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***Abstract*: In this paper we compare the spatial throughput of Aloha and Carrier Sense Multiple Access (CSMA) in Wireless multihop Ad-Hoc Networks. In other words we evaluate the gain offered by carrier sensing (CSMA) over the pure statiscal collision avoidance which is the basis of Aloha. We use a Signal-to-Interference-and-Noise Ratio (SINR) model where a transmission is assumed to be successful when the SINR is larger than a given threshold. Regarding channel conditions, we consider both standard Rayleigh and negligible fading. For slotted and non-slotted Aloha, we use analytical models as well as simulations to study the density of successful transmissions in the network. As it is very difficult to build precise models for CSMA, we use only simulations to compute the performances of this protocol. We compare the two Aloha versions and CSMA on a fair basis, i.e. when they are optimized to maximize the density of successful transmissions. For slotted Aloha, the key optimization parameter is the medium access probability, for non-slotted Aloha we tune the mean back-off time, whereas for CSMA it is the carrier sense threshold that is adjusted. Our study shows that CSMA always outperforms slotted Aloha, which in turn outperforms its non-slotted version.**

Index Terms

Medium Access Control, MANET, slotted and non-slotted Aloha, CSMA, Poisson point process, shot-noise, SINR, stochastic geometry.

I. Introduction

Multiple communication protocols are used to organize transmissions from several sources (network nodes) in such a way that scheduled transmissions are likely to be successful. Aloha is one of the most common examples of such a protocol. A major charcteristic of Aloha is its great simplicity: the core concept consists in allowing each source to transmit a packet and back-off for some random time before the next transmission, independently of other sources. The main idea of the Carrier-Sense Multiple Access technique (CSMA) is to listen before sending a packet. CSMA is perhaps the most simple and popular access protocol that integrates some collision avoidance mechanism.

Simple classical models allow one to analyze Aloha and CSMA (see [15], [4]). They show that CSMA significantly outperforms Aloha as long as the maximum propagation delays between network nodes remain small compared to the packet transmission delays. However these models are not suitable for a wireless multihop network context, as they do not take into account the specificity of the radio propagation of the signal. Consequently, they cannot capture the spatial reuse effect (i.e., the possibility of simultaneous successful wireless transmissions) which is a fundamental property of multihop wireless communications.

Intuitively, it could be inferred that the collision avoidance embedded in CSMA should provide a greater spatial throughput than Aloha's purely random technique. Despite the large number of studies which evaluate Aloha and CSMA, to the authors' best knowledge there is no "fair" comparison of the spatial throughput of the two schemes in wireless multihop ad-hoc networks[1]. The aim of this paper is to carry out such a comparison and to quantify the gain in spatial throughput of CSMA over Aloha. We also study the effect of the various parameters on the performances. To do so, we model the geographic locations of network nodes by a planar Poisson point process and use the standard power-law path-loss function of the Euclidean distance to model the mean attenuation of the signal power. Regarding radio channel conditions, we consider both standard Rayleigh and negligible fading. We use a SINR model in which each successful transmission requires that the receiver is covered by the transmitter with a minimum SINR.

For Aloha (both slotted and non-slotted), the above model lends itself to mathematical analysis as shown in [3], [6]. We adopt use and develop this approach and use simulations (which confirm the analytical results) to evaluate and optimize the performances of Aloha. The performance of the CSMA in the previous model is very complex thus we use simulations to study it.

The main contribution of this paper is the analysis and comparison of the performances of slotted, non-slotted Aloha and CSMA, all optimized to maximize the rate of successful transmissions, *under various radio propagation assumptions* (path-loss exponent, fading conditions). Our *main findings of this analysis are*:

[1][17] is the only similar study we know of but we explain in this paper why the comparison presented in [17] is not, in our opinion, "fair" according to us.

- CSMA always outperforms slotted Aloha, which in turn outperforms non-slotted Aloha. In a moderate path-loss scenario (path-loss exponent equal to 4), without fading and the SINR level required for capture equal to 10, CSMA offers approximately a 2.4 times larger rate of successful transmissions than slotted Aloha and approximately a 3.2 times larger rate than non-slotted Aloha.
- The advantage of using CSMA is slightly reduced by increasing path-loss decay.
- This advantage is significantly reduced by the existence of fading since CSMA is much more sensitive to channel randomness than Aloha. In particular, for Rayleigh fading the above comparison of CSMA to slotted and non-slotted Aloha gives the ratios 1.7 and 2.3, respectively.
- The advantage of using CSMA increases with the SINR capture level.
- The above observations are valid when the transmissions are roughly scheduled to nearest neighbors and all the three MAC schemes are optimally tuned. This optimal tuning results in scheduling each node for transmission for about 8%, 6% and 4% of the time, for CSMA, slotted and non-slotted Aloha, respectively. These values do not depend on the network density, provided the nearest-neighbor receiver scheduling is used.
- The optimal tuning of CSMA is obtained by fixing the carrier-sensing power level (used to detect if the channel is idle) to about 8% of the useful signal power received at the nearest neighbor distance. This makes the transmissions successful with a high probability (from 0.8 to 0.95). Both smaller and larger values of the carrier-sensing threshold lead to essentially suboptimal performance of CSMA and sometimes even comparable to that of slotted Aloha. This might explain the apparent contradiction of our results to those of [17], which indicate similar performance of Aloha and CSMA.

This paper also *contributes to the development of the mathematical tools for Aloha by showing that the so-called spatial contention factor cf [12], appearing in the Laplace-transform characterization of the interference, is larger in non-slotted Aloha than in slotted Aloha under the same channel assumptions, by a factor that depends in a simple, explicit way only on the path-loss exponent; cf. Fact 3.2*. We also suggest the usage of the Bromwich contour inversion integral, developed in [1], to evaluate the coverage probability in the no-fading case; cf. Fact 3.6.

In this paper we will not take into account second order factors such as the back-off strategy in CSMA or guard intervals in slotted Aloha. We will briefly discuss these factors at the end of the paper to show that they can not change the order of magnitude of the comparison between Aloha and CSMA.

The remaining part of this paper is organized as follows. In Section I-A) we recall some previous studies of Aloha and CSMA. Section II introduces the model: distribution of nodes, channel and capture assumptions. It also describes in more detail the three MAC protocols studied in this paper. In Section III we present our analysis tools. Section IV provides our findings regarding the performance of the MAC protocols considered. The conclusions are presented in Section V.

A. Related Work

Aloha and Time Division Multiple Access (TDMA) are the oldest multiple access protocol. Aloha, which is the "mother" of random protocols, was born in the early seventies, the seminal work describing Aloha [2] being published in 1970. Since that time it has become widespread in various implementations. The essential simplicity of Aloha also allows for simple analysis. A first, and now widely taught result regarding the ratio of successful transmissions (cf. e.g [4, 4.2]) was obtained assuming an aggregate, geometry-less process of transmissions following a temporal Poisson process, with some overlapping of two or more packet transmissions necessarily leading to a collision. In this model, the ratio of successful transmissions can reach $1/(2e) \approx 18\%$, when the scheme is optimized by appropriate tuning of the mean back-off time (intensity of the Poisson process). It was also shown, that this performance can be multiplied by 2 in *slotted-Aloha*, when all the nodes are synchronized and can send packets only at the beginning of some universal time slots.

