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Abstract

In this paper, a method for the finite time estimation of the switching times in linear switched systems is proposed. The approach is based on algebraic tools and distribution theory. Switching time estimates are given by explicit algebraic formulae that can be implemented in a straightforward manner using standard tools from computational mathematics. Simulations illustrate the proposed techniques.

Keywords: Linear systems, switched systems, switching time estimation, distribution theory.

1. Introduction

Many systems encountered in practice exhibit switchings between several subsystems, inherently by nature, such as when a physical plant has the capability of undergoing several operational modes, or as a result of the controller design, such as in switching supervisory control.

Switched systems can be seen as higher–level abstractions of hybrid systems, obtained by neglecting the details of the discrete behavior. A switched system is composed of a family of dynamical (linear or nonlinear) subsystems and a rule, called the switching law, that orchestrates the switching between them (see [24, 36] for surveys). In the recent years, there has been an increasing interest in the control problems of switched systems due to their significance from both a theoretical and practical point of view. Important results for switched systems have been achieved for problems including stability [2, 10, 12, 25, 40], stabilization [26, 29, 41, 43], tracking [11] or controllability [35, 42].

Observability and state estimation is a key problem for such systems because both the active mode and the continuous state have to be estimated and this during a finite time interval. The notion of state estimation for switched systems was introduced in [1]. Observability notions for some classes of hybrid systems such as switched linear systems has been discussed and characterized in recent works such as [4], [33], [39].

The problem is to recover from available measurements the state of the system and/or the switching signal, and eventually the switching time. Different observation and identification methods have been performed during the last years ([3, 5, 7, 13, 14, 15, 16, 21, 22, 23, 27, 28, 32]). Usually, the hybrid observer consists of two parts: an index estimator of the current active sub-model and a continuous observer that estimates, asymptotically in most cases, the continuous state of the hybrid system.

The aim of this paper is to estimate in "real-time" the switching time sequence of some class of switched linear systems with the knowledge (full or partial) of the continuous state only. The possibility to have *finite time* estimate for that kind of systems is clearly important, not to say crucial. The approach considered here takes root in recent works developed in [19] for parameter identification of linear time-invariant systems. This method is based on algebraic tools (differential algebra, module theory and operational calculus) and results in finite time estimates given by explicit algebraic formula that can be implemented in a straightforward manner using standard tools from computational mathematics. Those results have been extended to the problems of closed-loop parametric estimation for continuous-time linear systems in [20], parametric estimation of systems with delayed and structured entries in [9], state estimation of linear systems with time-varying parameters in [38] or with delays in [8], fault diagnosis in [18], nonlinear systems with unknown inputs in [6] or some class of linear infinite dimensional systems governed by partial differential equations in [31]. This approach was also applied in [17] for the estimation of the index corresponding to the current active subsystem, and the state variable of this subsystem. In this paper, the algebraic approach is extended to the problem of the finite time identification of the switching occurrences. Using some properties of the distribution theory, the switching time estimation is given by an explicit formula, as a function of the integral of the output, in order to attenuate the influence of measurement noises.

This paper is organized as follows: Section 2 gives the problem formulation. The main result is derived in Section 3. First, the switching time identification of one commutation between two subsystems is analyzed. Then, the result is extended to the case of commutations among an arbitrary number of subsystems. The last section provides a simulation that illustrates the proposed approach.

2. PROBLEM STATEMENT

In this work, we study a class of linear hybrid systems called switched linear systems (see [39] for a definition), i.e. systems whose evolution is determined by a collection of linear models (Q subsystems) with continuous state $x \in \mathbb{R}^n$ connected by switches among a number of discrete states $q \in I_Q \triangleq \{1, \ldots, Q\}$, modeled by linear ordinary differential equations of the following form:

$$\dot{x} \quad = \quad A_q x, \tag{1}$$

where $A_q \in \mathbb{R}^{n \times n}$ are constant matrices. For the sake of convenience and without loss of generality, it is assumed that at each time $t \in \mathbb{R}$ only one discrete event can act on the system.

