Switching time estimation for linear switched systems: an algebraic approach

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***Abstract*— This paper aims at estimating the switching time for linear switched systems, i.e. the time instant when a sub-model is switched on while another one is switched off. Assuming that the state of the active sub-models is known, a distribution point of view is adopted to get a real-time estimation of these switching times. Real-time means that an explicit algorithm computes on-line these time instants in a fast and efficient way. Simulations illustrate the proposed techniques which is easily implementable.**

***Keyword*—Linear systems, switched systems, hybrid systems, switching time estimation, distribution theory.**

I. INTRODUCTION

Many systems encountered in practice exhibit switchings between several subsystems, inherently by nature, such as when a physical plant has the capability of undergoing several operational modes, or as a result of the controller design, such as in switching supervisory control.

Switched systems may be viewed as higher–level abstractions of hybrid systems, obtained by neglecting the details of the discrete behavior. Roughly speaking, a switched system is composed of a family of dynamical (linear or nonlinear) subsystems and a rule, called the switching law, that orchestrates the switching between them. In the recent years, there has been increasing interest in the control problems of switched systems due to their significance from both a theoretical and practical point of view. Important results for switched systems have been achieved for problems including stability [2], [8], [28], controllability [24], [29], stabilization [18], [30], tracking [9], ... See also [17], [25] for surveys.

Observability and state estimation is a key problem for such systems, where discrete and continuous parts are mixed. The vector-valued output is a function of the continuous state. The observation problem is to determine the continuous state and the active ODE driving the current continuous state evolution. In [1], the notion of state estimation for switched systems is introduced. Observability notions for some classes of hybrid systems such as switched linear systems has been discussed and characterized in recent works, see e.g. [4], [22], [27]. The problem is to recover from available measurements the state of the system and/or the switching signal, and eventually the switching time. Different observation and identification methods have been proposed during the last years ([3], [5], [10], [15], [16], [19], [21]). In most of the cases, the hybrid observer consists of two parts: an index estimator of the current active sub-model and a continuous observer that estimates, asymptotically in most cases, the continuous state of the hybrid system.

The aim of this paper is to estimate in "real-time" the switching time sequence with the knowledge of the continuous state of some class of switched linear systems. The possibility to have *finite time* estimate for that kind of systems is clearly important, not to say crucial. The approach considered here is based on an idea recently published by M. Fliess and H.-S. Ramírez [13] for parameter identification of linear time-invariant systems. This method is based on tools which are of algebraic flavor: differential algebra, module theory and operational calculus. The estimators are non asymptotic: solutions are provided in finite time by explicit algebraic formulae and result in relatively simple and fast algorithms, with straightforward implementation using standard tools from computational mathematics. Those results have been extended to the problems of closed-loop parametric estimation [14], state estimation of linear systems with time-varying parameters [26] or with delays [7], fault diagnosis [12], nonlinear systems with unknown inputs [6] or some class of linear infinite dimensional systems governed by partial differential equations [20]. This approach was also applied in [11] for the estimation of the index corresponding to the current active subsystem, and the state variable of this subsystem, *via* quite robust methods with respect to corrupting noises. Here, explicit computation of the switching times is obtained: the switching time estimation is given by an explicit formula, as a function of the integral of the output, in order to attenuate the influence of measurement noises. The proposed method exhibits the following features:

- the switching times can be efficiently identified on-line and in finite time,
- computations can be carried out by a computer and in a very fast manner.

The rest of the paper is organized as follows: the problem formulation is given in Section II. Then, the main result is derived in Section III last section provides a simulation example in order to highlight the efficiency of the proposed approach.

II. PROBLEM STATEMENT

Consider the class of *linear switched system* made of subsystems modeled linear ordinary differential equations

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(LODE) of the following form:

$$\dot{x} = A_q x, \tag{1}$$

where $q \in I_Q \triangleq \{1,\ldots,Q\}$ is the discrete state, $x \in \mathbb{R}^n$ is the continuous state and $A_q \in \mathbb{R}^{n\times n}$. For the sake of convenience and without loss of generality, it is assumed that at each time $t \in \mathbb{R}$ only one discrete event can act on the system. It is also assumed that there is **no state jump**, i.e. that if t_k is a switching instant

$$\lim_{t\to t_k^-} x(t) = \lim_{t\to t_k^+} x(t)$$

The switching function driving the change of mode is defined by:

$$\begin{aligned} \sigma(t) : \mathbb{R} &\to I_Q \\ x &\mapsto \sigma(t) \end{aligned} \tag{2}$$

where $\sigma(t) \in I_Q$ corresponds to the index associated with the current active LODE. The switching function is supposed to have a finite number of discontinuities on a finite time interval. For example, $\sigma(t)$ has k discontinuities at time instants $t_1, t_2, \ldots t_k$ within the time interval $[t_0, T_{\text{final}}[$.

