Hair Interactions

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Abstract

Processing interactions is one of the main challenges in hair animation. Indeed, in addition to the collisions with the body, an extremely large number of contacts with high friction rates are permanently taking place between individual hair strands. Simulating the latter is essential: without hair self-interactions, strands would cross each other during motion or come to rest at the same location, yielding unrealistic behavior and a visible lack of hair volume.

This chapter reviews the most recent advances to tackle the specific problems of hair collision detection and response. The solutions presented here range from simple approximations that provide hair with a volumetric behavior in real-time to dedicated algorithms for efficiently yet robustly detecting collisions between hair guides and for generating a realistic response to hair interactions.

1. Introduction

Human hair is a composite, deformable material made of more than 100 000 individual fibers called hair strands. These thin tubular structures are elastic: after motion, they tend to come back to a rest shape, which is related to their individual natural curliness and to the set of external forces applied to them. Global hair motion and even the shape hair takes at rest highly depend on the nature of the multiple interactions taking place between hair strands: collisions and contacts between hair strands of different orientations cause hair to occupy a pretty high volume, especially in the case of irregular, curly or fuzzy hair. Due to this larger volume, tangled or fuzzy hair in motion is much more subject to air damping than smooth and disciplined hair.

The nature of interactions between hair strands is very complex. This is largely due to the surface of individual hair strands, which is not smooth but composed of tilted scales (see Figure 1). This irregular surface causes anisotropic friction inside hair, with an amplitude that strongly depends on the orientation of the scales and of the direction of motion [Zvi86]. Moreover, hair is very triboelectric, meaning it can easily release static charges by mere friction. This phenomenon, which has been measured in the case of combed hair, most probably impacts the hair-hair friction rates.

Because of the extremely large number of strands that compose a full head of hair, processing hair interactions is known as one of the main challenges in hair animation. Until the late nineties, most hair animation methods tackled hair collisions with the body, but were not processing self-interactions at all. This often resulted into an obvious lack of hair volume. The first methods that detected interactions between hair wisps spent more than 80% of the simulation time in this process. More recently, several interesting solutions that make hair interactions much more practical were developed: some of them mimic the effect of hair interactions globally, using a structure that stores the volumetric density of hair. Others achieve more accurate results by developing efficient algorithms for detecting collisions between hair-wisps and by setting up realistic models for response and friction forces.

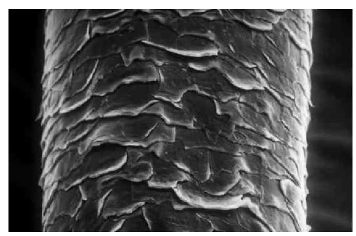

Figure 1: *An electron micrograph of a hair fiber that shows the structure of the outer cuticle surface, which is composed of thin overlapping scales [Rob94].*

This chapter presents those of these recent advances in which the authors participated: Section 2 briefly reviews the two main approaches for animating hair, namely modeling hair as a continuum or as a set of individual hair wisps. The associated methods for processing hair interactions with the body are presented and the issues raised by hair self-interactions are introduced. Section 3 presents a practical real-time solution, applicable in any hair animation system, which gives hair a volumetric behavior without requiring to detect individual interactions between the animated guide-strands. We then focus on more accurate methods, applicable for generating high quality animation of long hair: Section 4 reviews some recent methods for efficiently, yet robustly detecting the interactions between guide-strands. Section 5 discusses the anisotropic models that were set up to model response to these interactions. In particular, we describe a validated model for friction forces. In conclusion, we emphasize the steps forwards made in the last few years, but also the issues that were not tackled yet, showing that improving the efficiency and visual realism of hair animation is going to remain a hot research topic for a while.

2. Hair animation and interaction processing

2.1. Continuous versus wisp-based hair models

Hair animation was made practical in the early nineties [RCT91] by the idea of animating only a subset of the hair strands (typically one or two hundreds), which we will call here the *guide-strands*. This is made possible by the local spatial coherence of hair motion. Once the guide-strands have been animated (using for instance spring and masses, projective dynamics or chains of articulated rigid bodies), their position is used to generate the remaining hair strands at the rendering stage.