The objective of this paper is to estimate on line the switching times of the system (1) using the dynamics of the continuous state only. The method that will be developed will involve high order time derivatives of x. Because of the presence of non smooth dynamics, derivation has to be understood in the distribution sense.

2.1. Distribution Framework

We recall here some standard definitions and results from distribution theory developed in [34], and fix the notations to be used in the sequel. The space of C^∞-functions having compact support in an open subset Ω of $\mathbb{R}$ is denoted by $\mathscr{D}(\Omega)$, and $\mathscr{D}'(\Omega)$ is the space of distributions on Ω, i.e., the space of continuous linear functionals on $\mathscr{D}(\Omega)$.

When concentrated at a point $\{\tau\}$, the Dirac distribution $\delta(t-\tau)$ is written δ_τ.

Functions are considered through the distributions they define and are therefore indefinitely differentiable. Hence, if y is a continuous function except at a point a with a finite jump σ_a, its derivative writes

$$\dot{y} = dy/dt + \sigma_a \delta_a, \tag{2}$$

where dy/dt is the distribution stemming from the usual derivative of y.

A distribution is said to be of order r if it acts continuously on C^r-functions but not on C^{r-1}-functions. Measures and functions are of order 0. By virtue of Schwartz Theorem (see [34]), $\alpha \times T = 0$ for any smooth function α whose derivatives of appropriate order vanish on the support of a given distribution $T \in \mathscr{D}'(\Omega)$. In particular, one has for the Dirac distribution[1]:

$$f(t)\cdot\delta_{t_i} = f(t_i)\cdot\delta_{t_i}, \ \ \forall f \in C^\infty(\mathbb{R}) \ \text{ if } f \text{ is continuous at } t_i. \tag{3}$$

3. Switching time identification

The goal of this work is to obtain an algebraic relation of the measured variables which involves only known parameters in order to explicitly obtain the unknown switching times. To do so, first, a time-varying state transformation is used to get an intermediate differential algebraic relation which parameters are not depending anymore on the switching times but on a Dirac distribution at the switching instant. Then, an annihilating algebraic manipulation based on (3) is provided to get the desired differential algebraic relation where the unknown switching times explicitly appear. Finally, using iterative integral operators, the final expression of the switching instant is obtained in terms of time integrals of the state variables.

3.1. The case of two subsystems and one commutation

In this case, $q \in \{1,2\}$. Assume that the system (1) switches from a subsystem to the other one at time t_c. The aim is to get a finite time estimation of the switching time t_c. For the sake of simplicity, the method is first detailed assuming that all the state

[1]The existence of the product $f(t)\cdot\delta_{t_i}$ is ensured as soon as f is a $C^\infty-$function.

is measured (some hints for the case of partial state measurement are given in Section 3.4).

The dynamical behavior of the system can be written as follows:

$$\dot{x} = \Gamma(t)x \qquad \Gamma(t) \in \{A_1, A_2\}. \tag{4}$$

Under the change of variable $z = e^{Gt}x$, where G will be defined later, the system (4) is transformed into:

$$\dot{z} = M(t)z$$

with

$$M(t) = G + e^{Gt}\Gamma(t)e^{-Gt}. \tag{5}$$

Then, the matrix G is chosen such that:

$$M_1(t) + M_2(t) = 0 \tag{6}$$

where

$$M_i(t) = G + e^{Gt}A_i e^{-Gt}, \quad i = 1,2$$

Since G and e^{Gt} commute, equation (6) implies that

$$G = -\frac{A_1 + A_2}{2} \tag{7}$$

and

$$\dot{z} = \sigma(t)M_1(t)\,z \tag{8}$$

with $\sigma(t) \in \{-1, 1\}$ and, without loss of generality, $\sigma(0+) = 1$ if $\Gamma(0+) = A_1$.

Remark 1. When the matrices A_1 and A_2 commute, $M_1(t) = -M_2(t) = \frac{A_1 - A_2}{2}$ are constant. This case was treated in [37].

