by being fair at each instant.

Definition 2.3 *We call* $E[Z(s, X, \alpha)]$ *the expected instantaneous degree of* α*-fairness under* s.

In Section 2.1 we introduce the expected long-term fairness in which the expected amount of resource is assigned fairly.

Definition 2.4 *We say that a utility is linear in the resource if it has the form:*

$$U_i(x_i, s_i) := s_i q_i(x_i).$$

For example, consider transmission between a mobile source and a base station, and assume
(i) that the base station is in the origin ($x = 0$) but at a height of one unit, whereas all mobiles are on the ground and have height 0. Thus, the distance between the base station and a mobile located on the ground at point x is $\sqrt{1 + ||x||^2}$.
(ii) that the Shannon capacity can be used to describe the utility. If the resource that is shared is the frequency then the utility has the linear form:

$$\begin{aligned} U(C, x) &:= Cq(x) \\ \text{where } q(x) &= \log\left(1 + \frac{P(x^2+1)^{-\beta/2}}{\sigma^2}\right) \end{aligned}$$

Note: if the power and not the frequency, were taken to be the resource then we would not obtain the linear form of the resource.

In the linear case, we write Z_i as:

$$Z_i(s, x, \alpha) := s_i^{1-\alpha} v_i(x), \qquad v_i := \frac{(q_i(x_i))^{1-\alpha}}{1-\alpha}$$

2.1 Fairness over time: Instantaneous Versus Long term α-fairness

Next we consider the case where $x_i(t)$, $i = 1, ..., n$, may change in time.

Definition 2.5 *We define an assignment to be instantaneous α-fair if at each time t each mobile is assigned a resource so as to be α-fair at that instant.*

Consider the instantaneous α-fair allocation and assume that time is discrete. We thus compute the instantaneous α-fair fair assignment over a period of T slot as the assignment that maximizes (for $\alpha \neq 1$)

$$\sum_{i=1}^{n} \frac{(U_i(x_i(t), s_i(t)))^{1-\alpha}}{1-\alpha}.$$

for every $t = 1, ..., T$. This is equivalent to maximizing

$$\sum_{t=1}^{T} \sum_{i=1}^{n} \frac{(U_i(x_i(t), s_i(t)))^{1-\alpha}}{1-\alpha}. \tag{2}$$

For $\alpha = 1$ the same is true but where we replace

$$\frac{(U_i(x_i(t), s_i(t)))^{1-\alpha}}{1-\alpha}$$

by

$$\log[U_i(x_i(t), s_i(t))]$$

We make the following surprising observation: The optimization problem (2) corresponds to the α-fair assignment problem in which there are nT players instead of n players, where the utility of player $i = kn + j$ $(k = 0, ..., T-1, j = 1, ..., n)$ is defined as

$$\mathcal{U}_i(x_i, s_i) = U_j(x_j(k+1), s_j(k+1))$$

. Thus the expected instantaneous fairness criterion in the stationary and ergodic case regards assignments at different time slots of the same player as if it were a different player at each time slot!

Note that when considering the proportional fair assignment, then the resulting assignment is the one that maximizes

$$\prod_{i=1}^{n} \prod_{t=1}^{T} U_i(x_i(t), s_i(t))$$

Definition 2.6 *Assume that the state process $X(t)$ is stationary ergodic. Let λ_i be the stationary probability measure of $X(0)$. The long term α-fairness index of an assignment*

$s \in \mathbf{S}$ of a stationary process $X(t)$ is defined as

$$
\begin{aligned}
\overline{Z}_\lambda(s) &:= \sum_{i=1}^{n} \overline{Z}_\lambda^i(s) \\
\textit{where } Z_\lambda^i(s) &= \frac{\Big(E_\lambda\left[U_i(X_i(0), s_i(X(0)))\right]\Big)^{1-\alpha}}{1-\alpha}
\end{aligned}
$$

An assignment s is long-term α-fair if it maximizes $Z_\lambda(s)$ over $s \in \mathbf{S}$.

As we see, instead of attempting to have a fair assignment of the resources at every t, it is the expected utility in the stationary regime that one assigns fairly according to the the long-term fairness. Under stationarity and ergodicity conditions on the process $X(t)$ this amounts in an instantaneous assignment of the resources in a way that the time average amount allocated to the users are α-fair.

2.2 Fairness over time: T-scale α-fairness

Next we define fairness concepts that are in between the instantaneous and the expected fairness. They are related to fairness over a time interval T. Either continuous time is considered or discrete time where time is slotted and each slot is considered to be of one time unit. Below, we shall understand the integral to mean summation when ever time is discrete.

Definition 2.7 *The T-scale α-fairness index of an assignment $s \in \mathbf{S}$ is defined as*

$$
\begin{aligned}
Z_T(s) &:= \sum_{i=1}^{n} Z_T^i(s) \\
\textit{where } Z_T^i &= \frac{\left[\frac{1}{T}\int_0^T U_i(X_i(t), s_i(X(t)))dt\right]^{1-\alpha}}{1-\alpha}
\end{aligned}
$$

The expected T-scale α-fairness index is its expectation. An assignment s is T-scale α-fair if it maximizes $Z_T(s)$ over $s \in \mathbf{S}$.

Definition 2.8 *The T-scale expected α-fairness index of an assignment $s \in \mathbf{S}$ is defined as*

$$
\begin{aligned}
Z_T(s) &:= \sum_{i=1}^{n} Z_T^i(s) \\
\textit{where } Z_T^i &= \frac{\left[\frac{1}{T}\int_0^T E[U_i(X_i(t), s_i(X(t)))]dt\right]^{1-\alpha}}{1-\alpha}
\end{aligned}
$$

We shall consider the following simple example of 2-scale fairness

Example 1 *Consider two time slots and two mobile stations. To whoever the first time slot will be allocated, that mobile would send or receive 25 units. At the second slot, a rate of 5 (resp. 10) units will be used if the slot is assigned to mobile 1 (resp. 2). We make the following observations. By [i,j] we shall denote the allocation that assigns slot 1 to mobile i and slot 2 to mobile j. The allocation [1,2] maximizes the global utility and moreover, the α-fair 2-scale utility for any α.*

Thus, we observe that the α-assignment is not monotone: The player with larger utilities received less at the α-fair utility, for all values of α!