Although Aloha was primarily designed to manage wireless networks, the lack of a geometric representation of node locations in the above model makes it unsuitable for wireless networks. To the authors' best knowledge, it is in the paper by Nelson and Kleinrock [18] that Aloha was first explicitly studied in a wireless context. The authors showed that under ideal circumstances with slotted Aloha the "expected fraction of terminals in the network that are engaged in successful traffic in any slot does not exceed 21%". Despite the very simple on-off wireless propagation model used in the paper, this result, as we will show, is surprisingly close to the results that can be obtained using more recent and more sophisticated, physical propagation and interference models (cf. [3], [12]) in the case of the fading-less channel model with the mean path-loss decay equal to 3.5. The key element of this latter approach is the explicit formula of the Laplace transform of the interference created by a Poisson pattern of nodes using Aloha. This analysis was recently extended to non-slotted Aloha in [6]. We adopt this approach and slightly extend it in the present paper.

In the widely referenced paper [8] another simplified propagation model was used to study local interactions of packet transmissions and the stability of spatial Aloha.

CSMA was studied in the 70s in [15] and in the 90s in articles such as [14]. In these studies, the spatial reuse is usually not considered. However a few articles such as [7], [9], [11] take it into account by modeling carrier sensing with a graph. Nodes within carrier sense are linked vertices in this graph. However this model only approximates the carrier-sensing and the capture effect. Other simplified models of the carrier sensing and capture effect are proposed in [16], [21].

At the end of the 90s, an original and well referenced study tried to capture the behaviour of the IEEE 802.11 distributed medium access algorithm [5]. Although this study represented a step forward in the analysis of the IEEE 802.11 collision avoidance mechanism, [5] did not include an accurate model to capture interference. Thus the spatial throughput of IEEE 802.11 can not be analyzed with this model. Although numerous papers are actually using models close to that of [5], they are all unable to compute the spatial throughput of IEEE 802.11.

In contrast to [5], [20] studies the behaviour of a CSMA network using a more realistic model for interference and for the capture of packets. However [20] can not obtain closed formulas and [20] is actually a semi analytical model based on a Markov chain. Moreover this model can only handle a few dozen nodes. Thus it can not easily compute average performance or investigate the effect of the network parameters. New models have recently appeared such as [19]. These models use the Matern hard core process to model the pattern of simultaneously transmitting nodes in a CSMA network. These models, which allow the spatial throughput to be evalutated, have many flaws. First, CSMA is not accurately modeled by the Matern hard core process. Secondly the interference is also only approximated. Lastly the formulas obtained in these models to obtain the throughput are complex and it is difficult to use them to optimize the protocol when we vary the network parameters. Despite the many papers trying to analyze the performance of CSMA with spatial reuse, we believe that none of these papers offers a method for precise and straightforward evaluation of the gain from using the collision avoidance mechanism of

CSMA, in the same framework (infinite Poisson ad-hoc network) in which spatial Aloha can been analyzed. Thus, for this paper we chose to rely on simulations to estimate the performance of CSMA. We believe that, for our purpose, this approach offers a faster, more accurate method which is also easily to implement. We also want to recall the original geometric approach, also by Nelson and Kleinrock, presented in [17]. Their seminal paper presents a comparison of the performance of Aloha and CSMA in the geometric setting with the simple on-off wireless propagation model. Such a comparison is also the goal of our present study which however uses a more realistic propagation and interference model (see above). Our conclusions appear to *differ* from those of [17], where the performance of CSMA is found comparable to Aloha. We show that CSMA, with an appropriately tuned sensing threshold, can essentially outperform Aloha. The reason for this difference is presumably not due to the different wireless channel models, but primarily because of a sub-optimal tuning of the CSMA in [17], consisting of too small a sensing range (taken to be equal to the transmission range). In that sense [17] does not provide a fair comparison of the spatial throughput of Aloha and CSMA whereas, we believe, our paper does. [13] also compares Aloha and CSMA but only in terms of outage probability; [13] does not derive the density of successful transmission.

II. Models

In this section we present the models, which will be used to evaluate and compare the performance of the CSMA and Aloha MAC schemes.

A. Distribution of Nodes and Channel model

The model that we use here was proposed in [3]; we call it *the Poisson Bipole model*. It assumes that the nodes of a Mobile Ad hoc NETwork (MANET) are distributed on the infinite plane according to a homogeneous, planar Poisson point process of intensity λ nodes per unit surface area (say per square meter). Each node of this network transmits a packet to its own dedicated receiver located within a distance r meters from it, which is *not* a part of the Poisson point process. In this paper we choose $r = a/\sqrt{\lambda}$, for some constant $a > 0$, i.e. of the *order of the mean distance to the nearest neighbor* in a Poisson point process of intensity λ. This choice mimics the nearest neighbor scenario. We also assume that every node has always a pending packet to send. We believe that this assumption represents the behaviour of a loaded network and allows us to compute the maximum throughput of the network in a multihop context.

Using the formalism of the theory of point processes, we will say that a snapshot of the MANET can be represented by an independently marked Poisson point process (P.p.p) $\widetilde{\Phi} = \{(X_i, y_i)\}$, where the *locations of nodes* $\Phi = \{X_i\}$ form a homogeneous P.p.p. on the plane, with an intensity of λ, and where the mark y_i denotes the location of the receiver for node X_i. We assume here that one receiver is associated with only one transmitter and that, given Φ, the vectors $\{X_i - y_i\}$ are i.i.d with $|X_i - y_i| = r$.

We assume that whenever node $X_i \in \Phi$ transmits a packet it emits a unit-power signal that is propagated and reaches any given location y on the plane with power equal to $F/l(|X_i - y|)$.

$$l(u) = (Au)^{\beta} \quad \text{for } A > 0 \text{ and } \beta > 2 \tag{2.1}$$

and $|\cdot|$ denotes the Euclidean distance on the plane. Regarding the distribution of the random variable F, called for simplicity fading, we will consider two cases:

- constant $F \equiv 1$, called *the no fading case*,
- exponential F of parameter 1; this corresponds to the *Rayleigh fading* in the channel.