The determinant $\Delta_1(t)$ of $M_1(t)$ satisfies the following property:

$$2^n\Delta_1(t) = \det(2M_1(t)) = \det(M_1 - M_2) = \det(A_1 - A_2) = \text{cte}. \tag{9}$$

Assume henceforth that the matrix $(A_1 - A_2)$ is full rank. Then, $M_1(t)$ is invertible and one can define $W_1(t) := \mathrm{Ad}(M_1(t)) = \Delta_1(t)M_1^{-1}(t)$. Equation (8) becomes:

$$W_1(t)\dot{z} = \sigma(t)\,\Delta_1(t)\,z. \tag{10}$$

Note that every term in (10) is known but the evolution of $\sigma(t)$. Since $\sigma(t)$ is a constant outside the set $\{t_c\}$, the time derivative of the product $\sigma(t)g(t)$ is well-defined as soon as $g(t)$ is a smooth function for $t = t_c$. Denoting $\sigma_c = \sigma(t_c+) - \sigma(t_c-) = \pm 2$, one has:

$$\begin{aligned} \dot{\sigma}(t) &= \sigma_c\,\delta_{t_c} \\ (\sigma(t)\dot{}g(t)) &= \dot{\sigma}(t)g(t) + \sigma(t)\dot{g}(t) = \sigma_c\,g(t_c)\delta_{t_c} + \sigma(t)\dot{g}(t). \end{aligned} \tag{11}$$

By derivation of (10), one obtains:

$$\dot{W}_1(t)\dot{z} + W_1(t)\ddot{z} = \dot{\sigma}(t)\,\Delta_1(t)\,z + \sigma(t)\,\Delta_1(t)\,\dot{z} = \dot{\sigma}(t)\,\Delta_1(t)\,z + \Delta_1(t)\,M_1(t)\,z$$

$$W_1(t)\ddot{z} + \dot{W}_1(t)\dot{z} - \Delta_1(t)\,M_1(t)\,z = \dot{\sigma}(t)\,\Delta_1(t)\,z. \tag{12}$$

The equation (12),using property (11), can be expressed as:

$$\begin{aligned} &\sum_{i=0}^{2} K_i(t)\,z^{(i)} = \gamma_c \delta_{t_c} \\ &K_0(t) = -\Delta_1(t)\,M_1(t) \\ &K_1(t) = \dot{W}_1(t) \\ &K_2(t) = W_1(t) \\ &\gamma_c = \sigma_c\,\Delta_1(t_c)z(t_c) \end{aligned} \tag{13}$$

Thus, using the change of variables $z = e^{Gt}x$, one obtains a differential systems with a left-hand side that only contains known quantities. The right-hand side involves a Dirac distribution that will be annihilated using property (3), as shown in the next section.

3.2. Explicit computation of the switching instant

Take any function with the following properties:

(i) $f(t,t_c)\delta_{t_c} = 0$,

(ii) $f(0,t_c) = \dot{f}(0,t_c) = 0$.

Multiplying (13) by $f(t,t_c)$, one obtains:

$$f(t,t_c)\left(\sum_{i=0}^{2} K_i(t)\,z^{(i)}\right) = 0. \tag{14}$$

Integrating (14) from 0 to $t > t_c$ leads to

$$\int_0^t f(\tau,t_c)\left(W_1(\tau)\ddot{z} + \dot{W}_1(\tau)\dot{z} - \Delta_1(\tau)\,M_1(\tau)z\right)d\tau = 0. \tag{15}$$

Integration by parts gives:

$$\begin{aligned} \int_0^t f(\tau,t_c)W_1(\tau)\ddot{z}(\tau)d\tau &= [f(\tau,t_c)W_1(\tau)\dot{z}(\tau)]_{\tau=0}^{\tau=t} - \int_0^t \left(\dot{f}(\tau,t_c)W_1(\tau) + f(\tau,t_c)\dot{W}_1(\tau)\right)\dot{z}(\tau)d\tau \\ &= f(t,t_c)W_1(t)\dot{z}(t) - \int_0^t \dot{f}(\tau,t_c)W_1(\tau)\dot{z}(\tau)d\tau - \int_0^t f(\tau,t_c)\dot{W}_1(\tau)\dot{z}(\tau)d\tau \end{aligned}$$