Example 2 *(Example 1 continued) We now change a single utility in the last example: assume that if mobile 2 receives the first slot then it earns 10^2 units.*

(i) Now the global optimal solution is the assignment [2,2].

(ii) The proportional fair solution ($\alpha = 1$) is [2,1].

(iii) The maxmin fair assignment is [1,2].

We depict in Figure 1 the performance index of the assignments [1,2] and [2,1]. We see that the max-min fair assignment [2,1] is 2-scale α-fair for all α larger than 1.36, whereas the assignment [1,2] is α fair for $\alpha \in [1, 1.36]$.

For $\alpha < 1$ the two best assignments are [2,1] and [2,2]. The former is optimal over $\alpha \in [0.17, 1]$ and the latter over $\alpha \in [0, 0.17]$. This is seen from Figure 2.

Assume that the state processes is stationary ergodic. Then for any assignment $s \in \mathbf{S}$ we would have by the Strong Law of Large Numbers:

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T U_i(X_i(t), s_i(X(t)))dt = E_\lambda \left[U_i(X_i(0), s_i(X(0))) \right]$$

P-a.s. Hence, for every i and s, we have P-a.s.

$$\begin{aligned} \lim_{T\to\infty} Z_T^i(s) &= \lim_{T\to\infty} \frac{\left[\frac{1}{T}\int_0^T U_i(X_i(t), s_i(X(t)))dt\right]^{1-\alpha}}{1-\alpha} \\ &= \frac{\left(E_\lambda \left[U_i(X_i(0), s_i(X(0)))\right]\right)^{1-\alpha}}{1-\alpha} \\ &= \overline{Z}_T^i(s). \end{aligned}$$

Assume that U_i is bounded. Then Z_T^i is bounded uniformly in T. The bounded convergence then implies that

$$\lim_{T\to\infty} E[Z_T^i(s)] = \overline{Z}_T^i(s). \tag{3}$$

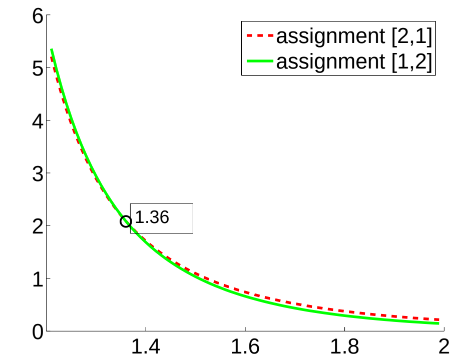

Figure 1: Performance index of [2,1] (dashed line) and [1,2] (solid line) assignments as a function of α (horizontal axis)

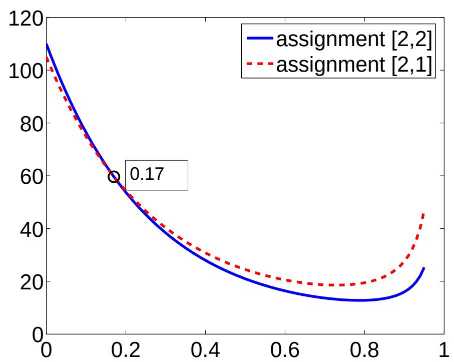

Figure 2: Performance index of [2,1] (dashed line) and [2,2] (solid line) assignments as a function of α (horizontal axis)

Theorem 2.1 *Assume that the convergence in (3) is uniform in s. Let $s^*(T)$ be the T-scale α fair assignment and let s^* be the long term α-fair assignment. Then the following holds:*

- $s^* = \lim_{T\to\infty} S^*(T)$
- *For any* $\epsilon > 0$, s^* *is an* ϵ*-optimal assignment for the* T*-scale criterion for all* T *large enough.*
- *For any* $\epsilon > 0$, $s^*(T)$ *is an* ϵ*-optimal assignment for the long term fairness for all* T *large enough.*

Proof. According to [9], any accumulation point of $s^*(T)$ as $T \to \infty$ is an optimal solution to the problem of maximizing $\overline{Z}_T$ over S. Due to the strict concavity of $\overline{Z}_T$ in s it has a unique solution and it is coincides with any accumulation point of $s^*(T)$. This implies the first statement of the theorem. The other statements follow from Appendices A and B in [9]. $\diamond$

2.3 Fairness over different time scales: Multiscale fairness

We consider real time (RT) and non-real time (NRT) traffic. Resource allocation policy for RT traffic is *instantaneous-fair*, while for the NRT traffic, it is *expected-fair*. The available resources are divided amongst the RT and NRT traffic so as to guarantee a minimum quality of service (QoS) requirement for the RT traffic and to keep service time as short as possible for the NRT traffic.

The real time traffic would like the allocation to be instantaneously α-fair. For $\alpha > 0$, this guarantees that at any time it receives a strictly positive allocation.

The non-real time traffic does not need to receive at each instant a positive amount of allocation. It may prefer the resources to be assigned according to the T-scale α-fair assignment where T may be of the order of the duration of the connection. Moreover, different non real time applications may have different fairness requirements. For instance, bulk FTP transfer can prefer fairness over time scale longer than a time scale for some streaming application.

In order to be fair, we may assign part (say half) of the resource according to the instantaneous α-fairness and the rest of the resources according to the T-scale α-fairness. We thus combine fairness over different time scales.

We may now ask how to choose what part of the resource would be split according to the instantaneous assignment and what part according to the T-scale assignment. We propose to determine this part using the same α-fair criterion.