The general problem to be solved is: "Is it possible to get in real time (when they occur) the switching times t_i using the available past and current measurements?". In this paper, the study is restricted to the case when $Q = 2$ and when the whole continuous states are measured. In that case, system (1) can be rewritten as follows:

$$\begin{aligned} \dot{x} &= \Gamma(t)x, && (3)\\ \Gamma(t) &= A_1 H(t) + A_2(1-H(t)) && (4) \end{aligned}$$

where A_1, $A_2 \in \mathbb{R}^{n\times n}$ are constant matrices. Without loss of generality[1], assume that $q = 1$ within some given interval $t \in [t_{i-1}, t_i[$ and $q = 2$ on $t \in [t_i, t_{i+1}[$. Then, the function H is given by:

$$H(t) = \begin{cases} 1, & t_{i-1} \le t < t_i \\ 0, & t_i \le t < t_{i+1} \end{cases}$$

Suppose that the previous switching time was found: thus t_{i-1} is known. The objective is to have a finite time estimation of the switching time t_i when the system commutes from the mode $q = 1$ to the mode $q = 2$. This will be done by using higher order time derivatives of x. Because of the presence of non smooth dynamics, derivation has to be understood in the distribution sense.

Distribution Framework

We recall here some standard definitions and results from distribution theory [23], and fix the notations to be used in the sequel. The space of C^∞-functions having compact support in an open subset Ω of $\mathbb{R}$ is denoted by $\mathscr{D}(\Omega)$, and $\mathscr{D}'(\Omega)$ is the space of distributions on Ω, i.e., the space of continuous linear functionals on $\mathscr{D}(\Omega)$.

When concentrated at a point $\{\tau\}$, the Dirac distribution $\delta(t-\tau)$ is written δ_τ.

[1]This will be explained later (see subsection III-A).

Functions are considered through the distributions they define and are therefore indefinitely differentiable. Hence, if y is a continuous function except at a point a with a finite jump σ_a, its derivative writes

$$\dot{y} = dy/dt + \sigma_a \delta_a, \tag{5}$$

where dy/dt is the distribution stemming from the usual derivative of y.

A distribution is said to be of order r if it acts continuously on C^r-functions but not on C^{r-1}-functions. Measures and functions are of order 0. By virtue of Schwartz Theorem [23], $\alpha \times T = 0$ for any smooth function α whose derivatives of appropriate order vanish on the support of a given distribution $T \in \mathscr{D}'(\Omega)$. In particular, one has for the Dirac distribution[2]:

$$\begin{aligned} f(t)\cdot\delta_{t_i} &= 0,\ \forall f \in C^\infty(\mathbb{R}) \text{ with } \ f(t_i) = 0 \\ f(t)\cdot\delta_{t_i} &= f(t_i)\cdot\delta_{t_i},\ \ \forall f \in C^\infty(\mathbb{R}) \ \text{ with } f(t_i) \neq 0 \end{aligned}$$

III. MAIN RESULT

A. Change of variable

Assume that A_1 and A_2 are two square commuting matrices, and consider the following change of variable:

$$z = e^{Gt}x \tag{6}$$

where $G = -\frac{A_1+A_2}{2}$. One has:

$$\begin{aligned} \dot{z} &= Ge^{Gt}x + e^{Gt}\dot{x} = Gz + e^{Gt}\Gamma(t)x \\ &= (G + e^{Gt}\Gamma(t)e^{-Gt})z \end{aligned}$$

which leads to

$$\dot{z} = M(t)z \tag{7}$$

where

$$M(t) = (G + e^{Gt}A_1e^{-Gt})H(t) + (G + e^{Gt}A_2e^{-Gt})(1-H(t))$$

Since A_1 and A_2 are two square commuting matrices, one gets:

$$\begin{aligned} M(t) &= (G+A_1)H(t) + (G+A_2)(1-H(t)) \\ &= \frac{A_1-A_2}{2}H(t) + \frac{A_2-A_1}{2}(1-H(t)) \\ &= -AH(t) + A(1-H(t)) \end{aligned}$$

with $A = \frac{A_2-A_1}{2}$. The choice of the change of coordinates (6) was motivated by the fact that the subsequent computations involve $M^2 = A^2$ which is a known quantity independent of the switching time.