and

$$\begin{aligned} \int_0^t \dot{f}(\tau,t_c)W_1(\tau)\dot{z}(\tau)d\tau &= \left[\dot{f}(\tau,t_c)W_1(\tau)z(\tau)\right]_{\tau=0}^{\tau=t} - \int_0^t \left(\ddot{f}(\tau,t_c)W_1(\tau) + \dot{f}(\tau,t_c)\dot{W}_1(\tau)\right)z(\tau)d\tau \\ &= \dot{f}(t,t_c)W_1(t)z(t) - \int_0^t \ddot{f}(\tau,t_c)W_1(\tau)z(\tau)d\tau - \int_0^t \dot{f}(\tau,t_c)\dot{W}_1(\tau)z(\tau)d\tau. \end{aligned}$$

Hence

$$
\begin{aligned}
&\int_0^t f(\tau,t_c)\left(W_1(\tau)\ddot{z}+\dot{W}_1(\tau)\dot{z}\right)d\tau \\
&= f(t,t_c)W_1(t)\dot{z}(t)-\dot{f}(t,t_c)W_1(t)z(t)+\int_0^t \ddot{f}(\tau,t_c)W_1(\tau)z(\tau)d\tau+\int_0^t \dot{f}(\tau,t_c)\dot{W}_1(\tau)z(\tau)d\tau
\end{aligned}
\tag{16}
$$

Using the property

$$
\int_0^t\int_0^{t_{\nu-1}}\cdots\int_0^{t_1}x(\tau)dt_{\nu-1}\cdots dt_1 d\tau=\int_0^t\frac{(t-\tau)^{\nu-1}}{(\nu-1)!}x(\tau)d\tau,
$$

integrating one more time (15) from 0 to $t>t_c$ and using (16), one obtains:

$$
\begin{aligned}
&f(t,t_c)W_1(t)z(t)-2\int_0^t \dot{f}(\tau,t_c)W_1(\tau)z(\tau)d\tau-\int_0^t f(\tau,t_c)\dot{W}_1(\tau)z(\tau)d\tau \\
&\quad+\int_0^t (t-\tau)\left(\ddot{f}(\tau,t_c)W_1(\tau)+\dot{f}(\tau,t_c)\dot{W}_1(\tau)-\Delta_1(\tau)M_1(\tau)\right)z(\tau)d\tau=0.
\end{aligned}
\tag{17}
$$

An estimate of t_c can be obtained from (17). Take for instance the function $f(t,t_c)=t^2(t-t_c)$ that satisfies the properties (i)-(ii) and

$$
\begin{aligned}
\dot{f}(t,t_c)&=t(3t-2t_c),\\
\ddot{f}(t,t_c)&=6t-2t_c.
\end{aligned}
$$

Using the result of (17), the estimate of t_c is given by the following formula:

$$
D(t,z,M_1)t_c=N(t,z,M_1)
$$

with

$$
\begin{aligned}
N(t,z,M_1)&=t^3W_1(t)z(t)-\int_0^t(6\tau^2W_1(\tau)+\tau^3\dot{W}_1(\tau))z(\tau)d\tau\\
&\quad+\int_0^t(t-\tau)(6\tau W_1(\tau)+3\tau^2\dot{W}_1(\tau)-\Delta_1(\tau)M_1(\tau))z(\tau)d\tau\\
D(t,z,M_1)&=t^2W_1(t)z(t)-\int_0^t(4\tau W_1(\tau)+\tau^2\dot{W}_1(\tau))z(\tau)d\tau\\
&\quad+\int_0^t(t-\tau)(2W_1(\tau)+2\tau\dot{W}_1(\tau))z(\tau)d\tau
\end{aligned}
\tag{18}
$$

Note that $N(t,z,M_1)$ and $D(t,z,M_1)$ are column vectors of dimension n. So

$$
t_c=\frac{N_g(t,z,M_1)}{D_g(t,z,M_1)},
\tag{19}
$$

for any $1\le g\le n$, where $N_g(t,z,M_1)$ and $D_g(t,z,M_1)$ are the components of $N(t,z,M_1)$ and $D(t,z,M_1)$, respectively.