B. Successful Transmission

It is natural to assume that transmitter X_i *successfully transmits* a given packet of length B to its receiver y_i within the time interval $[u, u + B]$ if

$$\text{SIR} = \frac{F/l(|X_i - y_i|)}{\bar{I}} \geq T\,, \tag{2.2}$$

where T is some signal-to-interference (SIR) threshold and where $\bar{I}$ is the *average interference* suffered by the receiver y_i during this packet transmission interval

$$\bar{I} = \frac{1}{B}\int_u^{u+B} I(t)\,dt\,, \tag{2.3}$$

with

$$I(t) = \sum_{X_j \in \Phi, X_j \neq X_i} F_{j,y_i}/l(|X_j - y_i|)\mathbb{1}(X_j \text{ transmits at time } t)\,. \tag{2.4}$$

Note that taking (2.2) as the successful transmission condition, we ignore any external noise. This is a reasonable assumption if the noise is significantly smaller than the interference power

$\bar{I}$, which is the case in our setting. We should remark however, that if necessary, it would not be difficult to extend both the simulation scenario and the analysis to the case with non-null constant or even random noise power.

C. MAC Protocols

We will assume a saturated traffic model, i.e, that each node always has a packet to transmit to its receiver. The times at which any given node can transmit are decided by the Medium Access Protocol (MAC). In this paper we study three MAC protocols: CSMA, slotted Aloha and non-slotted Aloha.

1) CSMA: The basic rule of CSMA is very simple: *each node ready to transmit a packet listens first to the channel and transmits only if it finds the channel idle. Otherwise it waits for the channel to be idle and further postpones its transmission attempt for an additional random "back-off" time used to select a single node among the nodes blocked by the previous transmission*. We assume that this random "back-off" time is very small and we do not consider it in this study. This assumption is true if the ratio of the propagation plus detection time over the transmission time of the packet is very small. We discuss at the end of the article how to introduce corrective terms if propagation and detection times are not negligle.

To decide whether the channel is idle, the sender node computes the interference it receives I'. If $I' \leq \theta$, where θ is the *carrier-sense threshold* then the channel is "idle" otherwise it is busy. The carrier-sense threshold θ is the main, and in our model, the only parameter that will be tuned to maximize the density of successful transmissions and thus optimize the performance of the CSMA.

2) Slotted Aloha: Slotted Aloha supposes that all the network nodes are perfectly synchronized to some time slots (each of the length B of the packet, common for the whole network) and transmit packets according to the following rule: *each node, at each time slot independently tosses a coin with some bias* p which will be referred to as the *Aloha medium access probability* (Aloha MAP); *it sends the packet in this time slot if the outcome is heads and does not transmit otherwise*. The Aloha MAP p is the main parameter to be tuned to optimize the access (see a precise description of the stationary space-time model in [6]).

3) Non-Slotted Aloha: In non-slotted Aloha all the network nodes independently, without synchronization, send packets (of the same duration B) and then back off for some exponential random time of mean ε. In a more formal description of this mechanism one assumes that, given a pattern of network nodes, the temporal patterns of their retransmission are independent (across the nodes) renewal processes with the generic inter-arrival time equal to $B + E$ where E is exponential (back-off) with mean ε. A precise description of this stationary space-time model, called the *Poisson-renewal model* of non-slotted Aloha can be found in [6]. The analysis of this Poisson-renewal model of Aloha is feasible although it does not lead to simple closed formulas. In [6] another model, called the *Poisson rain model*, of non-slotted Aloha has been proposed. The main difference with respect to the scenario considered above is that the nodes X_i and their receivers y_i are not fixed in time. Instead, we may think of these nodes as being "born" at some time T_i transmitting a packet during time B and "disappearing" immediately after. The joint space-time distribution of node locations and transmission instances $\Psi = \{(X_i, T_i)\}$ is modeled by a homogeneous Poisson p.p. in $2+1$ dimensions with intensity $\lambda_s = \lambda B/(\varepsilon + B)$. It might be theoretically argued that the Poisson rain model is a good approximation of the Poisson-renewal model when the density of nodes λ is large, and the time instances at which a given node retransmits are very sparse. Indeed, the performance of the Poisson-renewal model is shown in [6] to be very close to that of the Poisson rain model. Thus, in our analytical study of non-slotted Aloha we will use the results regarding the latter for simplicity, while in our simulations we use the former.

D. Network Performance under a Given MAC

MAC protocols are supposed to create some space-time patterns of active (transmitting) nodes that increase the chances of successful transmissions. MAC optimization consists in finding the right trade-off between the density of active nodes and the probability that the individual transmissions are successful.

The first step of the analysis of the above trade-off problem consists in evaluating how much a given MAC protocol contends to the channel; i.e., how many packets it attempts to send per node and per unit of time. In homogeneous models this can be captured by the *average fraction of time a typical node is authorized to transmit*. We will denote this metric by τ. By space-time homogeneity, $\tau\lambda$ is the spatial density of active nodes at any given time and thus τ can also be interpreted as the probability that a typical node of the MANET is active at a given time. In what follows we will call it the*channel occupation* parameter. The way it depends on the basic (tunable) MAC parameters will be explained later on.

A complete evaluation of the performance of a MAC protocol must establish the fraction of successful authorized transmissions. We will denote by p_c the *probability that a typical transmission by a typical node is successful* (given this node was authorized by the MAC to transmit). We call it the *coverage probability* for short. By (2.2) we have

$$p_c = \mathbf{P}^0\{\, F \geq l(r)T\bar{I}\,\}\,, \tag{2.5}$$

where the probability $\mathbf{P}^0$ corresponds to the distribution of the random variables for a typical node during its typical transmission; this can be formalized using the Palm theory for point processes. This expression will be the basis of our analytical evaluation of the coverage probability for both slotted and non-slotted Aloha in Section III-C. We can notice that $\bar{I}$ is independent of F in (2.5) because our MAC schemes do not schedule transmissions according to the channel conditions at the receivers.

We define the optimal performance of a given MAC scheme as the situation where the mean number of successful transmissions per unit of surface and unit of time $\tau\lambda p_c$, called the *density of successful transmissions*, is maximized. For a given MANET density λ, this is equivalent to maximizing τp_c, which can be interpreted as the probability that a typical node is transmitting at a given time and this transmission is successful. Following this interpretation, we call τp_c *the mean throughput per node*. It will be analytically evaluated for both Aloha schemes and estimated by simulations for Aloha and CSMA MAC.

III. Analysis Tools

A. Simulation Scenarios

Our simulations are carried out in a square of 1000 m$\times$1000 m in which we generate a Poisson sample of MANET nodes with intensity $\lambda = 0.001$ nodes per square meter. For each MANET node we generate the location of its receiver uniformly on the circle of radius $r = a\sqrt{1000}$ m centered on this node. To avoid side effects, we consider a toroidal metric on this square. (Recall that, roughly speaking, rectangular torus is a rectangle whose opposite sites are "identified".) Given this metric we consider the distance dependent path-loss model (2.1) with some given path-loss exponent β and $A = 1$.