More precisely, two main families of approaches were developed for modeling hair: The first ones, more appropriate for smooth, fluid hair, consider hair as a continuum [AUK92, DTKT93, HMT01, CJY02, BCN03] and thus use interpolation between the animated guide-strands for generating a full head of hair. The second ones, which achieve their best results for wavy of curly hair, model hair as a set of disjoint wisps [CSDI99, KN00, PCP01, KH01, BKCN03, WL03, CCK05]. The animated guide-strands are assimilated to wisp skeletons and extrapolation is used for generating extra hair-strands within each wisp. Recently, Bertails [BAC*06] bridged the gap between the two kinds of approaches by allowing the guide-strands to be used both for interpolation or approximation depending on the type of hair and on the current distance between neighboring guide-strands. This model captures hair that looks like a continuum near the head while well identified wisps can be observed at the tip.

In the remainder of this chapter, we will discuss hair interactions independently of the hair model used among the approaches above: hair will be considered as a set of individual hair guides, each of them more or less explicitly modeling a volume of hair around it. Interactions will be detected and treated based on the position and motion of these guide-strands.

2.2. Processing hair interactions with the body

The first step towards processing hair interactions is to adequately model hair collisions and contacts with obstacles, starting with the body of the animated character. Since hair is animated using guide-strands, the latter and the wisp volumes around them (if any) should be prevented from penetrating inside the body. The latter is often approximated using sets of ellipsoids or stored in a spatial partitioning grid to accelerate this detection. Since hair is a very soft material, modeling a one way response is sufficient: the body can be considered as infinitely rigid and heavy compared with hair, so the collision has no effect on the subsequent body shape and motion. Moreover, hair is a very soft and light material: it does not bounce after collision, but rather experiment a strong static friction with the parts of the body it is in contact with. Collision response can thus be treated using methods set up for other very light material, such as clothing: when a penetration is detected, the guide-strand or the associated wisp volume is re-positioned as to be in resting contact with the body. The guide-strand is either given the velocity of this body part, or a static friction force is set up between them.

The remainder of the chapter focuses on the part of interaction processing most specific to hair and much more difficult to handle than collisions with obstacles: we are now addressing the challenging problem of self-interactions.

2.3. The issues raised by hair self-interactions

The interactions that occur between hair-strands are very difficult to simulate, For the following reasons:

Firstly, in real hair, the friction between neighboring strands of similar orientation plays an important part: it dissipates some kinetic energy and damps the overall motion. This phenomenon cannot be simulated properly in virtual hair, where only a few guide-hair distributed on the scalp are animated. The only way to capture this part of self-interaction is to add some internal damping - which should depend on the type of hair and is quite difficult to tune - on the individual motion of a guide strand.

Secondly, strands are very thin, so standard collision detection methods based on penetration cannot be used: strands or even small wisps of hair of different orientations might cross each other between two simulations steps and go to rest in the wrong positions, this interaction remaining unnoticed.

Lastly, once a collision between hair guides or hair wisps of different orientation have been detected, the response model should account for the complex state of surface of a hair strand: the tilted scales that cover a strand result in strongly anisotropic static friction. Moreover, these friction forces are dominant: due to the lightness on a hair strand, the colliding strands will most probably remain in contact. One of the challenges of hair self-interactions it thus to define a response model that prevents strands from crossing each other while avoiding to generate any bouncing. The latter, often noticeable in hair animation systems, gives an overall unstable behavior to the full hair, due to the extremely large number of local collisions that occur at each time step, even when hair is at rest.