Remark 2. When the measured signal $z(t)$ is perturbed by noise, (17) can be integrated one or several more times in order to reduce the noise effect. In this case, the expression of $N(t,z,M_1)$ and $D(t,z,M_1)$ in (18) becomes

$$\begin{aligned} N(t,z,M_1) &= \int_0^t \tau^3 W_1(\tau)z(\tau)d\tau - \int_0^t (t-\tau)(6\tau^2 W_1(\tau) + \tau^3 \dot{W}_1(\tau))z(\tau)d\tau \\ &+ \int_0^t \frac{1}{2}(t-\tau)^2(6\tau W_1(\tau) + 3\tau^2 \dot{W}_1(\tau) - \Delta_1(\tau)M_1(\tau))z(\tau)d\tau \\ D(t,z,M_1) &= \int_0^t \tau^2 W_1(\tau)z(\tau)d\tau - \int_0^t (t-\tau)(4\tau W_1(\tau) + \tau^2 \dot{W}_1(\tau))z(\tau)d\tau \\ &+ \int_0^t \frac{1}{2}(t-\tau)^2(2W_1(\tau) + 2\tau \dot{W}_1(\tau))z(\tau)d\tau \end{aligned} \tag{20}$$

Hereafter, the method is extended to the problem of the identification of an arbitrary number of switching times between an arbitrary number of subsystems.

3.3. The case of Q subsystems and S commutations

Assume that every pair of matrices A_i and A_j ($i,j \in I_Q$ and $i \neq j$) in system (1) is such that ($A_i - A_j$) is full rank. It has been seen that an estimator $E_{i,j}$ that computes the quantities $D(t,z,M_{i,j})$ and $N(t,z,M_{i,j})$, with $M_{i,j} = G_{i,j} + e^{G_{i,j}t} A_i e^{-G_{i,j}t}$, $\quad G_{i,j} = -\frac{A_i + A_j}{2}$ can be used to determine a switching time $t_{i,j}$ that occurs between the two subsystems i and j (either from mode i to mode j or from mode j to mode i).

Hence, in order to identify all the switches among Q modes, one can use $C_Q^2 = \frac{Q(Q-1)}{2}$ estimators in parallel. Then, the output signals of each estimator can be analyzed as follows to determine the occurrence of a switch and its associated mode. Indeed, assume that the system (1) is in the mode i for $t \in [t_0, t_{i,j}[$ and in mode j for $t \in [t_{i,j}, T[$, where times t_0 and T stands for other switch occurrences. Then, one has:

1. $D_g(t,z,M_{i,j}) = 0$ and $N_g(t,z,M_{i,j}) = 0$ for $t \in [t_0, t_{i,j}[$: in practice, this property is checked on a few sampling periods before the switching instant;
2. $D_g(t,z,M_{i,j})$ and $N_g(t,z,M_{i,j})$ are straight lines for $t \in [t_{i,j}, T[$: in practice, several points have to be used to detect that the slope of the line is constant.
3. for $t \in [t_{i,j}, T[$ the ratio of $\frac{N_g(t,z,M_{i,j})}{D_g(t,z,M_{i,j})}$ is constant and equal to $t_{i,j}$: to detect when this ratio is constant, successive values within a boundary of a given thickness are needed.

Properties 1. and 2. arise from the fact that Dirac distributions in equation (13) are integrated twice. Thus, the estimator that fulfills those three conditions provides the switching instant as well as the index of the subsystems between which the switch occurs. Furthermore, with the knowledge of the first active mode, the sequence of all active models can be estimated.