Differentiating (7) , one has:

$$\begin{aligned} \ddot{z} - A^2z &= -2A\delta_{t_{i-1}}z(t_{i-1}), \qquad t_{i-1} \le t < t_i \\ \ddot{z} - A^2z &= -2A\delta_{t_{i-1}}z(t_{i-1}) + 2A\delta_{t_i}z(t_i), \quad t_i \le t < t_{i+1} \end{aligned}$$

[2]The existence of the product $f(t)\cdot\delta_{t_i}$ is ensured as soon as f is a $C^\infty-$function

which can be rewritten in the following compact form:

$$\ddot{z} - A^2 z = \sum_{g=i-1}^{i} \gamma_g \delta_{t_g} z(t_g), \quad t_{i-1} \le t < t_{i+1} \tag{8}$$

where γ_g is obviously defined. Note that a similar expression is obtained if the initial system is $q = 2$, but with the γ_g having opposite signum. As it will be seen hereafter, this has no influence because the right hand side of (8) will be canceled by suitable algebraic manipulations. Thus the proposed method does not need the knowledge of the current mode.

B. Explicit computation of the switching instant

Recall that $(t - t_i)\delta_{t_i} = 0$ and multiply (8) by

$$f(t, t_{i-1}, t_i) = (t - t_{i-1})^2 (t - t_i). \tag{9}$$

One obtains:

$$(t - t_{i-1})^2 (t - t_i)(\ddot{z} - A^2 z) = 0$$

Integrating from t_{i-1} to $t > t_i$ leads to

$$\int_{t_{i-1}}^{t} (\tau - t_{i-1})^2 (\tau - t_i)(\ddot{z}(\tau) - A^2 z) d\tau = 0 \tag{10}$$

The function (9) has also been chosen in such a way that $f(t_{i-1}, t_{i-1}, t_i) = f'(t_{i-1}, t_{i-1}, t_i) = 0$. Then, integration by parts leads to:

$$\begin{aligned}
&\int_{t_{i-1}}^{t} f(\tau, t_{i-1}, t_i)\ddot{z}(\tau) d\tau \\
&= [f(\tau, t_{i-1}, t_i)\dot{z}]_{\tau=t_{i-1}}^{\tau=t} - \int_{t_{i-1}}^{t} f'(\tau, t_{i-1}, t_i)\dot{z}(\tau) d\tau \\
&= f(t, t_{i-1}, t_i)\dot{z} - \int_{t_{i-1}}^{t} f'(\tau, t_{i-1}, t_i)\dot{z}(\tau) d\tau
\end{aligned}$$

and

$$\begin{aligned}
&\int_{t_{i-1}}^{t} f'(\tau, t_{i-1}, t_i)\dot{z}(\tau) d\tau \\
&= \left[f'(\tau, t_{i-1}, t_i) z\right]_{\tau=t_{i-1}}^{\tau=t} - \int_{t_{i-1}}^{t} f''(\tau, t_{i-1}, t_i) z(\tau) d\tau \\
&= f'(t, t_{i-1}, t_i) z - \int_{t_{i-1}}^{t} f''(\tau, t_{i-1}, t_i) z(\tau) d\tau
\end{aligned}$$

Hence

$$\begin{aligned}
&\int_{t_{i-1}}^{t} f(\tau, t_{i-1}, t_i)\ddot{z}(\tau) d\tau \\
&= f(t, t_{i-1}, t_i)\dot{z} - f'(t, t_{i-1}, t_i) z + \int_{t_{i-1}}^{t} f''(\tau, t_{i-1}, t_i) z(\tau) d\tau
\end{aligned} \tag{11}$$

Taking one more time the integral from t_{i-1} to $t > t_i$ leads to the following relation:

$$\begin{aligned}
&\int_{t_{i-1}}^{t} [f(\tau, t_{i-1}, t_i)\dot{z}(\tau) - f'(\tau, t_{i-1}, t_{i-1}, t_i) z(\tau)] d\tau \\
&+ \int_{t_{i-1}}^{t} \int_{t_{i-1}}^{\tau_1} (f''(\tau, t_{i-1}, t_i) - A^2 f(\tau, t_{i-1}, t_i)) z(\tau) d\tau d\tau_1 = 0
\end{aligned} \tag{12}$$

