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Adaptive geometry compression based on 4-point interpolatory subdivision schemes with labels

Hui Zhang∗†, Jun-Hai Yong† and Jean-Claude Paul† ‡
†School of Software, Tsinghua University, Beijing 100084, P. R. China
‡CNRS, France

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We propose an adaptive geometry compression method with labels based on 4-point interpolatory subdivision schemes. It can work on digital curves of arbitrary dimensions. With the geometry compression method, a digital curve is adaptively compressed into several segments with different compression levels. Each segment is a 4-point subdivision curve with a subdivision step. Labels are recorded in data compression to facilitate merging those segments in data decompression. In the meantime, we provide high-speed 4-point interpolatory subdivision curve generation methods for efficiently decompressing the compressed data. For an arbitrary positive integer k, formulae of the number of the resultant control points of a 4-point subdivision curve after k subdivision steps are provided. Some formulae for calculating points at the kth subdivision step are presented as well. The time complexity of the new approaches is $O(n)$, where n is the number of the points in the given digital curve. Examples are provided as well to illustrate the efficiency of the proposed approaches.

Keywords: Geometry compression; Subdivision scheme; 4-point subdivision; Interpolatory subdivision; High-speed curve generation

AMS Subject Classification: 68P30; 94B27

1 Introduction

It is a common practice to compress data before they are archived. With ubiquitous applications of computers and network, gigantic amount of data are continuously generated. In the meantime, the increasing demand for communication and data exchange over network beats the limitation of the network band. Data compression becomes more and more important and receives more and more attentions [7, et al]. While data compression has a long history and has achieved a high level of sophistication, some new tools are eager to

∗Corresponding author. Email: huizhang@tsinghua.edu.cn

be discovered to fill the gap between the requirement and the ability of data compression. Geometry compression is relatively new, and becomes a hot topic in a short time after it appeared [6,7,9–12,14, et al]. Wavelet transforms [7, et al], multiresolution [6, et al] and various trees [14, et al] are frequently used in geometry compression.

Almost all geometry compression methods focus on how to compress three-dimensional meshes. In this paper and [13], we propose new geometry compression methods based on 4-point interpolatory subdivision schemes. With our new methods, a digital curve of an arbitrary dimension is compressed into one or several subdivision curve segments. The advantages of our methods are at least as follows.

- It is able to work on a digital curve of arbitrary dimensions. And any sequence of data can be considered as a digital curve of certain dimensions.
- The set of the inner control points of the resultant subdivision curves is exactly a subset of the points of the compressed curve.
- It is possible to simplify the pattern recognition of some digital curves into the pattern recognition of the subdivision curve segments after data compression. The inner control points of the resultant subdivision curves may be considered as the key points of the given digital curves, since they can be used to reproduce the given digital curves after data decompression.

Our work of this paper and [13] gives contributions to the area about subdivision curves and surfaces as well. The first subdivision scheme for generating subdivision curves was proposed by Chaikin [2] in 1974. The 4-point interpolatory subdivision [4] appeared in 1987. Recently, research on subdivision schemes for generating curves and surfaces becomes popular in graphical modeling [3,9, et al], animation [15, et al] and CAD/CAM [8, et al] because of their stability in numerical computation and simplicity in coding. Much work on subdivision surfaces is carried out in several important topics such as Boolean operations [1], mesh editing [15], and adaptive tessellation [9]. And a lot of work [5, et al] has been carried out on the 4-point subdivision schemes as well. In this paper and [13], we provide approaches for the high-speed 4-point interpolatory subdivision curve generation to speed up the data decompression.

[13] is presented at IWICPAS (the International Workshop on Intelligent Computing in Pattern Analysis/Synthesis). Some further work based on [13] is carried out in this paper. For data compression, the geometry compression methods are replaced by the new ones with labels in this paper. In the new geometry compression methods, on the one hand, labels are used to identify the open case and the close case. On the other hand, we use labels to facilitate the merging process in data decompression, i.e., labels are used to identify the segments which should be merged during the data decompression. For data decompression, some more formulae for calculating the control points at the

kth subdivision step with respect to the original control points are provided. With the new formulae, the relationship between the control points at the kth subdivision step and the original control points may become more clear.

The remaining part of the paper is arranged as follows. A brief review of the 4-point interpolatory subdivision curve is given in Section 2. Data compression methods with labels are provided in Section 3. The high-speed generation approaches are provided in Section 4 for the open 4-point interpolatory subdivision curve and the closed 4-point interpolatory subdivision curve, respectively, to speed up the data decompression. Section 5 uses some examples to illustrate the efficiency of the proposed approaches. Some concluding remarks are given in the last section.

2 4-Point Interpolatory Subdivision Schemes

In this section, we briefly go through the 4-point interpolatory subdivision schemes given by [4]. Initially, a set of points $\mathbf{M}_0 = \{\mathbf{P}_{0,0}, \mathbf{P}_{1,0}, \cdots, \mathbf{P}_{n_0-1,0}\}$ is given, where $\mathbf{P}_{i,0}(i = 0, 1, \cdots, n_0 - 1)$ are points, and n_0 is the number of the points. The subdivision is preformed in a recursive procedure. At each subdivision step, some points before the subdivision are inherited, and some new points are inserted into the point set such that the number of the points usually becomes larger and larger. Let $\mathbf{M}_k = \{\mathbf{P}_{0,k}, \mathbf{P}_{1,k}, \cdots, \mathbf{P}_{n_k-1,k}\}$ be the resultant point set after the kth $(k = 0, 1, 2, \cdots)$ subdivision step, where n_k is the number of the points in $\mathbf{M}_k$. All points $\mathbf{P}_{i,k}$ in $\mathbf{M}_k$ are called control points as well.