The implementation of the numerical algorithm which allows to identify all the switching instants is done on a sliding window as follows:

---

function **detection of the switching time** t_c

read $x(kT_e)$

T_e: sample time;

h: number of chosen points to verify the constraints with $h.T_e$ less than the dwell time of the system;

$t_s = 0$: left bound of the integration window;

$m = length(x)$: length of the sampled state x;

$z_{i,j}(kT_e) = e^{-\frac{A_i+A_j}{2}} x(kT_e)$: new state

for $k = h : m$

Reset all the components of $N(t,z,M_{i,j})(k)$ and $D(t,z,M_{i,j})(k)$ to zero

for $i = t_s : k$

Approximate numerically the integrals as sums using the trapezoidal rule

end

Compute the value of $N(t,z,M_{i,j})(k)$ and $D(t,z,M_{i,j})(k)$ in an integration window with (18)

if an estimator satisfies the three detection criterions

update the integration window: $\mathbf{t_s} \longleftarrow \mathbf{k} - \mathbf{h}$;

print this estimator and the active model.

else

break;

end

print the switching time

end

The numerical implementation of the above obtained algorithm has also to address the following points:

- integration window: when none of the $C_Q^2 = \frac{Q(Q-1)}{2}$ estimator detect the commutation, the algorithm will calculate the value of $N(t,z,M_{i,j})(k)$, $D(t,z,M_{i,j})(k)$ in a new integration window ($k \longleftarrow k+1$) whose width is one sample time larger than the old one (the lower bound of the integration is not changed while the upper bound is increased of one sampling), until a switching instant is detected with one of the estimator.

- update: when t_c is detected, one has to update the scheme by resetting the integral computation and replacing the lower bound of the integration with t_c in order to identify the next switching time.

3.4. A hint for an extension to systems with partial state measurement

Consider the following system:

$$\begin{aligned} \dot{x} &= \Gamma(t)x \qquad \Gamma(t) \in \{A_1, A_2\} \\ y &= Cx \end{aligned}$$

where $y \in \mathbb{R}^p$ is the output vector of available measurements. Assume it is possible to find an output relation given by:

$$\dot{Y} = \bar{\Gamma}(t)Y + \Theta\left(\delta_{t_c}, \dot{\delta}_{t_c}, \ldots, \delta_{t_c}^{(l)}\right), \qquad \bar{\Gamma}(t) \in \left\{\bar{A}_1, \bar{A}_2\right\} \tag{21}$$

where $Y = \left[y^T, \ldots, y^{(r)^T}\right]^T$ is a vector composed of the output and a finite number of its time derivatives, $\bar{A}_1$, $\bar{A}_2$ are square matrices, and where Θ is a square matrix that depends on the Dirac distribution and a finite number of its time derivatives. Using the same procedure as in Section 3.1, it is possible to obtain a differential system similar to (13):

$$\sum_{i=0}^{2} \bar{K}_i(t)\,\bar{z}^{(i)} = \bar{\Theta}\left(\delta_{t_c}, \dot{\delta}_{t_c}, \ldots, \delta_{t_c}^{(\bar{l})}\right)$$

where $\bar{z} = e^{\bar{G}t}Y$. Then, setting $\omega = \int^r \bar{z}$ leads to:

$$\sum_{i=0}^{2} \bar{K}_i(t)\,\omega^{(i+r)} = \bar{\Theta}\left(\delta_{t_c}, \dot{\delta}_{t_c}, \ldots, \delta_{t_c}^{(\bar{l})}\right).$$

Following the lines of Section 3.2, with a suitable choice of the function $f(t,t_c)$ (that allows for the annihilation of $\bar{\Theta}$) and a sufficient number of integrations, one can obtain an algebraic relation of the form:

$$D(t,\omega,\bar{M}_1)t_c = N(t,\omega,\bar{M}_1),$$

that provides an on-line estimation of t_c, since ω is available for measurement (it depends on the output y and a finite number of its integrals).
To completely solve the problem of switching time identification of systems with partial state measurement using the proposed approach, one has to study the possibility to obtain the relation (21). For this, observability and identifiability properties of the system (4) have to be investigated. This problem is out of the scope of this paper (whose aim is to introduce an algebraic methodology for fast identification of switching times) and will be studied in future work.