Using the properties

$$\int_0^t \int_0^{t_{\nu-1}} \cdots \int_0^{t_1} x(\tau) dt_{\nu-1} \cdots dt_1 d\tau = \int_0^t \frac{(t-\tau)^{\nu-1}}{(\nu-1)!} x(\tau) d\tau$$
$$\int_0^{t-t_i} \int_0^{t_{\nu-1}-t_i} \cdots \int_0^{t_1-t_i} x(\tau) dt_{\nu-1} \cdots dt_1 d\tau = \int_0^{t-t_i} \frac{(t-\tau)^{\nu-1}}{(\nu-1)!} x(\tau) d\tau$$

yields

$$\int_{t_i}^t \int_{t_i}^{t_{\nu-1}} \cdots \int_{t_i}^{t_1} x(\tau) dt_{\nu-1} \cdots dt_1 d\tau = \int_{t_i}^t \frac{(t-\tau)^{\nu-1}}{(\nu-1)!} x(\tau) d\tau.$$

Thus, one has:

$$\begin{aligned}
&f(t, t_{i-1}, t_i) z - 2\int_{t_{i-1}}^{t} f'(\tau, t_{i-1}, t_i) z(\tau) d\tau \\
&+ \int_{t_{i-1}}^{t} (t - \tau)(f''(\tau, t_{i-1}, t_i) - A^2 f(\tau, t_{i-1}, t_i)) z d\tau = 0
\end{aligned} \tag{13}$$

Since

$$\begin{aligned}
&f(t, t_{i-1}, t_i) = (t - t_{i-1})^2 (t - t_i) = t(t - t_{i-1})^2 - (t - t_{i-1})^2 t_i \\
&f'(t, t_{i-1}, t_i) = (t - t_{i-1})^2 + 2(t - t_{i-1})(t - t_i) \\
&f''(t, t_{i-1}, t_i) = 4(t - t_{i-1}) + 2(t - t_i)
\end{aligned}$$

the estimate of t_i is given by the following formula:

$$D(t, t_{i-1}, z) t_i = N(t, t_{i-1}, z)$$

with

$$\begin{aligned}
N(t, t_{i-1}, z) &= t(t - t_{i-1})^2 z(t) \\
&- \int_{t_{i-1}}^{t} \left(2(\tau - t_{i-1})^2 + 4\tau(\tau - t_{i-1})\right) z(\tau) d\tau \\
&+ \int_{t_{i-1}}^{t} (t - \tau)\left(4(\tau - t_{i-1}) I + 2\tau I - A^2 \tau(\tau - t_{i-1})^2\right) z(\tau) d\tau \\
D(t, t_{i-1}, z) &= (t - t_{i-1})^2 z(t) - \int_{t_{i-1}}^{t} 4(\tau - t_{i-1}) z(\tau) d\tau \\
&+ \int_{t_{i-1}}^{t} (t - \tau)(2I - A^2(\tau - t_{i-1})^2) z(\tau) d\tau
\end{aligned} \tag{14}$$

where $I \in \mathbb{R}^{n \times n}$ is the unit matrix. Note that $D(t, t_{i-1}, z)$ and $N(t, t_{i-1}, z)$ are column vectors of dimension n. So

$$t_i = \frac{N_j(t, t_{i-1}, z)}{D_j(t, t_{i-1}, z)},$$

for any $1 \le j \le n$, where $N_j(t, t_{i-1}, z)$ and $D_j(t, t_{i-1}, z)$ are the components of $N(t, t_{i-1}, z)$ and $D(t, t_{i-1}, z)$, respectively.

In the above expression, t_{i-1} has been estimated with a similar procedure and is known. The first switching time t_1 is obtained with $t_0 = 0$ and the following relation:

$$\begin{aligned}
&D(t, z) t_1 = N(t, z) \\
&N(t, z) = t^3 z(t) - \int_0^t 6\tau^2 z(\tau) d\tau + \int_0^t (t - \tau)(6\tau I - A^2 \tau^3) z(\tau) d\tau \\
&D(t, z) = t^2 z(t) - \int_0^t 4\tau z(\tau) d\tau + \int_0^t (t - \tau)(2I - A^2 \tau^2) z(\tau) d\tau
\end{aligned} \tag{15}$$

Thus, one can obtain the exact formula for the estimation of the switching times t_i. Remark: one could also choose any function $f(t, t_{i-1}, t_i)$ with the following properties:

(i) $f(t, t_{i-1}, t_i)\delta_{t_i} = 0$,
(ii) $f(t_{i-1}, t_{i-1}, t_i) = \dot{f}(t_{i-1}, t_{i-1}, t_i) = 0$.