The 4-point interpolatory subdivision curves can be classified into categories: the open case or the close case. At the kth subdivision step, the points inherited from $\mathbf{M}_{k-1}$ are $\mathbf{P}_{i,k-1}$, where $i = 1, 2, \cdots, (n_{k-1} - 2)$ for the open case, and $i = 0, 1, \cdots, (n_{k-1} - 1)$ for the close case. The point $\mathbf{P}_{j,k}$ to be inserted between $\mathbf{P}_{i,k-1}$ and $\mathbf{P}_{i+1,k-1}$ at the kth subdivision step is

$$\mathbf{P}_{j,k} = (w + 0.5)(\mathbf{P}_{i,k-1} + \mathbf{P}_{i+1,k-1}) - w(\mathbf{P}_{i-1,k-1} + \mathbf{P}_{i+2,k-1}), \tag{1}$$

for each $i = 1, 2, \cdots, (n_{k-1} - 3)$ under the open case, and

$$\begin{aligned} \mathbf{P}_{j,k} = (w + 0.5)(\mathbf{P}_{(i\%n_{k-1}),k-1} + \mathbf{P}_{((i+1)\%n_{k-1}),k-1}) \\ -w(\mathbf{P}_{((i-1)\%n_{k-1}),k-1} + \mathbf{P}_{((i+2)\%n_{k-1}),k-1}), \end{aligned} \tag{2}$$

for each $i = 0, 1, \cdots, (n_{k-1} - 1)$ under the close case, where the weight w is a given real number. Usually, the value of w is suggested to be $\frac{1}{16}$. In Equation (2), the modulus symbol (%) is used such that each subscription is in the set $\{0, 1, \cdots, n_{k-1} - 1\}$. Note that from $\mathbf{M}_{k-1}$ to $\mathbf{M}_k$, the points $\mathbf{P}_{0,k-1}$ and

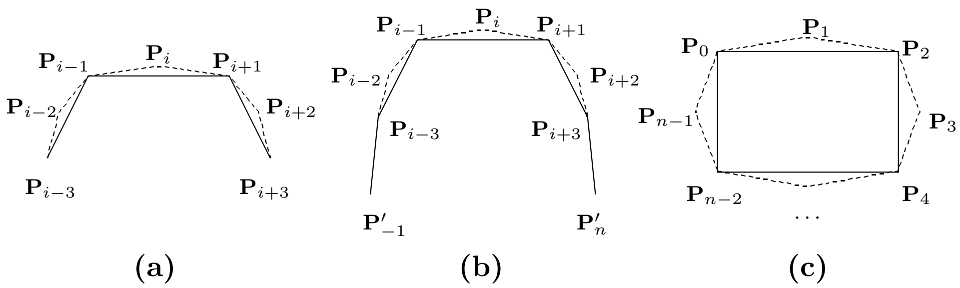


Figure 1. Principle of geometry compression: **(a)** mask of a removable point, **(b)** boundary case, and **(c)** close case.

$\mathbf{P}_{n_{k-1}-1,k-1}$ are discarded for the open case after the subdivision, which is called the shrink property of an open 4-point interpolatory subdivision curve. When $k \to \infty$, the point set $\mathbf{M}_\infty$ becomes a limit subdivision curve. $\mathbf{M}_\infty$ is an open curve under the open case, and a closed curve under the close case.

3 Data Compression

In this section, we propose geometry compression methods with labels for digital curves based on the above schemes. Here, a digital curve is an open polygonal curve or a closed polygonal curve (i.e. a polygon). Thus, we need the label L_{open} here to identify the open polygonal curve, and the label L_{close} to identify the closed polygonal curve. In either case, the digital curve is represented by n vertices $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$ with a label L_{open} or L_{close}. The principle of the geometry compression is, as shown in Figure 1(a),

- that for the open case, we do not need to store $\mathbf{P}_i$ which satisfy

$$\|\mathbf{P}_i - [(w+0.5)(\mathbf{P}_{i-1} + \mathbf{P}_{i+1}) - w(\mathbf{P}_{i-3} + \mathbf{P}_{i+3})]\|_2 \le e, \tag{3}$$

  where $i = 3, 4, \cdots, (n-4)$, and e is the given error tolerance;
- and that for the close case, we do not need to store $\mathbf{P}_i$ which satisfy

$$\left\|\mathbf{P}_i - \left[(w+0.5)(\mathbf{P}_{(i-1)\%n} + \mathbf{P}_{(i+1)\%n}) - w(\mathbf{P}_{(i-3)\%n} + \mathbf{P}_{(i+3)\%n})\right]\right\|_2 \le e, \tag{4}$$

  where $i = 1, 3, 5, \cdots$, $i \le (n-1)$, and e is the given error tolerance.