Historically, the continuous and wisp-based approaches have tackled hair self-interactions in dramatically different ways:

- **Volumetric interactions:** Continuum approaches such as Hadap's and Bando's methods relied on fluid-like internal viscosity to model hair friction and to prevent self-intersections is a rather global way [HMT01, BCN03]: no collision is detected between individual hair strands, but the latter interact (as fluid particles would do), depending on the local hair density and on the relative hair motion around them.
- **Guide-strands interactions:** In contrast, processing hair self-collision in discontinuous, wisp-based approaches has been done through the actual detection of penetration between moving hair wisps [PCP01]. This allows a more accurate modeling of the discontinuities that can be observed during fast motion of long, human hair: in these approaches, wisps of hair defined around a guide-strand are prevented from crossing each other and two wisps of different orientations can be in resting contact.

We believe that the general approach chosen for handling hair interactions can be chosen quite independently from the hair model, would it be a continuum model, an disjoint set of hair wisps, or something inbetween.

The remainder of this chapter presents the specific solution the authors have developed for tackling the problem of hair interactions. This chapter is not aimed as providing a state of the art in the area: the interested reader can find a recent survey on hair animation and rendering techniques in [WFK*06]. The volumetric method for hair interactions presented in Section 3 belongs to the volumetric interactions approach: it provides a real-time alternative to fluid-like interactions when a coarser approximation is sufficient. Methods for improving the efficiency of collision detection and the realism of collision response in the interacting guide-strands approach are detailed in Sections 4 and 5.

3. A volumetric approach for real-time self-interactions

The work presented in this section was first introduced in [BMC05], as a side application of a method for handling hair self-shadowing in real-time. We detail here the application of this approach to hair self-interactions.

3.1. A volumetric structure for hair

An acceptable approximation of hair self-interaction consists of considering that internal collisions mainly result into the preservation of hair volume [LK01]. Starting from this assumption, hair density information is very useful: If the local density of hair is over a fixed threshold (corresponding to the maximum quantity of hair that can be contained within a cell), the hair strands should undergo external forces that spread them out.

Bertails *et al.* [BMC05] use a light-oriented voxel grid to store hair density values. This enables them to efficiently compute *both lighting and mechanical interactions* inside the hair volume in real-time. Though very simple, this method yields convincing interactive results for animated hair, is very simple to implement, efficient and can easily be parallelized to increase performance.

More precisely, the volumetric structure used is based on a 3D light-oriented density map, which combines an optimized volumetric representation of hair with a light-oriented partition of space. This voxel structure stores the local hair density in space, computed from the number of guide-strand segments within a given cell. It is used to approximate the light attenuation through each cell of the grid: since the cells are sorted along the light direction, computing the accumulated translucency for each cell through the hair volume becomes straightforward.

3.2. Application to hair interaction

At each animation step, all guide-strand are moved to their new position and the density map is updated. Then, hair self-collisions are taken into account for the next simulation step by adding density-based interaction forces where needed: repulsive forces directed from the center to the border of a grid cell are generated. They are applied to each hair-guide element located in a cell whose density if over a threshold. This threshold value depends on the desired level of hair fuzziness.

Although this interaction method is extremely simple, it yields convincing results. In practice, it was tested with an accordingly simple, yet robust algorithm for animating the guide-strands: hair is composed of approximately a hundred wisps, each of which being simulated using three guide-strands modeled as chains of rigid links. The latter are animated using a fast and robust but non-accurate method [vO91]. The rendering technique is a hybrid between continuum and wisp-based methods: interpolation between the three guide-strands is used to generate a continuum of hair inside each deformable wisps. The overall method results into interactive hair animations that include self-interactions as well as self-shadowing, and generate visually convincing hair volume (see Figure 2). Furthermore, with this technique, handling hair self-collisions only requires 2.5% of the whole processing time.

Figure 2: *Interactive hair self-shadowing processed by accumulating transmittance values through a light-oriented voxel grid [BMC05]. (left) Animated smooth hair; (right) Animated curly hair.*

4. Detecting guide-strand interactions

Volumetric methods as the simple solution presented above are not sufficient for generating high quality animation of non-smooth hair: two hair wisps of different orientations may cross each other during motion despite of the volumetric forces they undergo. Most hair animation methods have thus relied on the distance between pairs of guide-strands or on the penetration between wisps of hair defined around them for accurately detecting hair self-interactions. In this chapter, we call these more accurate approaches *guide-strand interactions*.