Specifically we define the multiscale fairness as follows:

Definition 2.9 *The multiscale* α*-fairness index of an assignment* $s \in \mathbf{S}$ *is defined as*

$$
\begin{aligned}
Z_{T_1,\ldots,T_n}(s) &:= \sum_{i=1}^{n} Z^i_{T_i}(s) \\
\textit{where } Z^i_{T_i} &= \frac{\left[\frac{1}{T_i}\int_0^{T_i} U_i(X_i(t), s_i(X(t)))dt\right]^{1-\alpha}}{1-\alpha}
\end{aligned}
$$

The expected multiscale α-fairness index is its expectation. An assignment s is multiscale α-fair if it maximizes $Z_{T_1,...,T_n}(s)$ over $s \in \mathbf{S}$. We also say that multiscale α-fair assignment is $(T_1,...,T_n)$-scale fair assignment.

Example 3 *Let us consider an example of multiscale fairness. Say, we have in total N time slots. The allocation happens in a bundle of 6 slots, such that, either we allocate all of it to an outdoor user located at x_1 or fair share them amongst 3 indoor users located at (y_1, y_2, y_3) with $y_i \in (0, L)$, with any user getting two consecutive slots. Now the question is "Given that we fair-share among the indoor users, how do we fair share between the outdoor and the indoor users?".*

In this example, we assume any user gets a throughput $q \in [0,1]$. Let $\{U_1, U_2\}$, represent the utility of user 1 and sum utility of users $2-4$ and let $\{T_1, 1-T_1\}$ represent their respective assignment of resources. Now, utility of user 1,

$$U_1(T_1) = 6T_1 q_1(x_1).$$

Let $\bar{s} = \{s_1, s_2, 1-s_1-s_2\}$ represent the assignment of resources for the indoor users. Then, utility of users $2-4$ is

$$u_2(T_1, \bar{s}) = 6s_1(1-T_1)q_2(y_2),$$

$$u_3(T_1, \bar{s}) = 6s_2(1-T_1)q_3(y_3)$$

and

$$u_4(T_1, \bar{s}) = 6(1-s_1-s_2)(1-T_1)q_4(y_4)$$

Now the α-fair share $\bar{s}^* = \{s_1^*, s_2^*, 1-(s_1^*-s_2^*)\}$ is given by,

$$\bar{s}^* = \arg\max_{\bar{s}} \sum_{i=2}^{4} \frac{E[u_i(T_1, \bar{s})]^{1-\alpha_1}}{1-\alpha_1}$$

The sum utility of users $2-4$ is,

$$U_2(T_1) = \sum_{i=2}^{4} 6\bar{s}_i{}^*(1-T_1)q_i(y_i)$$

The α-fair share between the outdoor and indoor users is,

$$T_1^* = \arg\max_{T_1} \frac{E\left[U_1(T_1)\right]^{1-\alpha} + E\left[U_2(1-T_1)\right]^{1-\alpha}}{1-\alpha}$$

3 Instantaneous α-fairness for linear resources

In the case of linear resources the instantaneous α-fairness has a nice explicit expression.

Theorem 3.1 *(i) The α-fair share is given by*

$$s_i^* = \frac{v_i(x_i)^{1/\alpha}}{\sum_j v_j(x_j)^{1/\alpha}} = \frac{q_i(x_i)^{1/\alpha-1}}{\sum_j q_j(x_j)^{1/\alpha-1}}$$

(ii) The utility for mobile i under the fair assignment is then

$$U_i(s^*, x) = \frac{v_i(x_i)^{1/\alpha}}{\sum_j v_j(x_j)^{1/\alpha}} q_i(x_i) = \frac{q_i(x_i)^{1/\alpha}}{\sum_j q_j(x_j)^{1/\alpha-1}}$$

(iii) The optimal value Z is given by

$$Z = \frac{1}{1-\alpha} \sum_i \left(\frac{q_i(x_i)^{1+1/\alpha}}{\sum_j q_j(x_j)^{1/\alpha}} \right)^{1-\alpha}$$

Proof. We relax the constraint and use KKT condition. s is optimal if and only if there is some $\lambda > 0$ such that s maximize L^λ s.t. $s_i \geq 0$ for all i, where

$$L^\lambda = \sum_i s_i^{1-\alpha} v_i + \lambda(1 - \sum_i s_i)$$

Equating the derivative w.r.t. s_i to zero gives

$$\begin{aligned} s_i^{-\alpha} v_i(x_i) &= \frac{\lambda}{1-\alpha} \\ \text{so that } s_i &= \left(\frac{1-\alpha}{\lambda} v_i(x_i) \right)^{1/\alpha} \end{aligned}$$

Since the sum of s_i is 1, we conclude that

$$\frac{\lambda}{1-\alpha} = \left(\sum_j v_j(x_j)^{1/\alpha} \right)^{\alpha}$$

Substituting in the previous equation yields (i), which then implies the rest. ◇

Example 4 *Consider as an example a path loss $\beta = 2$ and let the base station be located one unit above the mobiles. We assume that $q_i(x)$ is proportional to the attenuation between the mobile and the base station: $q_i(x) = c_i q(x)$ where $q(x) = (1 + x^2)^{-1/2}$. For $\beta = \alpha = 2$ we have*

$$s_i^*(x) = \frac{c_i^{-1/2}(1 + x_i^2)^{-1/2}}{\sum_j c_j^{-1/2}(1 + x_j^2)^{-1/2}}.$$

Furthermore,

$$U_i(s^*, x) = s_i^*(x) q_i(x_i) = \frac{c_i^{1/2}(1 + x_i^2)}{\sum_j c_j^{-1/2}\sqrt{1 + x_j^2}}.$$