Typically β is larger than 2 and smaller than 6. 2 corresponds to free space propagation and 6 is for situations with a lot of obstacles and reflections. We will use the default value $\beta = 4$; in [10] the Walfishch-Ikegami model provides $\beta = 3.8$. However in some experiments, we try different values of β. For each pair of nodes we generate an independent copy of the exponential variable F in the case of Rayleigh fading or take $F \equiv 1$ in the no-fading case. Unless explicitly specified, our default value of the SIR threshold is $T = 10$ which is a widely used value.

For a given distribution of nodes we run the dynamic simulation for each of the three MAC schemes described in Section II-C with some particular choice of their main parameters: the carrier-sense threshold θ for CSMA, MAP p for slotted Aloha and mean back-off time for non-slotted Aloha. The packet transmission duration is always $B = 1$ unit of time. We count both the total number of packet transmissions and the number of successful transmissions during the simulation, whose total time is 4000 units of time. For CSMA, as already said, we ignore the time spent in back-off when a node, after having sensed the channel busy, finds the channel idle again before attempting another transmission. In the simulations we use very small back-off times to select the transmitting nodes and we neglect the time actually spent in these back-offs. Since each packet transmission takes $B = 1$ unit of time, dividing the number of transmissions by the simulation time and by the number of MANET nodes in the square, we obtain the one-network-sample estimators of, respectively, the average fraction τ of time a typical node is authorized to transmit and the mean throughput per node τp_c. We repeat the above experiment for 10 random choices of the network and take the empirical means of the above one-sample estimators. The error-bars in all simulation results correspond to a confidence interval of 95%. We use a home-made event-driven simulator specially dedicated to our simulation problem.

B. Analytical Results for Aloha MAC

The analytical results for Aloha are based on the (simple) calculation of the average fraction of time a typical node is authorized to transmit τ and a (more involved) calculation of the Laplace transform of the interference $\bar{I}$ that is the only variable of "unknown" distribution in the expression (2.5) of the coverage probability p_c.

1) Channel Occupation τ: It is straightforward to see that in slotted Aloha $\tau = p$. In the Poisson-renewal model of non-slotted Aloha $\tau = B/(B + \varepsilon)$; i.e., the ratio between the packet duration time and the mean inter-transmission time.

2) Interference Distribution: The basic observation allowing explicit analysis of the coverage probability for all our Poisson models of Aloha is that the distribution of the interference $\bar{I}$ under the Palm probability $\mathbf{P}^0$ in (2.5) corresponds to the distribution of the interference "seen" by an extra receiver added to the original MANET pattern (say at the origin) during an arbitrary period of time of length B (say in $[0, B]$). This is a consequence of Slivnyak's theorem.

Moreover, note that in the slotted Aloha MAC the interference $I(t) = I$ in (2.3) does not vary during the packet transmission and consequently $\bar{I} = I$. Furthermore, note that the pattern of nodes X_j, which emit at a given time slot and interfere with a given packet transmission (cf. expression (2.4)) is a Poisson p.p. of intensity $p\lambda$. This is a consequence of the independent MAC decisions of Aloha. The general expression of the Laplace transform $\mathcal{L}_I$ of I, which in this case is a Poisson shot-noise variable, is known explicitly. Here we recall the expressions for the special cases of interest.

Fact 3.1: *For the slotted Aloha model with path-loss function (2.1) and a general distribution of fading F with mean 1 we have:*

$$\mathcal{L}_I(\xi) = \exp\{-\lambda\tau A^{-2}\xi^{2/\beta}\kappa\}\,, \tag{3.6}$$

where $\kappa \geq 0$ is some constant depending only on the path-loss exponent and the distribution of the fading F. In particular

- $\kappa = \pi\Gamma(1 - 2/\beta)$ *in the no-fading scenario* $F \equiv 1$,
- $\kappa = 2\pi\Gamma(2/\beta)\Gamma(1 - 2/\beta)/\beta$ *with Rayleigh fading.*

The constant κ was evaluated in [3] for Rayleigh fading and in [12], for the no-fading scenario, where the name *spatial contention factor* was proposed for this constant. $\Gamma()$ is the classical gamma function.

Regarding the distribution of the averaged interference $\bar{I}$ in non-slotted Aloha, we have the following *new general* result.

Fact 3.2: *Assume the Poisson rain model of non-slotted Aloha with space-time intensity of packet transmissions $\lambda_s = \lambda\tau$ and the path-loss function (2.1). Assume a general distribution of fading F. Then the Laplace transform $\mathcal{L}_{\bar{I}}(\xi)$ of the averaged interference $\bar{I}$ is given by (3.6)* *with the spatial contention factor $\kappa = \kappa_{non-slotted}$ equal to*

$$\kappa_{non-slotted} = \frac{2\beta}{2+\beta}\kappa_{slotted}\,,$$

where $\kappa_{slotted}$ is the spatial contention factor evaluated for slotted Aloha under the same channel assumptions.

Proof: By (2.3), (2.4) and exchanging the order of integration and summation we express $\bar{I}$ in the following form

$$\bar{I} = \sum_{X_j\in\Phi, X_j\neq X_i} F_{j,y_i} H_j / l(|X_j - y_i|)\,,$$

where $H_j = \frac{1}{B}\int_u^{u+B} \mathbb{1}(X_j \text{ emits at time } t)\,dt$. In the Poisson rain model we have $\mathbb{1}(X_j \text{ emits at time } t) = \mathbb{1}(t - B \leq T_j \leq t)$, where T_j is the time at which X_j starts emitting. Integrating the previous function we obtain $H_j = h(T_j)$, where $h(s) = (B - |s|)^+/B$ and $t^+ = \max(0,t)$. Consequently, for the Poisson rain model represented by Poisson p.p. $\Psi = \{X_i, T_i\}$ (cf. Section II-C3) the averaged interference at the typical transmission receiver is equal in distribution to

$$\bar{I} \overset{distr.}{=} \sum_{X_j, T_j \in \Psi} F_j h(T_j)/l(|X_j|)\,,$$

where F_j are i.i.d. copies of the fading. Using the general expression for the Laplace transform of the Poisson shot-noise we obtain for the path-loss function (2.1)

$$\mathcal{L}_{\bar{I}}(\xi) = \exp\Big\{-2\pi\lambda_s \int_{-\infty}^{\infty}\int_0^{\infty} r\Big(1 - \mathcal{L}_F\big(\xi h(t)(Ar)^{-\beta}\big)\Big)\,dr\,dt\Big\}\,,$$

where $\mathcal{L}_F$ is the Laplace transform of F. Substituting $r := Ar(\xi h(t))^{-1/\beta}$ for a given fixed t in the inner integral we factorize the two integrals and obtain $\mathcal{L}_{\bar{I}}(\xi) = \exp\{-2\pi\lambda_s A^{-2}\xi^{2/\beta}\zeta\kappa\}$, where $\zeta = \int_{-\infty}^{\infty}(h(t))^{2/\beta}\,dt$ and $\kappa = \int_0^{\infty} r(1 - \mathcal{L}_F(r^{-\beta}))\,dr$. A direct calculation yields $\zeta = 2\beta/(2+\beta)$. This completes the proof. ∎

Remark 3.3: Regarding the ratio of the spatial contention parameters $\zeta = \zeta(\beta) = 2\beta/(2+\beta)$, that can be seen as the *cost of non-synchronization in Aloha* (cf Remark 3.5 below), note that in the free-space propagation model (where $\beta = 2$) it is equal to 1 (which means that the interference distribution, and so coverage probability, in slotted and non-slotted Aloha are the same). Moreover, $\zeta(\beta)$ increases with the path-loss exponent and asymptotically (for $\beta = \infty$) approaches the value 2. This was only conjectured in [6].