A naive implementation of guide-strand interactions would lead to $O(n^2)$ tests, where n is the total number of guide-strand segments (or wisp segments) in the hair model. Following Plante [PCP01], most methods use a pre-detection based on a regular 3D grid data structure, built around the character and its hair, to quickly get rid of most non-intersecting cases. Each grid cell contains a list of hair-guide elements (or wisp segments) whose bounding box intersects the cell. At each animation step, the grid is used for quickly determining a shorter list of segments susceptible to intersect. A mailbox parameter indicates the last time step when a given pair of such segments has been tested, ensuring that each pair is tested only once. The 3D grid data structure can also be used for optimizing collision detection between hair and the character model: to achieve this, each cell also references the polygons of the character model that intersect it.

4.1. Deformable versus cylindrical hair wisps

To account for the complex interactions observed in real hair during fast motion, Plante *et al.* represented hair using a fixed set of deformable, volumetric wisps [PCP01, PCP02].

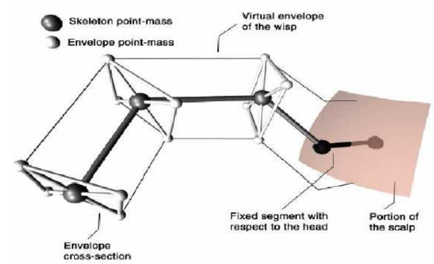

Figure 3: *Elements defining a deformable volumetric wisp [PCP01].*

Each wisp is structured into three hierarchical layers: a skeleton curve (called here guide-strand) that defines its large-scale motion and deformation, a deformable volumetric envelope that coats the skeleton and accounts for the deformation due to hair interaction within a wisp, and a given number of hair strands distributed inside the wisp envelope and which are generated only at the rendering stage (see Figure 3). More precisely, the deformable sections that shape a wisp of hair around its guide-strand are animated using 4 1D damped springs, attempting to capture the way a wisp of hair deforms when its moves and most often comes back to its initial size at rest. The wisp volume was defined as a quadratic surface envelop controlled by these cross-sections.

Using such a complex deformable wisp model for the detection of guide-strand interactions proved very time consuming: more than 70% of the simulation time was used in collision detection between hair wisps, despite of the space grid used to accelerate the process. In total, without taking hair rendering into account, about 3 hours of computations were required, in 2001, to compute 3 seconds of animation.

Bertails *et al.* [BKCN03] introduced an adaptive animation control structure, called the *Adaptive Wisp Tree* (AWT), which enables the dynamic splitting and merging of hair wisps. The AWT depends on a full hierachical structure for the hair, which can either be precomputed - for instance using a hierarchical hairstyle [KN02] - or computed on the fly. The AWT represents at each time step the wisps segments (or guide-strand segments) of the hierarchy that are actually simulated (called *active* segments). Considering that hair should always be more refined near the tips than near the roots, the AWT dynamically splits or merges hair wisps while always preserving a tree-like structure, in which the root coincides with the hair roots and the leaves stand for the hair tips.

In addition to limiting the number of active hair-wisp segments, one of the key benefits of the AWT for collision detection is that the splitting behavior of the wisps models their deformation: there is no need for the complex deformable wisp geometry used in [PCP01]. For collision processing, active wisp segments of the AWT are thus represented by cylinders, which greatly simplifies collision detection tests: detecting interactions simplifies into detecting the local minima of the distance between guide-strand and comparing its value to the sum of the wisp radii. With this method, ten seconds of animations could be computed, in 2003, in less than five minutes.

4.2. Handling curly hair and exploiting temporal coherence

The Super-Helix model that was recently introduced [BAC*06] is the first model that accurately simulates the dynamics of curly hair: unlike previous approaches, curly hair wisps are not modeled using a straight mass-spring skeleton around which wavy strands are drawn at the rendering stage, but are instead accurately modeled using wavy to fuzzy guide-strands, which have a piece-wise helical shape. Detecting interactions between such complex helical guide-strands is indeed more costly.