The points satisfying Equations (3) and (4) are called the removable points, which can be reproduced by the 4-point interpolatory subdivision schemes.

If the given digital curve is a closed curve with an even number of the vertices and each odd vertex $\mathbf{P}_i$, where $i = 1, 3, \cdots, (n-1)$, is a removable point, the digital curve can be compressed into a closed subdivision curve with the control points $\{\mathbf{P}_0, \mathbf{P}_2, \cdots, \mathbf{P}_{n-2}\}$. Thus, the compression ratio is $2:1$.

And the procedure can be recursively carried out, so the compression ratio can be higher than $2:1$. Otherwise, we compress the digital curve in the same way as the open case.

If the given digital curve is an open curve and $\{\mathbf{P}_a, \mathbf{P}_{a+2}, \cdots, \mathbf{P}_b\}$ are removable points, then the points $\{\mathbf{P}_{a-3}, \mathbf{P}_{a-2}, \cdots, \mathbf{P}_{b+2}, \mathbf{P}_{b+3}\}$ can be compressed into $\{\mathbf{P}'_{-1}\mathbf{P}_{a-3}, \mathbf{P}_{a-1}, \cdots, \mathbf{P}_{b+1}, \mathbf{P}_{b+3}, \mathbf{P}'_n\}$, where

$$\mathbf{P}'_{-1} = \frac{(w+0.5)(\mathbf{P}_{a-3}+\mathbf{P}_{a-1}) - \mathbf{P}_{a-2}}{w} - \mathbf{P}_{a+1} \tag{5}$$

and

$$\mathbf{P}'_n = \frac{(w+0.5)(\mathbf{P}_{b+3}+\mathbf{P}_{b+1}) - \mathbf{P}_{b+2}}{w} - \mathbf{P}_{b-1} \tag{6}$$

are two auxiliary points. Because of the shrink property, we need two auxiliary points $\mathbf{P}'_{-1}$ and $\mathbf{P}'_n$ to keep $\mathbf{P}_{a-3}$ and $\mathbf{P}_{b+3}$ after one subdivision step. Under this case, the compression ratio is $[(b-a)+7] : \frac{(b-a)+12}{2}$. For example, as shown in Figure 1(b), when $a = b = i$, the compression ratio is $7:6$.

After some removable points are removed, the digital curve becomes a subdivision curve segment or several subdivision curve segments. Each subdivision curve segment can be recursively compressed. Thus, a subdivision curve segment may be compressed into several subdivision curve segments again. In the inverse process, those subdivision curve segments should be decompressed and then merged together into the segment, on which the decompression process will be carried out again to recover the removable control points. In this paper, we use labels to identify the subdivision curve segments, which are from the same subdivision curve segment. Thus, the labels will facilitate the decompression process in coding. As shown in the following algorithms, the label $L_{merge}(k)$ is used to identify the subdivision curve segments, which should be decompressed and merged together into a segment at the data decompression process. Here, the value of k in the label $L_{merge}(k)$ will be used at the data decompression process. After merging the segments into a whole segment, k subdivision steps will be carried out on the whole segment at the data decompression process. For example, in the following algorithms, one branch will lead to a compressed data set $\{L_{merge}(k), \mathbf{S}_1, \mathbf{S}_2\}$. To decompress the data set, we need to decompress $\mathbf{S}_1$ and $\mathbf{S}_2$ first. And then, merge the two decompressed point sets into a control point set of a subdivision curve segment. k subdivision steps will be carried out on the subdivision curve segment to obtain the resultant decompressed data. And the label $L_{single}(k)$ is used to identify the result, which does not need merging, but needs similarly to be performed k

subdivision steps at the data decompression process.

Algorithm 1. Geometry compression with labels for the open case.
Input: the point set $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$, the error tolerance e, the weight w, and the current subdivision step k (with an initial value $k = 0$).
Output: a set of compressed data set $\mathbf{S}$ (with an empty initial value $\mathbf{S} = \mathbf{\Phi}$).

```
1 if (n ≤ 7) // note: the number of the vertices is too small for the compression.
  begin
        let M be a subdivision curve with {P_0, P_1, ···, P_{n-1}}, the subdivision
        step k and the label L_open;
        let S = {L_single(0), M};
        output S;
        go to Step 7;
  end
2 let a = 0;
  for (i = 3; i ≤ (n − 4); i+ = 2)
  begin
        if (P_i is a removable point according to Equation (3) )
        begin
              let a = i;
              go to Step 3;
        end
  end
3 if (a is zero) // note: no points could be compressed.
  begin
        let M be a subdivision curve with {P_0, P_1, ···, P_{n-1}}, the subdivision
        step k and the label L_open;
        let S = {L_single(0), M};
        output S;
        go to Step 7;
  end
  else if (a > 3) // note: the data will be compressed into several segments.
  begin
        let M_1 be a subdivision curve with {P_0, P_1, ···, P_{a-3}}, the subdivi-
        sion step 0 and the label L_open;
  end

4 let b = a;
  for (i = a + 2; i ≤ (n − 4); i+ = 2)
  begin
```