C. Coverage Probability

Evaluating p_c from (2.5) is straightforward in the case of Rayleigh fading. Indeed, with F independent of $\bar{I}$ one has $\mathbf{P}^0\{\, F \geq l(r)T\bar{I}\,\} = \mathbf{E}^0[\exp\{-l(r)T\bar{I}\}] = \mathcal{L}_{\bar{I}}(l(r)T)$. By Facts 3.1 and 3.2 we have the following result.

Fact 3.4: *For the Aloha model with the path-loss function (2.1) and Rayleigh fading*

$$p_c = \exp\Big\{-\lambda\tau r^2 T^{2/\beta}\kappa\Big\}\,, \tag{3.7}$$

where

- $\kappa = 2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)/\beta$ *for slotted Aloha and*
- $\kappa = 4\pi\Gamma(2/\beta)\Gamma(1-2/\beta)/(2+\beta)$ *for non-slotted Aloha.*

Remark 3.5: Note that due to our parametrization $r = a/\sqrt{\lambda}$ (which mimics the nearest-neighbor receiver model), the maximal mean throughput per node τp_c is achieved (in slotted or non-slotted Aloha with Rayleigh fading) for $\tau = \tau^* = \kappa^{-1}a^{-2}T^{-2/\beta}$ and it is equal to τ^*/e. In particular, by Fact 3.2, non-slotted Aloha achieves $\zeta = \zeta(\beta)$ times smaller maximal throughput than slotted Aloha, where ζ is the cost of non-synchronization in Aloha. The dependence of this cost on β is analyzed in Remark 3.3. Here, note only that the well-known result obtained for the simplified collision model with on-off path-loss function, and saying that slotted Aloha offers two times greater throughput than non-slotted Aloha (see [4, Section 4.2]) corresponds in our model to the infinite path-loss exponent; $\zeta(\infty) = 2$.

In the case of a general distribution of fading the evaluation of p_c from the Laplace transform $\mathcal{L}_{\bar{I}}$ is not so straightforward. Some integral formula, based on the Plancherel-Parseval theorem, can be used when F has a square integrable density. This approach however does not apply to the no-fading case $F \equiv 1$. Here we suggest another, numerical approach, based on the Bromwich contour inversion integral and developed in [1], which is particularly efficient in this case.

Fact 3.6: *For Aloha model with constant fading $F \equiv 1$ we have*

$$p_c = \frac{2\exp\{d/(Tl(r))\}}{\pi}\int_0^\infty \mathcal{R}\Big(\frac{1-\mathcal{L}_{\bar{I}}(d+iu)}{d+iu}\Big)\cos ut\,du\,, \tag{3.8}$$

where $d > 0$ is an arbitrary constant and $\mathcal{R}(z)$ denotes the real part of the complex number z. As suggested in [1], the integral in (3.8) can be numerically evaluated using the trapezoidal rule and the Euler summation rule can be used to truncate the infinite series; the authors also explain how to set d in order to control the approximation error.

D. Carrier-Sense Scaling in CSMA

As mentioned above, a similar analysis of the performance of the CSMA scheme is not possible. If fact, neither the channel contention described by τ nor the distribution of $\bar{I}$ under $\mathbf{P}^0$ is easy to evaluate for this scheme. Here we want only to comment on some scaling results (with the node density λ) regarding the performance of CSMA.

Note that in the noiseless scenario (cf SIR condition (2.2)), with nearest-neighbor-like distance $r = a/\sqrt{\lambda}$ from transmitter to receiver, and the path-loss function (2.1) the SIR is invariant with respect to a homothetic transformation of the model; i.e., dilating all the distances by some factor, say γ. However, the received powers (as interference I' measured by the transmitters) scale like $\gamma^{-\beta}$. By the well known scaling property of the homogeneous Poisson p.p. [2], this implies that the *performance of the CSMA scheme* (values of τ and the distribution of $\bar{I}$) *in our network model is invariant with respect to the MANET density provided the carrier-sense threshold θ varies with λ as $\theta = \theta(\lambda) = \theta(1)\lambda^{\beta/2}$.*

In order to present our simulation results for CSMA in a scale-free manner, in Section IV-A we plot the mean throughput τp_c if the function of the *modified carrier-sense* threshold is $\tilde{\theta} := \theta l(r)$ that can be seen as the power normalized by the *received* signal power (in contrast to θ that is normalized to the *emitted* signal power). This results in $\tilde{\theta} = \theta(1)\lambda^{\beta/2}(A\lambda^{-1/2})^{\beta} = \theta(1)A^{\beta}$ which does not depend on the density of the MANET.

Another way of presentating scale-free results is to express the carrier-sense threshold θ in terms of the *equivalent carrier-sense distance* R defined as the distance at which a unit of emitted power is attenuated to the value θ, i.e. satisfying $\theta = 1/l(R)$. In our path-loss model this relation makes $R = \theta^{-1/\beta}/A$. We will use this approach when comparing our optimal tuning of CSMA to that proposed in [17]; see Section IV-C.

IV. MAC Optimization and Comparison Results

In this section we present our findings regarding analysis and comparison of the performance of Aloha and CSMA.

[2]The dilation of a planar Poisson p.p. of intensity 1 by a factor $\gamma = \lambda^{-1/2}$ gives a Poisson p.p. of intensity λ.

A. MAC Performance Study

We study the mean throughput per node τp_c achieved by CSMA and Aloha under our default setting ($a = 1$, $\beta = 4, T = 10$) with and without fading, depending on the MAC parameters, which are carrier-sense threshold θ, MAP p and mean back-off time ε for, respectively, CSMA, slotted and non-slotted Aloha.

1) CSMA: Figure 1 presents the throughput τp_c achieved by CSMA versus the modified carrier-sense threshold $\tilde{\theta}$. Recall that $\tilde{\theta}$ is the carrier-sense threshold in ratio to the useful power at the received (at the distance $r = a/\sqrt{\lambda}$) [3]. This makes τp_c and $\tilde{\theta}$ independent of the MANET density; cf. Section III-D. Our first observations are as follows.