To handle collisions between hair clumps guided by Super-Helices in a both accurate and efficient way, our strategy is based on the two following ideas: 1) the use of *adaptive* cylindrical bounding envelopes around each hair wisp, whose number and size can automatically adapt during motion, depending on the geometry of the wisp, and 2) the *tracking of the closest points* between the skeletons (*i.e.*, the principal axes) of the bounding cylinders.

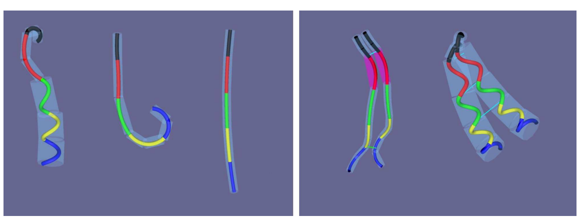

Figure 4: *Left: The three different adaptive representations for the bounding volume of a wisp segment. Right: Tracking the pairs of closest points between the skeletons of guide volumes (for smooth and curly hair) [Ber06].*

1. **Adaptive bounding envelopes**: the bounding volume of a helical element Q_i of the guide hair strand is composed of a single, large cylinder if the helix's spires are tight enough. In other cases (*i.e.* for straighter strands), we use one or two cylinders, oriented along the mean local tangent of the element, to approximate the volume of the wisp (see Figure 4).
2. **Tracking pairs of the closest points**: we adapted the algorithm of Raghupathi *et al.*, originally designed for detecting self-collisions in long and thin deformable objects [RCFC03], to the collision detection between guide hair volumes. Since guide hair volumes are composed of a set of cylinders, the method amounts to computing minimal distances between pairs of segments (the principal axes of the cylinders), as in [RCFC03]. For each pair of guide-strands, we first initialize a closest point pair near the root. At each time step, each closest point pair is updated by letting the closest points slide along the associated wisp, from the positions they had at the last time step. They stop in a location that locally minimizes the distance between the two wisp volumes. When this distance is under a threshold, new pairs of points are created at both sides of the initial pair, to track the possible multiple local minima. When two closest point pairs slide to the same location, they are merged together. At each time step, because of temporal coherence, only very few of these pairs need to be moved, so advancing them is very fast. Each time the distance between two guide volumes is locally smaller than the sum of their radii, collision is detected.

This algorithm ensures that at least one pair of closest points is maintained between two guide volumes, while keeping the number of tracked pairs between guide volumes low (merging occurs when two different pairs slide towards the same place). The algorithm has thus a n^2 complexity where n is the number of guide hair strands composing the hairstyle instead of the total number of segments composing hair, as it would be when using a naive algorithm.

The same adaptive wisp volumes and temporal coherence technique are used for detecting collisions between the hair and the body of the character. Distance tests are computed between segments and spheres, as the body is approximated by a unions of spheres. Using this technique, we obtained a total frame rate of only 3 seconds per frame for a dynamic hair style composed of a hundred of guide hair strands, including self-interactions and interactions with the body.

5. Response to guide-strand interactions

As already mentioned hair is a very soft and light material. Seen as a whole, it deforms rather than bouncing when it collides with a relatively rigid obstacle such as the character's body. Indeed, hair self-collisions should be very soft as well, and result into frictional rather than bouncing behaviors. Therefore, response to guide-strands interactions have been modeled using soft penalty forces together with friction forces.

5.1. Anisotropic response in wisp-based methods

As noted by Plante *et al.* [PCP01, PCP02], accounting for collisions between hair wisps is fairly different from modelling collisions between standard deformable bodies. Wisps are highly anisotropic, since they are just a virtual representation for a group of hair strands. While two perpendicular colliding wisps should be compressed in order to avoid intersection, interpenetration can be allowed between neighbouring wisps moving roughly in the same plane. In consequence, the authors proposed an anisotropic model for the interactions between hair wisps: Wisps of similar orientations are mostly submitted to viscous friction and penetrate each other, whereas wisps of different orientations actually collide in a very dissipative way.