if ($\mathbf{P}_i$ is not a removable point according to Equation (3))
go to Step 5;
else
let $b = i$;
end
5 if ($b < (n-4)$)
begin
call Algorithm 1 with the input $\{\mathbf{P}'_{-1}\mathbf{P}_{a-3}, \mathbf{P}_{a-1}, \cdots, \mathbf{P}_{b+1}, \mathbf{P}_{b+3}, \mathbf{P}'_n\}$, e, w and 1, where $\mathbf{P}'_{-1}$ and $\mathbf{P}'_n$ are calculated according to Equations (5) and (6), and obtain the set $\mathbf{S}_1$;
call Algorithm 1 with the input $\{\mathbf{P}_{b+3}, \mathbf{P}_{b+4}, \cdots, \mathbf{P}_{n-1}\}$, e, w and 0, and obtain the set $\mathbf{S}_2$;
if ($a > 3$)
let $\mathbf{S} = \{L_{merge}(k), \mathbf{M}_1, \mathbf{S}_1, \mathbf{S}_2\}$;
else
let $\mathbf{S} = \{L_{merge}(k), \mathbf{S}_1, \mathbf{S}_2\}$;
end
else
begin
if ($a > 3$)
begin
call Algorithm 1 with the input $\{\mathbf{P}'_{-1}\mathbf{P}_{a-3}, \mathbf{P}_{a-1}, \cdots, \mathbf{P}_{b+1}, \mathbf{P}_{b+3}, \mathbf{P}'_n\}$, e, w and 1, where $\mathbf{P}'_{-1}$ and $\mathbf{P}'_n$ are calculated according to Equations (5) and (6), and obtain the set $\mathbf{S}_1$;
let $\mathbf{S} = \{L_{merge}(k), \mathbf{M}_1, \mathbf{S}_1\}$;
end
else
begin
call Algorithm 1 with the input $\{\mathbf{P}'_{-1}\mathbf{P}_{a-3}, \mathbf{P}_{a-1}, \cdots, \mathbf{P}_{b+1}, \mathbf{P}_{b+3}, \mathbf{P}'_n\}$, e, w and $(k+1)$, where $\mathbf{P}'_{-1}$ and $\mathbf{P}'_n$ are calculated according to Equations (5) and (6), and obtain the set $\mathbf{S}$;
end
end
6 **output** $\mathbf{S}$;
7 **End** of Algorithm 1.

Algorithm 2. Geometry compression with labels for the close case.
Input: the point set $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$, the error tolerance e, the weight w, and the current subdivision step k (with an initial value $k = 0$).
Output: a set of compressed data set $\mathbf{S}$ (with an empty initial value $\mathbf{S} = \mathbf{\Phi}$).

1 if ($n < 6$) // note: the number of the vertices is too small for the compression.
begin
  let $\mathbf{M}$ be a subdivision curve with $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$, the subdivision step k and the label L_{close};
  Let $\mathbf{S} = \{L_{single}(0), \mathbf{M}\}$;
  go to Step 4;
end
2 if (n is odd)
begin
  call Algorithm 1 with the input $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$, e, w and k, and obtain the set $\mathbf{S}$;
  let $\mathbf{S}$ have the label L_{close};
  go to Step 4;
end
3 if (all points $\mathbf{P}_i$ (where $i = 1, 3, \cdots, (n-1)$) are removable points)
begin
  call Algorithm 2 with the input $\{\mathbf{P}_0, \mathbf{P}_2, \cdots, \mathbf{P}_{n-2}\}$, e, w and $(k+1)$, and obtain the set $\mathbf{S}$;
end
else
begin
  call Algorithm 1 with the input $\{\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_{n-1}\}$, e, w and k, and obtain the set $\mathbf{S}$;
  let $\mathbf{S}$ have the label L_{close};
end
4 **output** $\mathbf{S}$;
5 **End** of Algorithm 2.

In Algorithms 1 and 2, we only check whether the points in the given point set are removable points at most twice, and we do not check whether any auxiliary point produced by Equation (5) or (6) is a removable point. Therefore, although Algorithms 1 and 2 contain loops and recursive procedures, the time complexity of both Algorithms 1 and 2 is $\mathrm{O}(n)$.

4 Data Decompression

With the method introduced in Section 3, a digital curve is compressed into one or some subdivision curve segments. Hence, the problem here is how to obtain the points in $\mathbf{M}_k$, which is the point set after k subdivision steps are carried out from the initial control point set $\mathbf{M}_0$. According to the method in Section 2, in order to obtain $\mathbf{M}_k$, we need to calculate all the control points in $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_{k-1}$. Unfortunately, we do not need $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_{k-1}$ at all, but only $\mathbf{M}_k$. Thus, we need extra memory to store those unnecessary points, and experience shows that the time cost in this way increases sharply with respect to the subdivision step k. In this section, we will provide the high-speed generation approaches. One is for the open subdivision curve, and the other one is for the closed curve.

4.1 *Open curve generation*

In this subsection, we only consider the open 4-point subdivision curve. First, we need to obtain the value of n_k, which is the number of points in $\mathbf{M}_k$, such that we could allocate memory to store the coordinates of the points in $\mathbf{M}_k$ before computing the coordinates. According to Section 2, from $\mathbf{M}_{k-1}$ to $\mathbf{M}_k$, all the points except for the first and the last points in $\mathbf{M}_{k-1}$ are inherited, and the number of new points inserted into $\mathbf{M}_k$ is 3 less than the number of the points in $\mathbf{M}_{k-1}$. Thus, we obtain n_k with respect to n_{k-1} in Lemma 4.1.