C. *Numerical implementation of the Integration Window*

The numerical implementation of the above obtained result has to address the following problems:

1) sampling: for computation purpose the continuous signal is sampled (the sampling period is T_e and quantized (not taken into account here)). It is clear that the choice of T_e is of importance and should be less than the minimum dwell time (i.e. the minimum difference between two successive switching instants).

2) numerical integration: for the implementation of the algorithm, since it involves integrals (see (14)), one should choose an efficient numerical algorithm in order to compute the integrals. Here, a trapezoidal rule was chosen.

3) singularity: t_i is given by (14). As a result, a singularity (0 divided by 0) occurs at the beginning of the computation. In order to overcome this problem, one has to wait a very small amount of time before the computation is taken as valid. In practice, two samplings are sufficient.

4) initialization: let us recall that this algorithm needs to be initialized with a first computation of the first switching time t_1. For this, one has to use (15) instead of (14).

5) detection: the ratio that defines t_i by (14) is fluctuating when $t \in [t_{i-1}, t_i[$ and constant as soon as $t > t_i$: to detect when this ratio is constant, successive values within a boundary of a given thickness are needed. In practice 2 or 3 successive values have to be within the interval of thickness.

6) update: when t_i is detected, one has to update the scheme by resetting the integral computation and changing the lower bound of the integration in order to compute the next switching time (for this in the integral approximation.

The advantage of the implementation of the algorithm with an *integration window* is that the estimate can be done at each integration step.

IV. EXAMPLE

Consider a linear switched system consisting of two second-order linear time invariant systems given by:

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -3 & 0 \\ 5 & 0 \end{pmatrix}$$

The switching instants of this hybrid system are detected using the algorithm proposed in section III with $T_e = 0.001$. The switching times are assumed to occur at: $t_1 = 0.2(s)$, $t_2 = 0.8(s)$, $t_3 = 0.85(s)$, $t_4 = 1.8(s)$ and $t_5 = 3(s)$ and the first active model is subsystem 1.Since one needs three samplings to identify the switching time, the dwell time has to be at least larger than $0.003(s)$.

Figure 1 shows that the estimator performs well. Note that t_3 occurs shortly ($0.05(s)$) after t_2 but that the algorithm can detect it accurately.

Figure 2 gives the behavior of the state z.

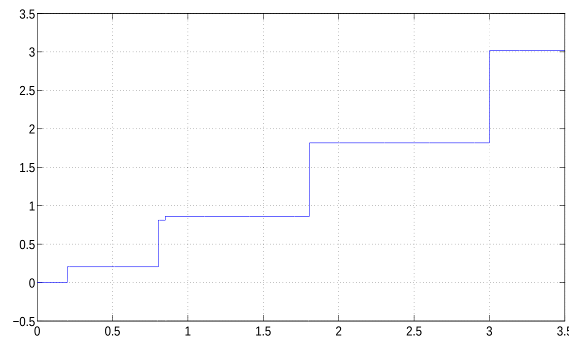

Fig. 1. Estimation of the switching times.

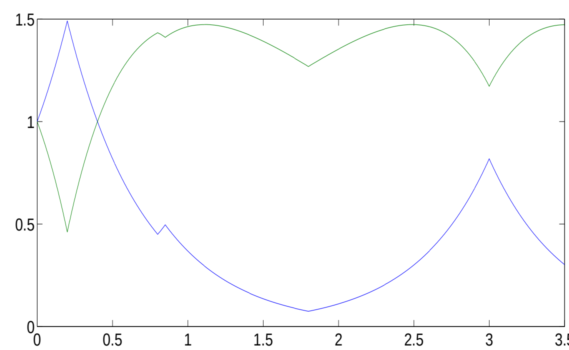

Fig. 2. State z

V. CONCLUSION AND PERSPECTIVES

In this paper, an algebraic approach for switching time estimation of hybrid systems has been introduced. An explicit algorithm which computes on-line the switching time instants in a fast way has been derived. In future works, this approach will be extended to more general cases when the switching system is composed of more than two subsystems (using a sufficient number of estimators in parallel) and with only partial state measurement outputs. Such techniques could also be used to estimate on-line and in real time both the switching instants and the switching function.

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