Figure 5: *The layered wisp model [PCP01] (right) captures both continuities and discontinuities observed in real long hair motion (left).*

As illustrated in Figure 5, this approach yields convincing results, even for fast motions: the model adequately captures the discontinuities that can be observed in long, thick hair, preserves hair volume and prevents crossing between hair wisps. Nevertheless, the high number of contacts that needed to be computed between the different wisps at rest caused some noticeable artifacts such as unstabilities when hair comes to rest.

The previous anisotropic collision response model was re-used and improved by the Adaptive Wisp Tree (AWT) method [BKCN03]: an AWT implicitly models some of the mutual hair interactions, since neighboring wisps with similar motions merge, thus efficiently yet robustly mimicking the static friction in real hair. This merging behavior also avoids subsequent collision processing between these wisps, thus increasing efficiency as well as gaining stability from the reduced number of primitives. Typically, an AWT simulation starts with a reduced number of hair wisps. While the character moves, these wisps refine where and when needed (see Figure 6), to merge again as soon as they can. When the character is back at rest, the simulation eventually ends up a single large hair wisps. This totally avoids the local unstabilities noted in previous approaches.

5.2. Setting up realistic penalty and friction forces

The recent work on Super-Helices tackled the problem of setting up more accurate response forces between interacting guide-strands [BAC*06]. Interactions between guide-hairs, and between hair and external objects (such as the body) are performed through penalty forces which include a normal elastic response together with a tangential friction force.

As in [Dur04], the normal penalty force is stabilized thanks to a quadratic regularization for small penetrations.

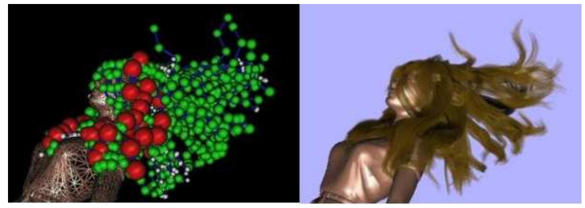

Figure 6: *Left: The structure of an AWT at a given animation step. Most of the parent wisps (in red) have split into medium-size wisps (in green), which eventually have split into small ones (in white). Right: Rendering of the same frame [BKCN03].*

From a regularization depth δ_{reg} (arbitrarily chosen), the normal reaction force $\boldsymbol{R_N}$ exerted between the two closest points of interacting guide-strands is computed as follows:

$$\begin{cases} \text{if } (gap \leq 0) & \boldsymbol{R_N} = \mathbf{0} \\ \text{if } (0 \leq gap \leq \delta_{reg}) & \boldsymbol{R_N} = \frac{k_c\, gap^2}{2\,\delta_{reg}}\, \boldsymbol{n_c} \\ \text{else} & \boldsymbol{R_N} = k_c\,(gap - \frac{\delta_{reg}}{2})\, \boldsymbol{n_c} \end{cases}$$

where $\boldsymbol{n_c}$ is the unitary vector giving the direction of collision (calculated as the cross product of the vectors defining the two closest segments), and k_c an arbitrary constant value.

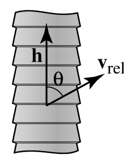

Figure 7: *Angle θ between the fiber orientation and its relative velocity w.r.t the external object in contact with the fiber.*

To simulated friction between wisps in contact or friction with an obstacle, the method extends viscous friction law in [CK05], defined as :

$$\mathbf{R_T} = -\nu\,(\mathbf{v_{rel}} - (\mathbf{v_{rel}}.\mathbf{n_c})\,\mathbf{n_c})$$

To account for the oriented scales covering individual hair fibers, the friction coefficient ν is multiplied by a sine function to account for the orientation of hair fibers with respect to their sliding motion over the external object: $\nu = \nu_0\,(1 + \sin(\theta/2))$, where angle θ is defined in Figure 7.