LEMMA 4.1 *If* $n_0 \geq 5$*, the number of the points in* $\mathbf{M}_k$ *is* $n_k = (n_{k-1} - 2) + (n_{k-1} - 3) = 2n_{k-1} - 5$*, for* $k = 1, 2, \cdots$.

According to Algorithms 1 and 2, no subdivision is necessary to be performed on a set of points which number is less than 5. Therefore, we do not consider the case when $n_0 < 5$. Recursively apply the above lemma, and we obtain n_k with respect to n_0 in Theorem 4.2.

THEOREM 4.2 *If* $n_0 \geq 5$*, the number of the points in* $\mathbf{M}_k$ *is* $n_k = 2k(n_0-5)+5$*, for* $k = 1, 2, \cdots$.

The remaining part of the subsection will provide the method for calculating the coordinates of the points in $\mathbf{M}_k$. It is based on the following important theorem. The theorem can be proved by the mathematical induction method according to Section 2.

THEOREM 4.3 *For* $k = 0, 1, 2, \cdots$*, we have*

1. $\mathbf{P}_{i\times 2^k+2,k} = \mathbf{P}_{i+2,0}$*, where* $i = 0, 1, 2, \cdots$*, and* $i \times 2^k + 2 < n_k$.
2. $\mathbf{P}_{(2^k)\times i+3,k}$

$$= \frac{w(-2^{(-k)}h+2^k w^k h+d^k(1-4w)-e^k(1-4w))}{(4w-1)h}\mathbf{P}_{i,0}$$
$$+ \frac{-(d^k(1+h-4w-4hw)+e^k(h-1+4w-4hw)+8\times 2^k w^{(k+1)}h-2^{(1-k)}h)}{4(4w-1)h}\mathbf{P}_{i+1,0}$$
$$+ \frac{-(-2h+d^k h+d^k-4d^k w+e^k h-e^k+4e^k w)}{2h}\mathbf{P}_{i+2,0}$$
$$+ \frac{8\times 2^k w^{(k+1)}h+d^k(4hw-1-h+4w)+e^k(1-h-4w+4hw)-2^{(1-k)}h}{4(4w-1)h}\mathbf{P}_{i+3,0}$$
$$+ \frac{-w(2^k w^k h-2^{(-k)}h-4e^k w+e^k+4d^k w-d^k)}{(4w-1)h}\mathbf{P}_{i+4,0}\ ,$$

where $w \in \left(0, \frac{1}{16}\right)$, $d = \frac{1+\sqrt{1-16w}}{4}$, $e = \frac{1-\sqrt{1-16w}}{4}$, $h = \sqrt{1-16w}$, $i = 0, 1, 2, \cdots,$ *and* $i + 4 < n_0$.

3. $\mathbf{P}_{(2^k)\times i+3,k}$

$$= \left(\frac{1}{12\times 2^k} - \frac{1}{12\times 8^k} - \frac{k}{8\times 4^k}\right)\mathbf{P}_{i,0}$$
$$+ \left(-\frac{2}{3\times 2^k} + \frac{1}{6\times 8^k} + \frac{1}{2\times 4^k} + \frac{k}{2\times 4^k}\right)\mathbf{P}_{i+1,0}$$
$$+ \left(1 - \frac{1}{4^k} - \frac{3k}{4^{(k+1)}}\right)\mathbf{P}_{i+2,0}$$
$$+ \left(\frac{2}{3\times 2^k} - \frac{1}{6\times 8^k} + \frac{k}{2\times 4^k} + \frac{1}{2\times 4^k}\right)\mathbf{P}_{i+3,0}$$
$$+ \left(-\frac{1}{12\times 2^k} + \frac{1}{12\times 8^k} - \frac{k}{8\times 4^k}\right)\mathbf{P}_{i+4,0}\ ,$$

where $w = \frac{1}{16}$, $i = 0, 1, 2, \cdots,$ *and* $i + 4 < n_0$.

4. $\mathbf{P}_{(2^k)\times i+1,k}$

$$= \frac{-w(-2^{(-k)}h+2^k w^k h+d^k(4w-1)+e^k(1-4w))}{(4w-1)h}\mathbf{P}_{i,0}$$
$$+ \frac{-(e^k(h-1+4w-4wh)+d^k(1+h-4w-4wh)-8\times 2^k w^{(k+1)}h+2^{(1-k)}h)}{4(4w-1)h}\mathbf{P}_{i+1,0}$$
$$+ \frac{-(d^k(h+1-4w)+e^k(h-1+4w)-2h)}{2h}\mathbf{P}_{i+2,0}$$
$$+ \frac{-(e^k(h-1+4w-4wh)+d^k(1+h-4w-4wh)+8\times 2^k w^{(k+1)}h-2^{(1-k)}h)}{4(4w-1)h}\mathbf{P}_{i+3,0}$$
$$+ \frac{w(-2^{(-k)}h+2^k w^k h+d^k(1-4w)+e^k(4w-1))}{(4w-1)h}\mathbf{P}_{i+4,0}\ ,$$

where $w \in \left(0, \frac{1}{16}\right)$, $d = \frac{1+\sqrt{1-16w}}{4}$, $e = \frac{1-\sqrt{1-16w}}{4}$, $h = \sqrt{1-16w}$, $i = 0, 1, 2, \cdots,$ *and* $i + 4 < n_0$.