4. EXAMPLE

Let us consider the buck-boost converter shown in Figure 1, the switched model consisting of two second-order linear time invariant systems is given by:

$$\begin{cases} \dot{x} & = A_1 x + B = \begin{pmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ \frac{U}{L} \end{pmatrix} \\ \dot{x} & = A_2 x = \begin{pmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{pmatrix} x \end{cases} \tag{22}$$

where $x = [V \;\; i]^T = [x_1 \;\; x_2]^T$ and one has a common output $y = x_2$.

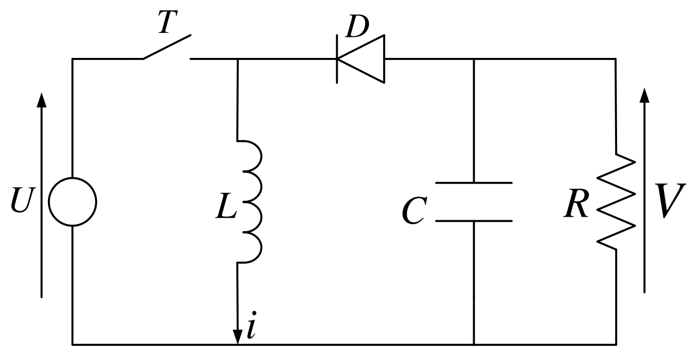

Figure 1: Buck-boost converter circuit.

The mode 1 represents the transistor T closed and the diode D blocked ($T = 1$, $D = 0$), while the mode 2 correspond with T OFF and D ON ($T = 0$, $D = 1$). This model is well being to the class defined in the section 3.4. Note that mode 1 is unstable, since the current increases linearly with time when T is closed, while mode 2 (T open) is stable at the origin.

One can rewrite this system as follows:

$$\begin{cases} \dot{Y} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} Y + \begin{pmatrix} \frac{U}{L} \\ \frac{U}{L}\delta_{t_c} \end{pmatrix} \\ \dot{Y} &= \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{pmatrix} Y \end{cases} \tag{23}$$

with $Y = [y \;\; \dot{y}]^T$ corresponds with the relation (21).

One has

$$\bar{G} = -\frac{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{pmatrix}}{2} = \frac{\begin{pmatrix} 0 & 1 \\ \frac{1}{LC} & \frac{1}{RC} \end{pmatrix}}{2}$$

to change the variable $\bar{z} = e^{\bar{G}t}Y$. Setting $\omega = \int_0^t \bar{z}(\tau)d\tau$, then ω is available for measurement (it is as a function of the output y and its integral). In order to annihilate $\Theta(\delta_{t_c})$, here one chooses the function $f(t,t_c) = t^2(t-t_c)^2$ satisfying with the properties:

(i) $f(t,t_c)\delta_{t_c} = f(t,t_c)\dot{\delta}_{t_c} = 0$,

(ii) $f(0,t_c) = \dot{f}(0,t_c) = 0$.

After twice integrations one obtains:

$$D_1(t,\omega,\bar{M}_1)t_c^2 - D_2(t,\omega,\bar{M}_1)t_c + N(t,\omega,\bar{M}_1) = 0 \tag{24}$$

with

$$N(t,\omega,\bar{M}_1) = t^4 W_1(t)\omega(t) - \int_0^t (8\tau^3 W_1(\tau) + \tau^4 \dot{W}_1(\tau))\omega(\tau)d\tau$$
$$+ \int_0^t (t-\tau)(12\tau^2 W_1(\tau) + 4\tau^3 \dot{W}_1(\tau) - \Delta_1(\tau)\bar{M}_1(\tau))\omega(\tau)d\tau$$
$$D_1(t,\omega,\bar{M}_1) = t^2 W_1(t)\omega(t) - \int_0^t (4\tau W_1(\tau) + \tau^2 \dot{W}_1(\tau))\omega(\tau)d\tau$$
$$+ \int_0^t (t-\tau)(2W_1(\tau) + 2\tau \dot{W}_1(\tau))\omega(\tau)d\tau$$
$$D_2(t,\omega,\bar{M}_1) = 2t^3 W_1(t)\omega(t) - \int_0^t (12\tau^2 W_1(\tau) + 2\tau^3 \dot{W}_1(\tau))\omega(\tau)d\tau$$
$$+ \int_0^t (t-\tau)(12\tau W_1(\tau) + 6\tau^2 \dot{W}_1(\tau))\omega(\tau)d\tau$$

Hence, one can calculate t_c explicitly from the equation (24).

