Skin Effect in Electromagnetism and Asymptotic Behaviour of Skin Depth for High Conductivity

Victor Péron, Gabriel Caloz, Monique Dauge, Erwan Faou

SKIN EFFECT IN ELECTROMAGNETISM AND ASYMPTOTIC BEHAVIOR OF SKIN DEPTH FOR HIGH CONDUCTIVITY

G. CALOZ, M. DAUGE, E. FAOU, AND V. PÉRON

Abstract. We study a three-dimensional model for the skin effect in electromagnetism, [1, 3, 4]. The 3-D case of the Maxwell equations in harmonic regime on a domain composed of a dielectric and of a highly conducting material is considered. We derive an asymptotic expansion with respect to a small parameter related to high conductivity. This expansion is theoretically justified at any order. The asymptotic expansion and numerical simulations in axisymmetric geometry exhibit the influence of the geometry of the interface on the skin effect.

We first prove uniform a priori estimates for a scalar transmission problem with constant coefficients on two subdomains, when the ratio between these coefficients is large. Then, we prove uniform a priori estimates for Maxwell transmission problem, [1, 4]. The technique is based on an appropriate decomposition of the electric field, which gradient part is estimated thanks to the first part. Next, we derive a multiscale asymptotic expansion for the solution of the harmonic Maxwell equations, with a regular interface between them. With the help of the uniform estimates, we prove the convergence of the asymptotic expansion as the conductivity tends to infinity. To measure the skin effect, we introduce a suitable function defined on the interface and generalizing the classical skin depth. We prove an asymptotic expansion at high conductivity for this function, which demonstrates the influence of the geometry of the interface on the skin depth, see [2].

Keywords: asymptotic analysis, electromagnetic waves, boundary layer, skin effect

Mathematics Subject Classifications (2000): 35J05, 78A45, 78M10, 78M35

References

[1] G. Caloz, M. Dauge, and V. Péron. Uniform estimates for transmission problems with high contrast in heat conduction and electromagnetism. Preprint IRMAR 09-40, Rennes, 2009. http ://hal.archives-ouvertes.fr/hal-00422315/en/

[2] M. Dauge, E. Faou, and V. Péron. Comportement asymptotique à haute conductivité de l'épaisseur de peau en électromagnétisme. Submitted. http ://hal.archives-ouvertes.fr/hal-00423607/en/

[3] H. Haddar, P. Joly, and H.-M. Nguyen. Generalized impedance boundary conditions for scattering problems from strongly absorbing obstacles: the case of Maxwell's equations. *Math. Models Methods Appl. Sci.*, vol. **18**, 10, 1787-1827, 2008.

[4] V. Péron. Modélisation mathématique de phénomènes électromagnétiques dans des matériaux à fort contraste. PhD Thesis. Université Rennes 1, 2009. http://tel.archives-ouvertes.fr/tel-00421736/fr/.

IRMAR, Université de Rennes 1, Campus de Beaulieu, F-35042 Rennes Cedex, France
E-mail address: gabriel.caloz@univ-rennes1.fr

IRMAR, Université de Rennes 1, Campus de Beaulieu, F-35042 Rennes Cedex, France
E-mail address: monique.dauge@univ-rennes1.fr

INRIA, Projet Ipso, ENS Cachan Bretagne, Avenue Robert Schumann, F-35170 Bruz Cedex, France
E-mail address: Erwan.Faou@inria.fr

INRIA Bordeaux Sud-Ouest, IMB, Université de Bordeaux 1, 351, cours de la Libération, F-33405 Talence Cedex, France
E-mail address: victor.peron@inria.fr