5. $\mathbf{P}_{(2^k)\times i+1,k}$

$$= \left(\frac{-1}{12\times 2^k} + \frac{1}{12\times 8^k} - \frac{k}{8\times 4^k}\right)\mathbf{P}_{i,0}$$
$$+ \left(\frac{2}{3\times 2^k} - \frac{1}{6\times 8^k} + \frac{k}{2\times 4^k} + \frac{1}{2\times 4^k}\right)\mathbf{P}_{i+1,0}$$
$$+ \left(1 - \frac{3k}{4^{(k+1)}} - \frac{1}{4^k}\right)\mathbf{P}_{i+2,0}$$
$$+ \left(-\frac{2}{3\times 2^k} + \frac{1}{6\times 8^k} + \frac{k}{2\times 4^k} + \frac{1}{2\times 4^k}\right)\mathbf{P}_{i+3,0}$$
$$+ \left(\frac{1}{12\times 2^k} - \frac{1}{12\times 8^k} - \frac{k}{8\times 4^k}\right)\mathbf{P}_{i+4,0}\ ,$$

where $w = \frac{1}{16}$, $i = 0, 1, 2, \cdots,$ *and* $i + 4 < n_0$.

Thus, according to Theorem 4.3 and Equation (1), we have the following algorithm for calculating the coordinates of the points in $\mathbf{M}_k$.

Algorithm 3. Calculating coordinates of points in $\mathbf{M}_k$ for the open case.
Input: $\mathbf{M}_0$ and the weight w with the assumption that $n_0 \geq 5$.
Output: $\mathbf{M}_k$.

1 calculate n_k according to Theorem 4.2;
2 allocate memory for $\mathbf{M}_k$ to store the coordinates of n_k points in $\mathbf{M}_k$;
3 for $(i = 0, i_0 = 2, i_k = 2; i_k < n_k; i++, i_0++, i_k+ = 2^k)$
  $\mathbf{P}_{i_k,k} = \mathbf{P}_{i_0,0}$ according to Theorem 4.3;
4 $\mathbf{P}_{0,k} = \mathbf{P}_{0,0}$; $\mathbf{P}_{1,k} = \mathbf{P}_{1,0}$; $\mathbf{P}_{n_k-1,k} = \mathbf{P}_{n_0-1,0}$; $\mathbf{P}_{n_k-2,k} = \mathbf{P}_{n_0-2,0}$;
5 for $(i = k; i >= 1; i--)$
begin
  $\mathbf{P} = (w + 0.5)(\mathbf{P}_{1,k} + \mathbf{P}_{2,k}) - w(\mathbf{P}_{0,k} + \mathbf{P}_{2^i+2,k})$;
  $\mathbf{P}_{2^{i-1}+2,k} = (w + 0.5)(\mathbf{P}_{2,k} + \mathbf{P}_{2^i+2,k}) - w(\mathbf{P}_{1,k} + \mathbf{P}_{2^{i+1}+2,k})$;
  $\mathbf{P}_{0,k} = \mathbf{P}_{1,k}$; $\mathbf{P}_{1,k} = \mathbf{P}$;
  for $(j = 2^i + 2^{i-1} + 2, m = 2; j < n_k - 3 - 2^{i-1}; j+ = 2^i, m+ = 2^i)$
  begin
    $\mathbf{P}_{j,k} = (w + 0.5)(\mathbf{P}_{2^i+m,k} + \mathbf{P}_{2^{i+1}+m,k}) - w(\mathbf{P}_{m,k} + \mathbf{P}_{3\times 2^i+m,k})$;
  end
  $\mathbf{P} = (w + 0.5)(\mathbf{P}_{n_k-2,k} + \mathbf{P}_{n_k-3,k}) - w(\mathbf{P}_{n_k-1,k} + \mathbf{P}_{n_k-3-2^i,k})$;
  $\mathbf{P}_{n_k-3-2^{i-1},k} = (w + 0.5)(\mathbf{P}_{n_k-3,k} + \mathbf{P}_{n_k-3-2^i,k}) - w(\mathbf{P}_{n_k-2,k} + \mathbf{P}_{n_k-3-2^{i+1},k})$;
  $\mathbf{P}_{n_k-1,k} = \mathbf{P}_{n_k-2,k}$; $\mathbf{P}_{n_k-2,k} = \mathbf{P}$;
end
6 **End** of Algorithm 3.