4 Application to spectrum allocation in random fading channels

We consider two users: fast-changing user and slowly-changing user. The users' channels are modeled by the Gilbert model. The users can be either in a good or in a bad state. The dynamics of the fast-changing user is described by a Markov chain $\{Y_1(t)\}_{t=0,1,\dots}$ with the following transition matrix

$$P_1 = \begin{bmatrix} 1-\alpha_1 & \alpha_1 \\ \beta_1 & 1-\beta_1 \end{bmatrix}.$$

Its stationary distribution is given by

$$\pi_1 = \begin{bmatrix} \frac{\beta_1}{\alpha_1+\beta_1} & \frac{\alpha_1}{\alpha_1+\beta_1} \end{bmatrix}.$$

The slowly-changing user is described by a Markov chain $\{Y_2(t)\}_{t=0,1,\dots}$ with the following transition matrix

$$P_2 = \begin{bmatrix} 1-\epsilon\alpha_2 & \epsilon\alpha_2 \\ \epsilon\beta_2 & 1-\epsilon\beta_2 \end{bmatrix}.$$

Its stationary distribution is given by

$$\pi_2 = \begin{bmatrix} \frac{\beta_2}{\alpha_2+\beta_2} & \frac{\alpha_2}{\alpha_2+\beta_2} \end{bmatrix}.$$

Note that the parameter ϵ does not have an effect on the stationary distribution but it influences for how long the slowly-changing user stays in some state. The smaller ϵ, the more seldom the user changes the states. If we choose $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then the fast-changing user and the slowly-changing user have the same stationary distribution.

We assume that state 1 is a bad state and state 2 is a good state. When the fast-changing user is in the bad state, its channel gain coefficient is h_{11} and when the fast-changing user is in the good state, its channel gain coefficient is h_{12}. Of course, we have $h_{11} < h_{12}$. Thus, the achievable throughputs in different states are given by

$$\begin{aligned} U_{11} &= s_{11} q_{11} = s_{11} \log\left(1 + \frac{h_{11} P_1}{\sigma^2}\right), \\ U_{12} &= s_{12} q_{12} = s_{12} \log\left(1 + \frac{h_{12} P_1}{\sigma^2}\right) \end{aligned}$$

where P_1 is the power applied by the fast-changing user.

Similarly, for the slowly-changing user we associate with the bad state (state 1) the channel gain h_{21} and with the good state (state 2) the channel gain h_{22}. Again we have $h_{21} < h_{22}$, and the achievable throughputs in different states are given by

$$\begin{aligned} U_{21} &= s_{21}q_{21} = s_{21}\log\left(1+\frac{h_{21}P_2}{\sigma^2}\right), \\ U_{12} &= s_{22}q_{22} = s_{22}\log\left(1+\frac{h_{22}P_2}{\sigma^2}\right) \end{aligned}$$

where P_2 is the power applied by the slowly-changing user.

First, we would like to analyze T-scale fairness and to see the effect of the time scale on the resource allocation. Specifically, we consider the following optimization criterion

$$\sum_{i=1}^{2}\frac{1}{1-\alpha}\left[\frac{1}{T}\sum_{t=0}^{T}U_i(t)\right]^{1-\alpha} \quad\rightarrow\quad \max_{s_1,s_2} \tag{4}$$

with $U_i(t) = s_i(t)q_{i,Y_i(t)}$ and $s_1(t)+s_2(t) = 1$.

Let us consider several options for the time horizon T:

Instantaneous fairness. If we take $T = 1$ we obtain the instantaneous fairness. Namely, the criterion (4) takes the form

$$\frac{1}{1-\alpha}\left[U_1^{1-\alpha}(0)+U_2^{1-\alpha}(0)\right] \quad\rightarrow\quad \max_{s_1,s_2}$$

The solution of the above optimization problem (follows from Theorem 1) is given by

$$s_i(0) = \frac{q_{i,Y_i(0)}^{(1-\alpha)/\alpha}}{q_{1,Y_1(0)}^{(1-\alpha)/\alpha}+q_{2,Y_2(0)}^{(1-\alpha)/\alpha}}$$

This allocation results in the following expected throughputs

$$\begin{aligned} \theta_1 &= \sum_{i,j}\frac{q_{1,i}^{1/\alpha}}{q_{1,i}^{(1-\alpha)/\alpha}+q_{2,j}^{(1-\alpha)/\alpha}}\pi_{1,i}\pi_{2,j}, \\ \theta_2 &= \sum_{i,j}\frac{q_{2,j}^{1/\alpha}}{q_{1,i}^{(1-\alpha)/\alpha}+q_{2,j}^{(1-\alpha)/\alpha}}\pi_{1,i}\pi_{2,j}. \end{aligned} \tag{5}$$

Mid-term fairness. Let us take the time horizon as a function of the underlying dynamics time parameter ϵ, that is $T = T(\epsilon)$, satisfying the following conditions: (a) $T(\epsilon)\rightarrow\infty$ and (b) $T(\epsilon)\epsilon\rightarrow 0$. The condition (a) ensures that

$$\frac{1}{T(\epsilon)}\sum_{t=0}^{T(\epsilon)}1\{Y_1(t)=i\}\rightarrow\pi_{1,i}, \quad \text{as} \quad \epsilon\rightarrow 0,$$

and the condition (b) ensures that

$$\frac{1}{T(\epsilon)} \sum_{t=0}^{T(\epsilon)} 1\{Y_2(t) = i\} \to \delta_{Y_2(0),i}, \quad \text{as} \quad \epsilon \to 0.$$