The Buck-Boost circuit has the following characteristics: $U = 24(V)$, $L = 100(\mu H)$, $C = 220(\mu F)$, $R = 20(\Omega)$. In order to get a good simulation, since the above parameters L and C are of order 10^{-4}, we use a time scaling $T = \frac{1}{\varepsilon}t$ with $\varepsilon = 10^{-4}$, that means we simulate $\frac{dx}{dT} = Ax + b$ instead of $\frac{dx}{dt} = \frac{1}{\varepsilon}(Ax + b)$. The switching instants of this system are detected using the proposed algorithm with $T_e = 0.001s$ with respect to the new time scaling. The switching times are assumed to occur at: $t_1 = 80(\mu s)$, $t_2 = 150(\mu s)$, $t_3 = 220(\mu s)$, $t_4 = 280(\mu s)$ (real time) (see Fig. 3) and the first active mode is 1. Note that in the simulation, one chooses 30 sampling periods to identify the switching time, so the dwell time of the system has to be at least larger than $3(\mu s)$ with respect to the real time. In Fig. 2 is reported the behavior of the state $x(t)$. Fig. 3 shows that the estimator scheme can detect all switching times t_c accurately.

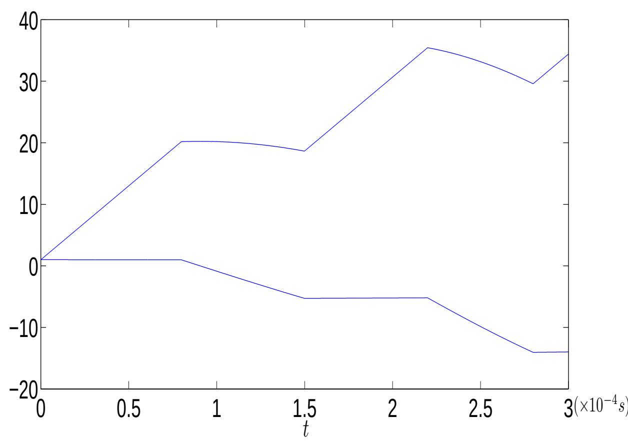

Figure 2: State x

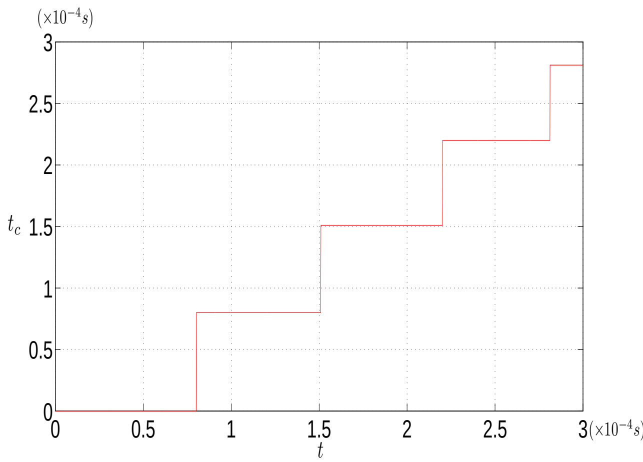

Figure 3: Identification of the switching times t_c

Note that the methodology of this algebraic approach can be applied to solve some practical problems, such as for instance fault detection, fault tolerant control issues [30].

5. Conclusion

In this paper, an algebraic approach for switching time estimation of a class of hybrid systems has been introduced. Using an approach based on algebraic tools and distribution theory, an explicit algorithm which computes on-line the switching time instants in a fast way has been derived for systems with full and partial state measurement. Future works are concerned with observability and identifiability issues.

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