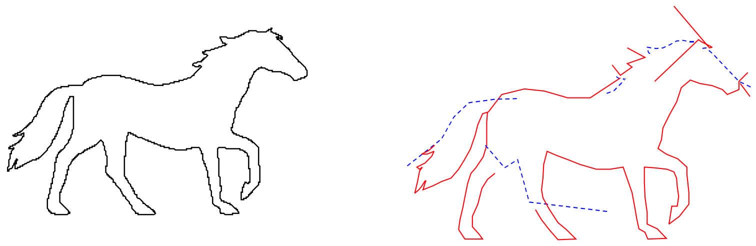
In Algorithm 3, we assume that $n_0 \geq 5$, so we have $n_k \geq 5$. In the algorithm, any point in $\mathbf{M}_k$, except for $\mathbf{P}_{0,k}$, $\mathbf{P}_{1,k}$, $\mathbf{P}_{n_k-1,k}$ and $\mathbf{P}_{n_k-2,k}$, are calculated only once. Therefore, the time complexity of Algorithm 3 is $\mathrm{O}(n_k)$, which is the lowest bound of calculating all points in $\mathbf{M}_k$.

4.2 *Closed curve generation*

This subsection focuses on the approach for the high-speed generation of the closed subdivision curve. According to Section 2, from $\mathbf{M}_{k-1}$ to $\mathbf{M}_k$, all the points in $\mathbf{M}_{k-1}$ are inherited, and the number of new points inserted into $\mathbf{M}_k$ is equal to n_{k-1}. Thus, we obtain the conclusions in Lemma 4.4 and Theorem 4.5.

LEMMA 4.4 *The number of the points in* $\mathbf{M}_k$ *is* $n_k = 2n_{k-1}$, *for* $k = 1, 2, \cdots$.

THEOREM 4.5 *The number of the points in* $\mathbf{M}_k$ *is* $n_k = n_0 \times 2^k$, *for* $k = 0, 1, \cdots$.



(a) **(b)**

Figure 2. Example 1: **(a)** before data compression, and **(b)** after data compression.

To calculate the coordinates of the points in $\mathbf{M}_k$, one important conclusion is drawn in Theorem 4.6.

THEOREM 4.6 *For* $k = 0, 1, 2, \cdots$, *we have* $\mathbf{P}_{i\times 2^k,k} = \mathbf{P}_{i,0}$, *where* $i = 0, 1, 2, \cdots$, *and* $i \times 2^k < n_k$.

Thus, according to Theorem 4.6 and Equation (2) in Section 2 for calculating new points, we obtain the following algorithm. Similar to Algorithm 3, the time complexity of the following algorithm is $\mathrm{O}(n_k)$.

Algorithm 4. Calculating coordinates of points in $\mathbf{M}_k$ for the close case.
Input: $\mathbf{M}_0$ and the weight w.
Output: $\mathbf{M}_k$.

1 calculate n_k according to Theorem 4.5;
2 allocate memory for $\mathbf{M}_k$ to store the coordinates of n_k points in $\mathbf{M}_k$;
3 for $(i = 0, i_k = 0; i_k < n_k; i++, i_k+= 2^k)$
    $\mathbf{P}_{i_k,k} = \mathbf{P}_{i,0}$ according to Theorem 4.6;
4 for $(i = k; i >= 1; i--)$
  begin
    for $(j = 2^{i-1}, m = -2^i; j \leq n_k - 2^{i-1}; j+= 2^i, m+= 2^i)$
      $\mathbf{P}_{j,k} = (w + 0.5)(\mathbf{P}_{(2^i+m)\%n_k,k} + \mathbf{P}_{(2^{i+1}+m)\%n_k,k}) - w(\mathbf{P}_{m\%n_k,k} + \mathbf{P}_{(3\times 2^i+m)\%n_k,k})$;
  end
5 **End** of Algorithm 4.

5 Examples

Experiment has been carried out on a lot of examples. Three examples are shown in Figures 2 and 3. The first example is a digital curve, which is the contour curve of a horse as shown in Figure 2. The original curve as shown in

Table 1. Performance of approaches on Example 2.

k	T_{no}(s)	T_{fo}(s)	T_{nc}(s)	T_{fc}(s)
3	0.00021	0.00018	0.00025	0.00021
4	0.00040	0.00033	0.00053	0.00043
5	0.00082	0.00064	0.0012	0.00088
6	0.0017	0.0011	0.0029	0.0017
7	0.0042	0.0023	0.0076	0.0034
8	0.011	0.0046	0.022	0.0070
9	0.035	0.0090	0.076	0.014
10	0.12	0.018	0.27	0.028
11	0.44	0.036	1.0	0.058
12	1.7	0.073	4.0	0.11

Figure 2(a) contains 5413 points. It is compressed into 11 subdivision curve segments, which total number of the control points is 167. The ratio of the numbers of points is $5413 : 167 \approx 32.4 : 1$. We alternate the red solid lines and blue dashed lines to identify different subdivision curve segments. Due to the shrink property of the 4-point interpolatory subdivision curves, some auxiliary points, which are out of the contour curve of the horse, are necessary as shown in Figure 2(b). In these examples in this section, the error tolerance is 0.007, and $w = \frac{1}{16}$. In Example 1, we consider the original digital curve as an open curve. The closed curve case is shown in Figure 3(d). The blue solid curve is the original digital curve, which contains 672 points. After the data compression, it becomes a closed subdivision curve with 21 control points. The ratio of the numbers of points is $512 : 16 = 32 : 1$.