This follows from the theory of Markov chains with multiple time scales (see e.g., [5]). It turns out to be convenient to take the following notation for the resource allocation: We denote by $s(t)$ the allocation for the fast-changing user and by $1-s(t)$ the resource allocation for the slowly-changing user. Thus, we have $s_1(t) = s(t)$ and $s_2(t) = 1 - s(t)$. We denote by $\bar{s}_{i,j} = E[s(t)|Y_1(t) = i, Y_2(t) = j]$. We note that since the fast-changing user achieves stationarity when $T(\epsilon) \to \infty$ we are able to solve (4) in stationary strategies. Then, the criterion (4) takes the form

$$\begin{aligned} \frac{1}{1-\alpha} \quad & \left[(\pi_{1,1}q_{1,1}\bar{s}_{1,Y_2(0)} + \pi_{1,2}q_{1,2}\bar{s}_{2,Y_2(0)})^{1-\alpha}\right. \\ & + \left.((1-\pi_{1,1}\bar{s}_{1,Y_2(0)} - \pi_{1,2}\bar{s}_{2,Y_2(0)})q_{2,Y_2(0)})^{1-\alpha}\right] \\ & \to \max_{\bar{s}_{1,Y_2(0)},\ \bar{s}_{2,Y_2(0)}} \end{aligned}$$

The above nonlinear optimization problem can be solved numerically. The expected throughputs in the mid-term fairness case are given by

$$\begin{aligned} \theta_1 &= (\pi_{1,1}q_{1,1}\bar{s}_{1,1} + \pi_{1,2}q_{1,2}\bar{s}_{2,1})\pi_{2,1} \\ &+ (\pi_{1,1}q_{1,1}\bar{s}_{1,2} + \pi_{1,2}q_{1,2}\bar{s}_{2,2})\pi_{2,2} \\ \theta_2 &= (1 - \pi_{1,1}\bar{s}_{1,1} - \pi_{1,2}\bar{s}_{2,1})q_{2,1}\pi_{2,1} \\ &+ (1 - \pi_{1,1}\bar{s}_{1,2} - \pi_{1,2}\bar{s}_{2,2})q_{2,2}\pi_{2,2} \end{aligned} \tag{6}$$

Long-term fairness. In the case of long-term fairness we set $T = \infty$ which results in the following criterion

$$\frac{1}{1-\alpha}\left[E[U_1]^{1-\alpha} + E[U_2]^{1-\alpha}\right] \quad \to \quad \max_{s_1,s_2}$$

Due to stationarity, we can solve the above optimization problem over sequences in stationary strategies. Namely, we have the following optimization problem

$$\begin{aligned} \frac{1}{1-\alpha} \quad & ((\pi_{1,1}\pi_{2,1}\bar{s}_{1,1} + \pi_{1,1}\pi_{2,2}\bar{s}_{1,2})q_{1,1} \\ + \ & (\pi_{1,2}\pi_{2,1}\bar{s}_{2,1} + \pi_{1,2}\pi_{2,2}\bar{s}_{2,2})q_{1,2})^{1-\alpha} \\ + \ & ((\pi_{2,1} - \pi_{1,1}\pi_{2,1}\bar{s}_{1,1} - \pi_{1,2}\pi_{2,1}\bar{s}_{2,1})q_{2,1} \\ + \ & (\pi_{2,2} - \pi_{1,1}\pi_{2,1}\bar{s}_{1,1} - \pi_{1,2}\pi_{2,2}\bar{s}_{2,2})q_{2,2})^{1-\alpha}] \\ & \to \max_{\bar{s}_{1,1},\ \bar{s}_{1,2},\ \bar{s}_{2,1},\ \bar{s}_{2,2}} \end{aligned}$$

The expected throughputs in the long-term fairness case are given by

$$\begin{aligned}
\theta_1 &= (\pi_{1,1}\pi_{2,1}\bar{s}_{1,1} + \pi_{1,1}\pi_{2,2}\bar{s}_{1,2})q_{1,1} \\
&+ (\pi_{1,2}\pi_{2,1}\bar{s}_{2,1} + \pi_{1,2}\pi_{2,2}\bar{s}_{2,2})q_{1,2} \\
\theta_2 &= (\pi_{2,1} - \pi_{1,1}\pi_{2,1}\bar{s}_{1,1} - \pi_{1,2}\pi_{2,1}\bar{s}_{2,1})q_{2,1} \\
&+ (\pi_{2,2} - \pi_{1,1}\pi_{2,1}\bar{s}_{1,1} - \pi_{1,2}\pi_{2,2}\bar{s}_{2,2})q_{2,2} \qquad (7)
\end{aligned}$$

Let us also consider the expected instantaneous fairness which is given by the criterion

$$\frac{1}{1-\alpha}\left[E[U_1^{1-\alpha}(t)] + E[U_2^{1-\alpha}(t)]\right] \quad \rightarrow \quad \max_{s_1,s_2}$$

which is equivalent to

$$\begin{aligned}
\frac{1}{1-\alpha} \quad & \left[\sum_{ij}\pi_{1,i}\pi_{2,j}\int_0^1 (sq_{1,i})^{1-\alpha}dF_{ij}(s)\right. \\
& \left. +\sum_{ij}\pi_{1,i}\pi_{2,j}\int_0^1 ((1-s)q_{2,j})^{1-\alpha}dF_{ij}(s)\right] \quad \rightarrow \quad \max_{F_{ij}}
\end{aligned}$$

where $F_{ij}(s)$ is the distribution for $s(t)$ conditioned on the event $\{Y_1(t) = i, Y_2(t) = j\}$. The above criterion is maximized by

$$F_{ij}(s) = \begin{cases} 0, & \text{if } s < q_{1,i}^{(1-\alpha)/\alpha}/(q_{1,i}^{(1-\alpha)/\alpha} + q_{2,j}^{(1-\alpha)/\alpha}), \\ 1, & \text{if } s \geq q_{1,i}^{(1-\alpha)/\alpha}/(q_{1,i}^{(1-\alpha)/\alpha} + q_{2,j}^{(1-\alpha)/\alpha}). \end{cases}$$

Thus, we can see that the expected instantaneous fairness criterion is equivalent to instantaneous fairness.