The parameters of interaction forces, as well as the other parameters of the Super-Helices model, can be set up using the actual study of real wisps of hair: The friction parameter ν_0 between hair and a given material is directly adjusted from real observations of sliding contacts between the hair clump and the material.

As Figures 8 and 9 show, the Super-Helices model results in realistic simulations which can be compared side by side with videos of real hair in motion.

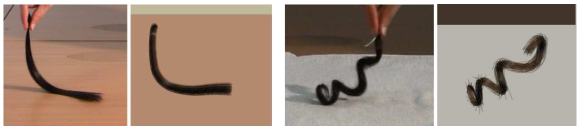

Figure 8: *Validation of the friction model in [BAC*06] on a sliding motion of a smooth (left) and curly (right) hair clump over different kinds of material (left: wooden surface, right: cotton).*

Figure 9: *Comparison between a real full head of hair and the model based on interacting Super-Helices [BAC*06], on a head shaking motion (straight and clumpy hair type).*

6. Conclusion

As we have shown, processing hair interactions requires a dedicated set of methods, due to the very specific nature of the hair material. Impressive advances were made in the last six years, from the first models able to handle hair self-collisions to efficient, robust and even partly validated methods. This chapter has detailed several specific solutions that range from the use of a volumetric approach when a very quick solution is required to realistic models that still keep the computational load to an acceptable rate.

In spite of all these advances, there still remains very challenging issues in the modeling of hair self-interactions: these interactions are indeed the origin of the complex collective behavior of hair. Especially they cause hair to group into clusters during motion; this phenomenon has never been accounted before (except in very simplified models, such as the AWT), as previous models usually assume that hair granularity is fixed by the number of simulated guide-strands. Moreover, hair interactions vary a lot according to external conditions such as moisture (wet hair being the extreme case), combing, or the use of cosmetic products. Lastly, hair triboelectricity has never been modelled in an accurate way.

Future research should include attempts to make volumetric methods such as the one presented in section 3 more accurate at low cost, by taking local hair directional distribution into account while setting up the response force. The approaches that seek for realism should probably extract the internal damping inside a hair wisp from the preliminary study of hair chunks actually modeled using a full set of interacting hair strands. This study should also bring more accurate criteria for splitting a wisp into sub-wisps or merging them, and could help characterizing the number of hair guides required according to the natural curliness and smoothness of a given hair type.

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References

[AUK92] ANJYO K., USAMI Y., KURIHARA T.: A simple method for extracting the natural beauty of hair. In *Proceedings of ACM SIGGRAPH 1992* (August 1992), Computer Graphics Proceedings, Annual Conference Series, pp. 111–120.

[BAC*06] BERTAILS F., AUDOLY B., CANI M.-P., QUERLEUX B., LEROY F., LÉVÊQUE J.-L.: Super-helices for predicting the dynamics of natural hair. In *ACM Transactions on Graphics (Proceedings of the SIGGRAPH conference)* (August 2006).

[BCN03] BANDO Y., CHEN B.-Y., NISHITA T.: Animating hair with loosely connected particles. *Computer Graphics Forum 22*, 3 (2003), 411–418. Proceedings of Eurographics'03.

[Ber06] BERTAILS F.: *Simulation de chevelures naturelles*. PhD thesis, Institut National Polytechnique de Grenoble, 2006.

[BKCN03] BERTAILS F., KIM T.-Y., CANI M.-P., NEUMANN U.: Adaptive wisp tree - a multiresolution control structure for simulating dynamic clustering in hair motion. In *ACM SIGGRAPH Symposium on Computer Animation* (July 2003), pp. 207–213.

[BMC05] BERTAILS F., MÉNIER C., CANI M.-P.: A practical self-shadowing algorithm for interactive hair animation. In *Graphics Interface* (May 2005). Graphics Interface'05.

[CCK05] CHOE B., CHOI M., KO H.-S.: Simulating complex hair with robust collision handling. In *SCA '05: Proceedings of the 2005 ACM SIGGRAPH/Eurographics symposium on Computer animation* (2005), pp. 153–160.