Remark 4.1: In the absence of fading the maximum throughput of 0.068 (unit-size packets per unit of time and per node) is attained by CSMA when the carrier-sense threshold is fixed roughly at the level of $\tilde{\theta} = \tilde{\theta}^* = 0.08$. This optimal tuning of the carrier-sense threshold seems to be quite insensitive to fading. However, the optimal throughput is significantly reduced by fading. Rayleigh fading of mean 1, reduces the CSMA throughput to 63.2% compared with the no fading scenario.

This latter observation is easy to understand as the channel-sensing is done at the emitter and that fading at the receiver is independent of fading at the emitter.

2) Aloha: Figure 2 presents the throughput τp_c with and without fading achieved by slotted Aloha versus the channel occupation time τ, which in this model is equal to the the MAP parameter p. The results of non-slotted Aloha are presented in Figure 3 with $\tau = 1/(1 + \varepsilon)$, where ε is the mean back-off time. The other parameters are as in the default setting. Here are our observations.

Remark 4.2: In the absence of fading the maximum throughput of 0.028 for the optimal MAP $p = p^* \approx 0.06$. As in CSMA, this optimal tuning seems to be quite insensitive to fading, which in the case of Rayleigh fading can be evaluated explicitly as $p = p^* = \kappa^{-1}T^{-2/\beta}$ (which gives $p^* = 0.064081$ in the default Rayleigh scenario). In contrast to CSMA, Rayleigh fading has a relatively small impact on the slotted Aloha throughput reducing it only to 92% of the throughput achieved in the no-fading scenario (in contrast to 63.2% in CSMA). Similar observations hold

[3]In other words, e.g. $\tilde{\theta} = 0.1$ means that the channel is considered by an emitter as idle if the total power sensed by it is at most 10% of the mean useful signal power received by its receiver.

for non-slotted Aloha, which in the Rayleigh fading scenario achieves $\zeta = 2\beta/(2+\beta) = 1.5$ times smaller throughput than the slotted version.

B. Impact of Model Parameters

In Figures 4, 5, 6, 7, 8 and 9 we can study the dependence of the *maximal throughput* achievable by the MAC schemes (at their respective optimal tunings) as a function of the path-loss exponent β, SIR threshold T and relative distance to the receiver a (recall that $a = r\sqrt{\lambda}$). It is clear that CSMA significantly outperforms both Aloha protocols for all choices of parameters. More detailed observations are as follows.

Remark 4.3: The higher path-loss exponent β is, the less advantage there is in using CSMA. When there is no fading, the increase of β from 3 to 6 reduces the gain in throughput of CSMA with respect to slotted Aloha from 2.6 to 2.1 and with respect to non-slotted Aloha from 3.5 to 3.2.

We can also see in Figure 4, that in the absence of fading, slotted Aloha attains the expected fraction of 21% of terminals engaged in successful traffic, foreseen in the seminal paper [18], for SINR threshold $T = 10$ and a moderate path loss exponent slightly larger than $\beta = 3.5$.

Remark 4.4: The existence of fading (see Figure 5) further diminishes the advantage of CSMA. In particular, Rayleigh fading reduces the gain in throughput of CSMA with respect to slotted Aloha to about 1.7 and for non-slotted Aloha to a factor between 2.5 and 2.1 (depending on β).

Studying the impact of the SINR threshold T we observe the following, see Figures 6 and 7.

Remark 4.5: The higher T is (and hence the smaller bit-error rate sustainable in each packet), the greater is the advantage of using CSMA. In particular, when there is no fading and for $\beta = 4$, increasing T from 1 to 11 results in the increase in the gain in throughput of CSMA with respect to slotted Aloha from 2.4 to 3.5 and this latter ratio remains stable for T larger than 11. For a similar comparison of CSMA to non-slotted Aloha the gains are from 1.8 to 2.6. In the case of Rayleigh fading the analogous gain factors of CSMA are, respectively, from 1.8 to 2.4 with respect to slotted Aloha and from 1.4 to 1.8 with respect to non-slotted Aloha.

Finally we study the impact of the relative distance to the receiver a (in ratio to the mean distance to the nearest neighbor in the network). Figures 8, and 9 show clearly that this distance should be kept as small as possible without disconnecting the network.

C. Optimal Tuning of Aloha and CSMA

For Aloha the optimal tuning of τ can be obtained analytically from (3.7) whereas the optimal tuning of CSMA is obtained by simulation. In Figure 10, we present the optimal values of τ versus β with Rayleigh fading for both Aloha and CSMA.

Remark 4.6: We observe that the more sophisticated the MAC scheme is, the more it can content to the channel when the MAC is tuned optimally. Additionaly the more sophisticated MACs also exhibit higher capture probabilities. In particular our simulations show the that this probability is close to 1 (between 0.8 and 0.95) for CSMA.
A practical conclusion that can be drawn from these observations is that *the carrier-sense threshold in CSMA should be chosen at the largest possible value at which the allowed transmissions are almost always successful*.

D. Nelson&Kleinrock's Model of CSMA Revisited

Remark 4.6 might explain why the significant superiority of CSMA with respect to Aloha was not observed in [17]. Let us be more precise and revisit this model.

The simple propagation model in [17] assumes a fixed transmission range R and *the same* carrier-sense range. In other words, any two successfully communicating nodes need to be within distance R from each other and no other transmission should occur in the distance R from the receiver.

In this model, an *ideal medium access scheme* suggested in [17] should be able to choose from the given pattern of nodes centers for a maximal number of hard (non-intersecting) disks of radius R. The asymptotic analysis of the performance of such an ideal scheme is done in [17] assuming an increasing density of nodes λ. Namely, if this density is large, then the optimal scheme should be able to chose the pattern of nodes close to the hexagonal packing, known to obtain the densest packing of hard disks of radius R. Such a packing attains the fraction of 0.90689 of the plane covered by the union of disks. Consequently, since there is no disk overlapping, it would choose the fraction

$$\tau_{ideal} = \frac{0.90689 \times \text{network area}}{\text{\# of nodes} \times \text{exclusion disk surface area}} = \frac{0.90689}{\lambda \pi R^2}$$

of the nodes of the network, whose density is λ nodes per unit of surface. This expression can be interpreted as the contention parameter of this ideal medium access scheme, which explains our notation. Since all transmissions allowed by this scheme are successful, we have $p_c = 1$ for it and the achieved throughput per node is $0.90689/(\lambda\pi R^2)$.

Regarding CSMA, the simple propagation model with transmission range equal to the carrier-sense range, assumed in [17], corresponds to a choice of nodes such that any selected node is not covered by the transmission range of any other selected node. This task is equivalent to the packing of hard disks of radius $R/2$. For some reason, that is partially explained in that paper, a slightly larger radius $1.2881R/2$ is taken. Similar to the ideal scheme, asymptotic analysis of the hexagonal pattern, gives the contention parameter of this CSMA scheme equal to

$$\tau_{CSMA} = \frac{2.214}{\lambda\pi R^2}\,.$$

Moreover, assuming that each authorized node chooses its receiver uniformly within the transmission range R, and calculating the fraction of the area within this range that is not covered by any other disk (no collision), the successful transmission probability is calculated as $p_c = 0.2034$. Consequently the throughput achieved by this CSMA is $\tau p_c = 0.4504/(\lambda\pi R^2)$.