Let us consider a numerical example. The parameters are given in Table 1. We consider three typical cases. For these three cases, we plot the expected throughput of the mobiles for various fairness criteria (see Figures 3-5). The first case corresponds to the symmetric scenario. Since in this scenario the users have the same stationary distributions and the same conditional Shannon capacities, the expected throughputs are the same when we use either instantaneous fairness criterion or long-term fairness criterion. Interesting, in the long-term fairness case, both users experience degradation in throughput when α increases. In the case of mid-term fairness, the expected throughput of the fast-changing user is higher as in the mid-term fairness criterion the utility of the fast-changing user is the α-fairness function of the expected throughput. In the second case, the fast-changing user has in general better channel conditions. In this case, different fairness criteria provide different resource allocation. We observe that instantaneous and mid-term fairness allocations are more sensitive with respect to the parameter α than the long-term fairness allocation. In the third scenario the slowly-changing user (user 2) is more often in the good channel state

Table 1: Case 1,2 & 3: Shannon capacity (q)/probability(π)

Case-1	state-1 (bad)	state-2 (good)
User-1	2/0.2	8/0.8
User-2	2/0.2	8/0.8
Case-2	state-1 (bad)	state-2 (good)
User-1	3/0.1	9/0.9
User-2	1/0.3	7/0.7
Case-3	state-1 (bad)	state-2 (good)
User-1	3/0.9	9/0.1
User-2	1/0.3	7/0.7

than the fast-changing user (user 1). Now, for all the criteria the slowly-changing user has better expected throughput.

Next, let us consider multiscale fairness over time. Specifically, (T_1, T_2)-scale fairness is defined by the following criterion

$$\frac{1}{1-\alpha}\left[\left(\frac{1}{T_1}\sum_{t=0}^{T_1} U_1(t)\right)^{1-\alpha} + \left(\frac{1}{T_2}\sum_{t=0}^{T_2} U_2(t)\right)^{1-\alpha}\right] \quad \rightarrow \quad \max_{s_1,s_2}$$

In this particular example, there are 6 possible combinations of different time scales. It turns out that in this example only the $(1,\infty)$-scale fairness gives a new resource allocation. The other combinations of time scales reduce to some T-scale fairness. Thus, let us first consider the multiscale fairness when we apply instantaneous fairness to the fast-changing user and long-term fairness to the slowly-changing user. The $(1,\infty)$-scale fairness corresponds to the following optimization criterion

$$\frac{1}{1-\alpha}\left[U_1(0)^{1-\alpha} + E[U_2(t)]^{1-\alpha}\right] \quad \rightarrow \quad \max_{s_1,s_2}$$

which is equivalent to

$$\begin{aligned}\frac{1}{1-\alpha} \qquad & \left[(q_{1,Y_1(0)}(\bar{s}_{Y_1(0),1}\pi_{2,1} + \bar{s}_{Y_1(0),2}\pi_{2,2}))^{1-\alpha}\right. \\ & \left. +(q_{2,1}(1-\bar{s}_{Y_1(0),1})\pi_{2,1} + q_{22}(1-\bar{s}_{Y_1(0),2})\pi_{2,2})^{1-\alpha}\right] \\ & \rightarrow \quad \max_{\bar{s}_{Y_1(0),1},\ \bar{s}_{Y_1(0),2}}\end{aligned}$$

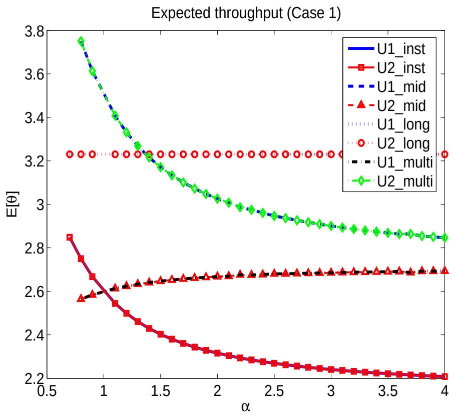

Figure 3: Throughput(θ) as a function of α for instantaneous, mid-term, long-term and (1,∞)-scale fairness criteria (Case 1).

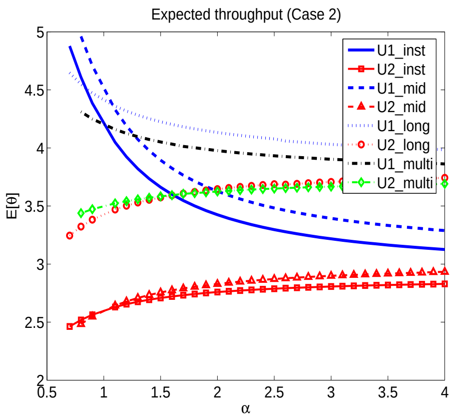

Figure 4: Throughput(θ) as a function of α for instantaneous, mid-term, long-term and (1,∞)-scale fairness criteria (Case 2).

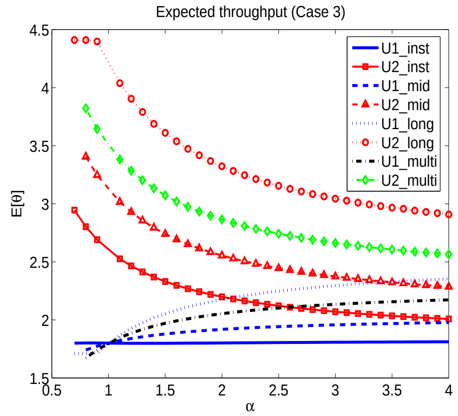

Figure 5: Throughput(θ) as a function of α for instantaneous, mid-term, long-term and (1,∞)-scale fairness criteria (Case 3).