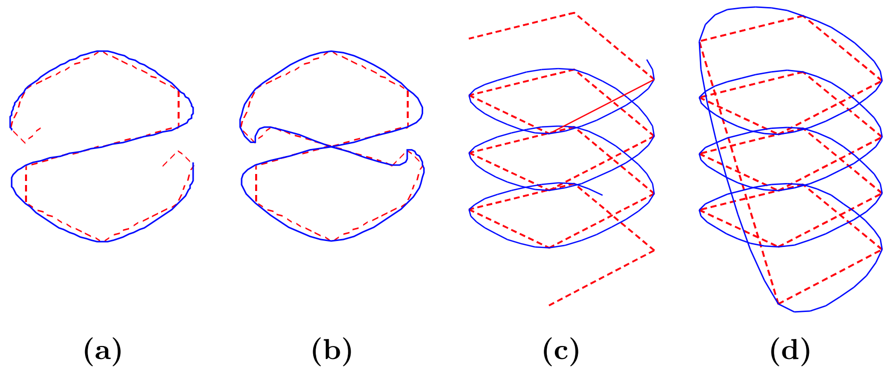


Figure 3. Open case **(a)** and close case **(b)** of Example 2; open case **(c)** and close case **(d)** of Example 3.

Two examples as shown in Figures 3 are used to illustrate the efficiency of the data decompression algorithms. The digital curves in Examples 2 and 3 are of two dimensions and three dimensions, respectively. In the open case of Example 3, the digital curve as shown in Figure 3 (c) is a three-dimensional

Table 2. Performance of approaches on Example 3.

k	T_{no}(s)	T_{fo}(s)	T_{nc}(s)	T_{fc}(s)
3	0.00038	0.00031	0.00042	0.00037
4	0.00086	0.00058	0.0098	0.00073
5	0.0017	0.0011	0.0021	0.0014
6	0.0036	0.0021	0.0054	0.0028
7	0.0084	0.0042	0.014	0.0058
8	0.026	0.0082	0.044	0.012
9	0.078	0.017	0.15	0.024
10	0.27	0.033	0.54	0.048
11	0.98	0.066	2.0	0.099
12	3.8	0.13	7.8	0.19

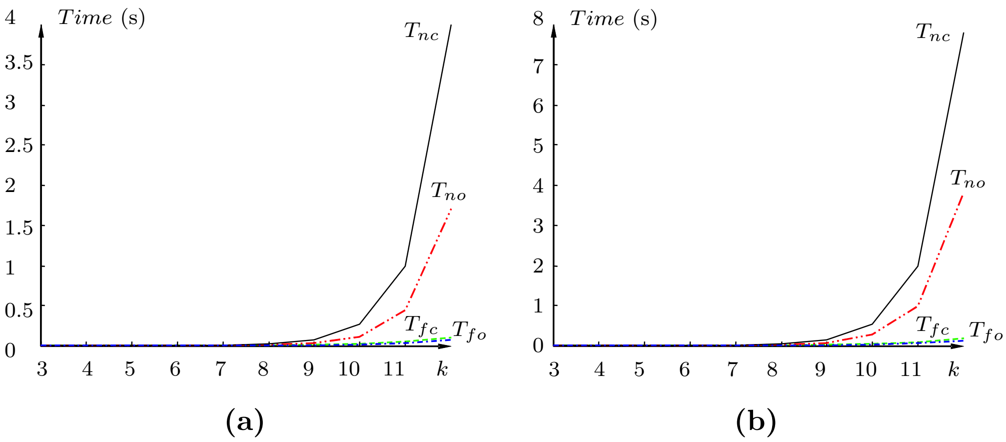


Figure 4. Performance of approaches on: **(a)** Example 2, and **(b)** Example 3.

spiral. The polygonal curves or the polygons formed by $\mathbf{M}_0$ are dashed in those two figures, and the solid curves are the results after several iterate subdivision steps. The numbers of the control points in $\mathbf{M}_0$ of Examples 2 and 3 are 14 and 21, respectively. Tables 1 and 2 give the time cost of the approaches on Examples 2 and 3, which are illustrated in Figure 4 as well. In the tables and the figure, k represents for the iterate subdivision steps. T_{no} and T_{nc} represent for the time cost with the method given by [4] on the open curves and the closed curves, respectively. T_{fo} and T_{fc} represent for the time cost by Algorithm 3 on the open curves and Algorithm 4 on the closed curves, respectively. All the data are calculated on a personal computer with 2.8 GHz CPU and 1G memory. The programming language is C++. As shown in Tables 1 and 2, the new approaches are much faster than the traditional method in [4].

6 Conclusions

This paper provides adaptive geometry compression methods with labels based on 4-point interpolatory subdivision schemes. It can work on digital curves of arbitrary dimensions, for example, d dimensions if the points are all of $d-$dimensions. The examples shown in Figures 2 and 3(d) show that the data compression ratios could be about 32 : 1. For decompressing the compressed data, this paper as well provides high-speed 4-point interpolatory subdivision curve generation methods such that decompression could be performed efficiently. As shown in the examples, the new approaches are able to reduce the time cost sharply. The high-speed 4-point interpolatory subdivision curve generation methods not only take advantages to data decompression, but also give great benefit to the real-time display and interaction of 4-point subdivision curves.

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