Note that apparently the *sub-optimal assumption of the carrier-sense range equal to the transmission range in the above model of CSMA leads to a relatively small successful transmission probability* $p_c = 0.2034$, close to that obtained by Aloha, which explains why there is no essential difference between the performance of these two schemes. Our optimally tuned CSMA model seems to be closer to the ideal scheme of [17], at least because the probability of successful transmission is much closer to 1.

Let us now try to compare the performance of our optimal CSMA and the two schemes of [17]. This is not straightforward, since unlike ours, the results of [17] scale in $1/\lambda$ are only valid asymptotically, when $\lambda \to \infty$ (due to the hexagonal approximation of the perfect packing). However, note that in the model of [17], the expression $N = \lambda\pi R^2$ corresponds to the expected number of nodes within the area contended (blocked) by one given authorized transmission. Consequently the constants $\rho = 0.90689 = \tau p_c N$ and $\rho = 0.4504 = \tau p_c N$ can be interpreted, respectively in the two models, as the expected number of successful transmissions per set of nodes contended (blocked) by one given authorized transmission. This kind of spatial efficiency can be evaluated in our model using the notion of the equivalent carrier-sense distance $R = \theta^{-1/\beta}/A = r\tilde{\theta}^{-1/\beta}$ introduced in Section III-D. Taking $N = \lambda\pi R^2$ with R calculated as such we obtain for our CSMA $\rho = \tau p_c N = \tau p_c \lambda \pi r^2 \tilde{\theta}^{-2/\beta}$. For the optimally tuned CSMA in the standard scenario $a = 1, T = 10, \beta = 4$ without fading we have $\rho = 0.07\pi 0.08^{-1/2} = 0.77750$ of successful transmissions per set of nodes contended (blocked) due to one given authorized transmission. This is a much better performance than $\rho = 0.4504$ for CSMA of [17], and in fact closer to $\rho = 0.90689$ achieved by the ideal scheme of [17].

E. Corrective terms

In this article we have not considered the effect of the back-off for CSMA, and for slotted Aloha we have ignored the guard times to avoid overlapping of the slots. In this sub-section we briefly study the effects of these parameters on performance. Let us call δ the ratio of maximum propagation time plus detection time in the network over the packet transmission time. Back-off in CSMA leads to wasting time and to collisions for nodes starting their transmissions within the same mini-slot of size $\delta\times$ packet transmission time. We know that the reduction of the throughput for CSMA is $\frac{1}{1+\sqrt{2\delta}}$ when the back-off is properly tuned, see [4] chapter 4. For slotted Aloha and with the same assumptions, the guard times lead to a reduction of approximately $\frac{1}{1+\delta}$. Thus for $\delta = 0.05, 0.02$ and 0.01 we obtain a throughput reduction of respectively $0.76, 0.83, 0.87$ for CSMA and $0.95, 0.98, 0.99$ for slotted Aloha. Thus, the throughput reduction is greater for CSMA than for slotted Aloha but these corrective terms do not change our main observation which gives a notably higher throughput to CSMA.

V. Conclusions

In this paper we compare slotted and non-slotted Aloha with CSMA in a Poisson ad-hoc network setting with SINR-based capture condition. We assume the usual power-law path-loss function and both Rayleigh and no-fading scenarios. To obtain a fair comparison between these protocols, their parameters are tuned to achieve the maximum successful transmission rates. Our analysis shows that CSMA always outperforms both slotted and non-slotted Aloha. However the gain obtained when using CSMA is slightly reduced by increasing path loss and more significantly by the existence of fading. We also show how to tune the carrier-sense threshold in CSMA so as to obtain its optimal throughput for an arbitrary network density. Our models concur with those of [17] even though some results may appear, at first glance, to be somewhat contradictory, because in [17] CSMA is not optimized.

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FIGURES

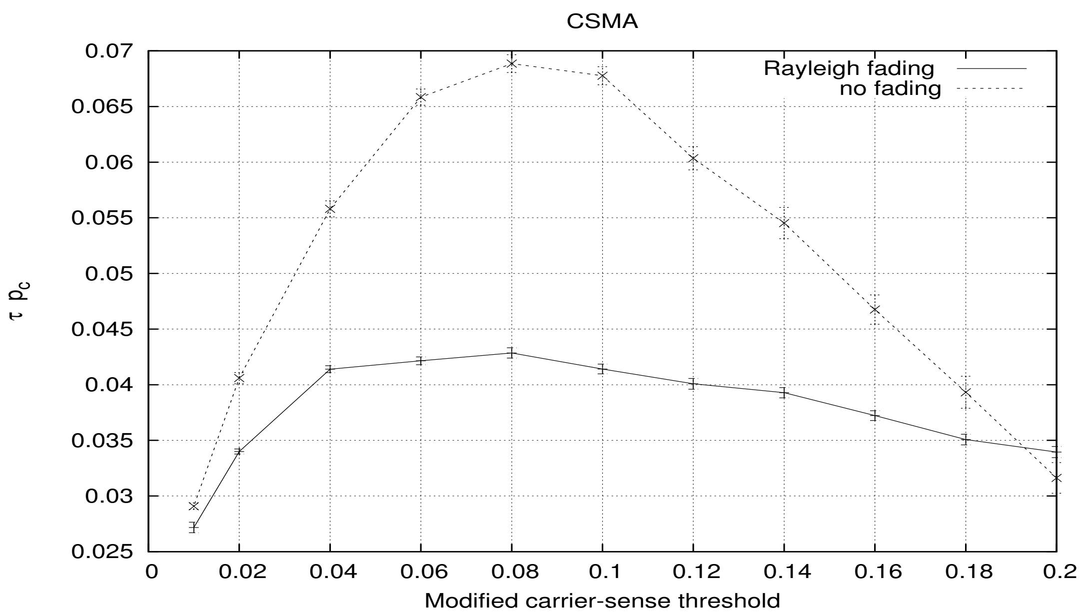

Fig. 1. Mean throughput per node τp_c versus modified carrier-sense threshold $\tilde{\theta}$ in CSMA; Rayleigh fading and no-fading scenario.