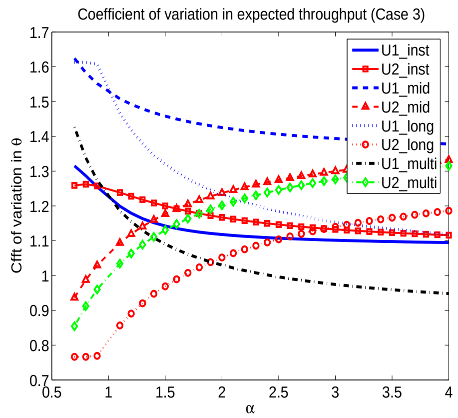

Figure 6: Coefficient of variation in expected throughput as a function of α for instantaneous, mid-term, long-term and (1,∞)-scale fairness criteria (Case 3).

The expected throughputs in the $(1,\infty)$-scale fairness case are given by

$$\begin{aligned}
\theta_1 &= q_{1,1}(\bar{s}_{1,1}\pi_{1,1}\pi_{2,1} + \bar{s}_{1,2}\pi_{1,1}\pi_{2,2}) \\
&+ q_{1,2}(\bar{s}_{2,1}\pi_{1,2}\pi_{2,1} + \bar{s}_{2,2}\pi_{1,2}\pi_{2,2}) \\
\theta_2 &= (q_{2,1}(1-\bar{s}_{1,1})\pi_{2,1} + q_{22}(1-\bar{s}_{1,2})\pi_{22})\pi_{1,1} \\
&+ (q_{2,1}(1-\bar{s}_{2,1})\pi_{2,1} + q_{22}(1-\bar{s}_{2,2})\pi_{22})\pi_{1,2}.
\end{aligned}$$

As we have mentioned above, the other combinations of time scales reduce to some T-scale fairness. In particular, $(1, T(\epsilon))$-fairness reduces to the instantaneous fairness, $(T(\epsilon), \infty)$-fairness reduces to long-term fairness, and $(T(\epsilon), 1)$-, $(\infty, 1)$- and $(\infty, T(\epsilon))$-fairness all reduce to mid-term fairness.

We also plot the expected throughputs for $(1,\infty)$-scale fair allocation for the numerical example with three cases (see Figures 3-5). We observe that in the symmetric case $(1,\infty)$-scale fairness criterion provides an allocation which is opposite to the allocation provided by the mid-term fairness criterion. This indicates that the T-scale fairness and multiscale fairness concepts provide a versatile framework for resource allocation which takes into account the dynamics of the users. From Figures 4 and 5 we conclude that multiscale fairness provide good sensitivity to the variation of the fairness parameter and good performance in expected throughput.

Coefficient of variation: We compute the coefficient of variation for short-term, mid-term, long-term and multiscale fairness. For this, we first compute the second moment of the throughput and then find the ratio of the standard deviation to its mean. For any user i, the coefficient of variation is

$$\Gamma_i = \frac{\sqrt{\mathbf{E}[\theta_i^2]}}{\mathbf{E}[\theta_{\mathbf{i}}]}$$

In Figure 6, we plot the coefficient of variation in throughput for the considered above various fairness criteria. It is very interesting to observe that except the $(1,\infty)$-scale fairness criterion all the other fairness criteria behave similarly with respect to the coefficient of variation. Only in the case of $(1,\infty)$-scale fairness the coefficient of variation decreases for sort-term fairness oriented user. This is a very desirable property of the multiscale fairness as a short-term fairness oriented user is typically a user with a delay sensitive application.

5 Application to indoor-outdoor scenario

Let Ω be the line segment $[-L, L]$, and let there be a wall at $x = 0$. Assume that the base station is located just to the left of the wall. Mobile 1 is at some point $x \le 0$ outdoor and user 2 remains always indoor and is located at some Y_t which is uniformly distributed over $[0, L]$. We let $q_i(x) = c_i q(x)$ with $c_1 = 1$ and c_2 is equal to some large fixed number. Thus the presence of the wall between the base station and mobile 2 is modeled by a multiplicative attenuation by some constant c_2. Assume that the mobility pattern of mobile 2 is uniform over the indoor part $[0, L]$.

We consider allocation of the fraction of time between the two mobiles.

5.1 Instantaneous Fairness

Example 4 (continued). We compute the expected utility for each user when assigning the channel so as to achieve instantaneous fairness. The expected utility for mobile 1 under the instantaneous optimal fairness $\mathbf{s}$ is given by

$$\begin{aligned} U_1(s^*, x) &= s^*(x)q_1(x), \text{ where } s^*(x) := \frac{a}{a+b} \\ a &:= c_1^{-1/2} \log\left(1 + \frac{1}{x^2}\right)^{-1/2} \\ b &:= c_2^{-1/2}\left[\log(1 + \frac{1}{L^2}) + \frac{2}{L}\tan^{-1}(L)\right]^{-1/2} \end{aligned}$$

Note that mobile 2 has a mobility pattern which is uniform over the indoor part [0,L] and hence its utility is given by $\frac{1}{L}\int_0^L \log(1 + \frac{1}{x^2})dx = \log(1 + \frac{1}{L^2}) + \frac{2}{L}\tan^{-1}(L)$, which is the second term in the denominator.

◇

In figure (7) and (8), we plot the scheduler and the instantaneous throughput for the indoor and outdoor user, as a function of α. We fix the location of the outdoor user at $x = -3$, path-loss $\beta = 3$. We set $L = 3$ for this example. The indoor user is located at some point which is uniformly distributed over $[0, L]$. When the fairness index α is small, we observe that the instantaneous throughput achieved is higher as the outdoor user is located at the boundary $(-L)$. But, as the fairness index increases, the throughput of the indoor and the outdoor user starts to converge. Notice that the scheduler starts to schedule the outdoor user more as α increases, which results in an increase in the outdoor users throughput.