[CJY02] CHANG J. T., JIN J., YU Y.: A practical model for hair mutual interactions. In *ACM SIGGRAPH Symposium on Computer Animation* (July 2002), pp. 73–80.

[CK05] CHOE B., KO H.-S.: A statiscal wisp model and pseudophysical approcahes for interactive hairstyle generation. *IEEE Transactions on Visualization and Computer Graphics 11*, 2 (March 2005).

[CSDI99] CHEN L., SAEYOR S., DOHI H., ISHIZUKA M.: A system of 3d hairstyle synthesis based on the wisp model. *The Visual Computer 15*, 4 (1999), 159–170.

[DTKT93] DALDEGAN A., THALMANN N. M., KURIHARA T., THALMANN D.: An integrated system for modeling, animating and rendering hair. *Computer Graphics Forum 12*, 3 (1993), 211–221.

[Dur04] DURVILLE D.: Modelling of contact-friction interactions in entangled fibrous materials. In *Procs of the Sixth World Congress on Computational Mechanics (WCCM VI)* (September 2004).

[HMT01] HADAP S., MAGNENAT-THALMANN N.: Modeling dynamic hair as a continuum. *Computer Graphics Forum 20*, 3 (2001), 329–338. Proceedings of Eurographics'01.

[KH01] KOH C., HUANG Z.: A simple physics model to animate human hair modeled in 2D strips in real time. In *Computer Animation and Simulation '01* (Sept. 2001), pp. 127–138.

[KN00] KIM T.-Y., NEUMANN U.: A thin shell volume for modeling human hair. In *Computer Animation 2000* (2000), IEEE Computer Society, pp. 121–128.

[KN02] KIM T.-Y., NEUMANN U.: Interactive multiresolution hair modeling and editing. *ACM Transactions on Graphics 21*, 3 (July 2002), 620–629. Proceedings of ACM SIGGRAPH 2002.

[LK01] LEE D.-W., KO H.-S.: Natural hairstyle modeling and animation. *Graphical Models 63*, 2 (March 2001), 67–85.

[PCP01] PLANTE E., CANI M.-P., POULIN P.: A layered wisp model for simulating interactions inside long hair. In *EG workshop of Animation and Simulation* (sep 2001).

[PCP02] PLANTE E., CANI M.-P., POULIN P.: Capturing the complexity of hair motion. *Graphical Models (Academic press) 64*, 1 (january 2002), 40–58.

[RCFC03] RAGHUPATHI L., CANTIN V., FAURE F., CANI M.-P.: Real-time simulation of self-collisions for virtual intestinal surgery. In *Proceedings of the International Symposium on Surgery Simulation and Soft Tissue Modeling* (2003), Ayache N., Delingette H., (Eds.), no. 2673 in Lecture Notes in Computer Science, Springer-Verlag, pp. 15–26.

[RCT91] ROSENBLUM R., CARLSON W., TRIPP E.: Simulating the structure and dynamics of human hair: Modeling, rendering, and animation. *The Journal of Visualization and Computer Animation 2*, 4 (1991), 141–148.

[Rob94] ROBBINS C. R.: *Chemical and Physical Behavior of Human Hair*, third ed. Springer-Verlag, New York, 1994.

[vO91] VAN OVERVELD K.: An iterative approach to dynamic simulation of 3-d rigid-body motions for real-time interactive computer animation. *The Visual Computer 7* (– 1991), 29–38.

[WFK*06] WARD K., F.BERTAILS, KIM T.-Y., MARSCHNER S., CANI M.-P., LIN M.: A survey on hair modeling: Styling, simulation and rendering. *IEEE Transaction on Visualization and Computer Graphics* (2006). To appear.

[WL03] WARD K., LIN M. C.: Adaptive grouping and subdivision for simulating hair dynamics. In *Pacific Graphics Conference on Computer Graphics and Applications* (October 2003), pp. 234–243.

[Zvi86] ZVIAK C.: *The Science of Hair Care*. Marcel Dekker, 1986.