5.2 Long term Fairness

Next we consider the long-term fairness. The long term allocation $s \in \mathbf{S}$ (which is a function of x and Y_t) is given by maximizing

$$Z(s) := \frac{\left[\frac{1}{L}\int_0^L dy s_1(x,y)q(x)\right]^{1-\alpha} + \left[\frac{1}{L}\int_0^L dy c_2 s_2(x,y)q(y)\right]^{1-\alpha}}{1-\alpha}$$

Theorem 5.1 *The long term α-fair policy is given by $s_2(x_2) = 1$ for $x_2 \leq l(\alpha)$ and is otherwise zero, where $l(\alpha)$ is the solution of the fixed point equation*

$$l(\alpha) = c_2^{1-\frac{1}{\alpha}} \left(\frac{q(l(\alpha))}{q(x)}\right)^{-\beta(1-\frac{1}{\alpha})}$$

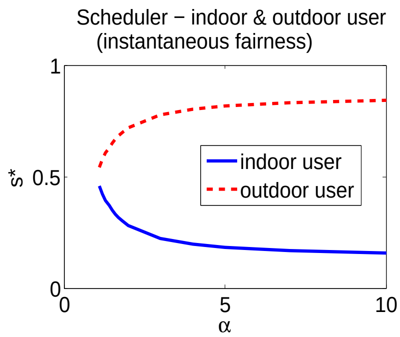

Figure 7: Scheduler s^* for the indoor and outdoor user with instantaneous fairness as a function of α for $\alpha > 1$. Wall attenuation 6 dB, path-loss $\beta = 3$, position of outdoor user $x = -3$.

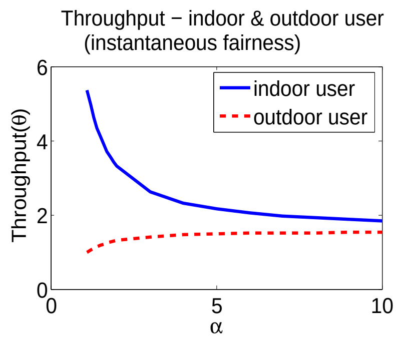

Figure 8: Throughput θ for the indoor and outdoor user with instantaneous fairness as a function of α for $\alpha > 1$. Wall attenuation 6 dB, path-loss $\beta = 3$, position of outdoor user $x = -3$.

where $q(x)$ is a monotone decreasing function of the form $x^{-\beta}$

Proof. It is easy to see that α-fair policy has to have the form mentioned in the theorem statement. If not, for example say there exists an optimal policy which allocates mobile 2 in two disjoint intervals. Then, one can construct a better policy by shifting the right most interval to the end of the left interval and this contradicts the optimality. Thus the optimization simplifies to one-dimensional optimization

$$\max_s Z(s) = \max_{l \in [0,L]} Z(s^l) \text{ where } s_2^l(x) = 1 \text{ for } \{x \leq l\}.$$

It is easy to see that

$$Z(s^l) = \frac{1}{1-\alpha}\left[\left(\frac{L-l}{L}\, q(x)\right)^{1-\alpha} + \left(\frac{1}{L}\int_0^l c_2\, q(y)dy\right)^{1-\alpha}\right].$$

The optimal $l(\alpha)$ is obtained by differentiating the above equation w.r.t l and equating to zero, which results in the fixed point equation

$$\big((L-l(\alpha))q(x)\big)^{-\alpha} q(x) - \left(c_2 \int_0^{l(\alpha)} q(y)dy\right)^{-\alpha} q(l(\alpha)) = 0.$$

Specially when $q(x) = x^{-\beta}$ then the fixed point equation simplifies to

$$l(\alpha) = c_2^{1-\frac{1}{\alpha}} \left(\frac{q(l(\alpha))}{q(x)}\right)^{-\beta(1-\frac{1}{\alpha})}$$

◇

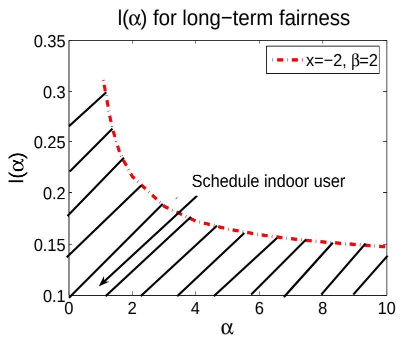

Figure 9: $l(\alpha)$ for long-term fairness as a function of α ($\alpha > 1$) and wall attenuation of 6 dB ,path-loss $\beta = 2$, position of outdoor user $x = -2$.

We plot in figure (9) a numerical example to observe how $l(\alpha)$ varies with α for $\alpha > 1$. In this example, we consider path loss $\beta = 2$, location of outdoor user $x = -2$ and wall attenuation of 6 dB. We observe that as α increases, the value of $l(\alpha)$ monotonically decreases and starts to saturate. It is interesting to note that the indoor user is scheduled when its *mobility* and the *fairness* of resource allocation, $(l(\alpha), \alpha)$, lie within the dashed region below the curve. Also, when the user exhibits higher mobility, the range of fairness applicable reduces.

6 Conclusion and Future Research

We have introduced T-scale fairness and multiscale fairness. The notion of T-scale fairness allows one to address in a flexible manner requirements of emerging applications (like YouTube) which demand quality of service requirement between strict real time traffic and best effort traffic. The notion of multiscale fairness allows one to use a single optimization criterion for resource allocation when different applications are present in the network. We have compared the new fairness notions with previously known criteria of instantaneous and long-term fairness criteria. We have illustrated the new notions by their application in wireless networks. Specifically, we have considered spectrum allocation when users with different dynamics are present in the system. We have demonstrated that the multiscale fairness provides a versatile framework for resource allocation. We have also considered the resource allocation in indoor-outdoor scenario and have observed how the spacial component influences the resource scheduling under different fairness criteria. In the near future we plan to investigate in detail how multiscale fairness criterion allocates resources when a number of applications with different QoS requirements are present in the network. It is also interesting to investigate T-scale fairness in the non-stationary regime.

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