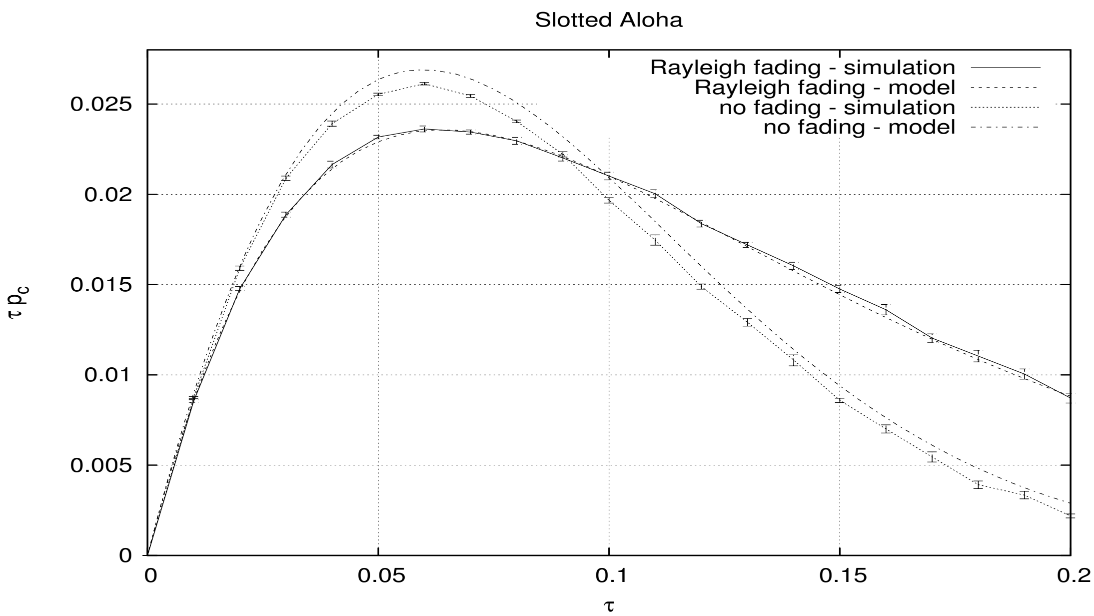

Fig. 2. Mean throughput per node τp_c versus channel occupation $\tau = p$ in slotted Aloha; Rayleigh fading and no fading scenario.

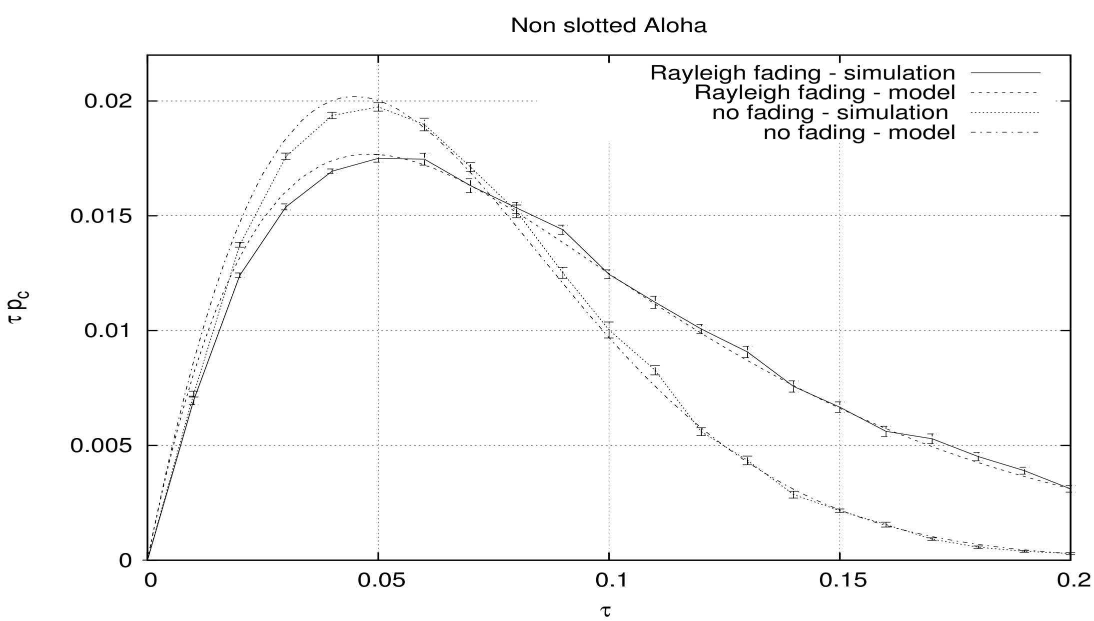

Fig. 3. Mean throughput per node τp_c versus channel occupation time $\tau = 1/(1+\epsilon)$ for non-slotted Aloha; Rayleigh fading and no fading scenario.

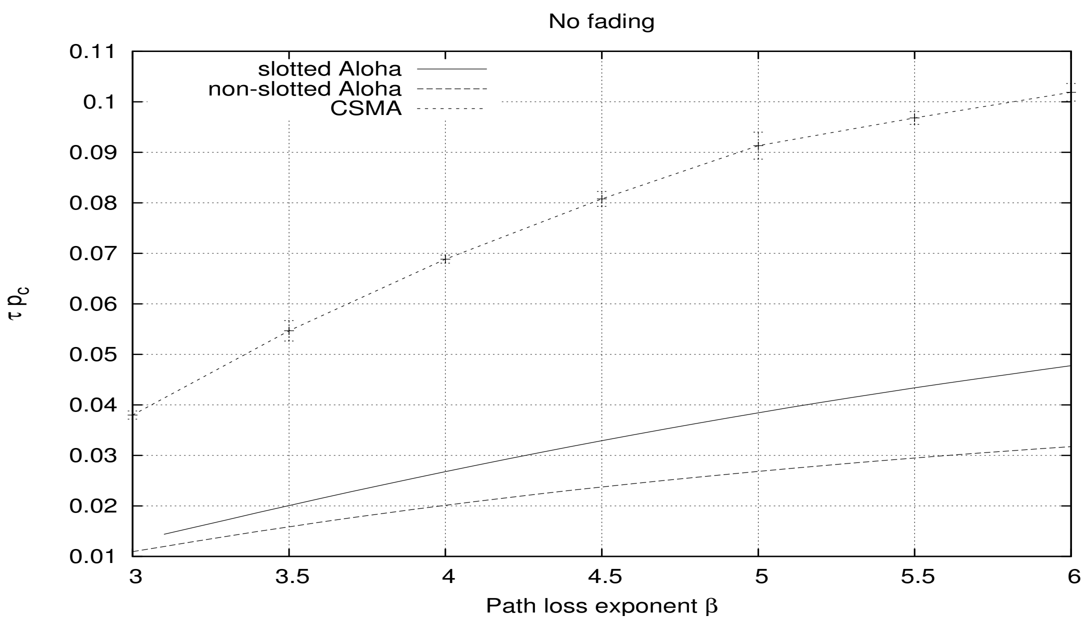

Fig. 4. Maximal achievable mean throughput per node τp_c versus path-loss exponent β in the absence of fading.

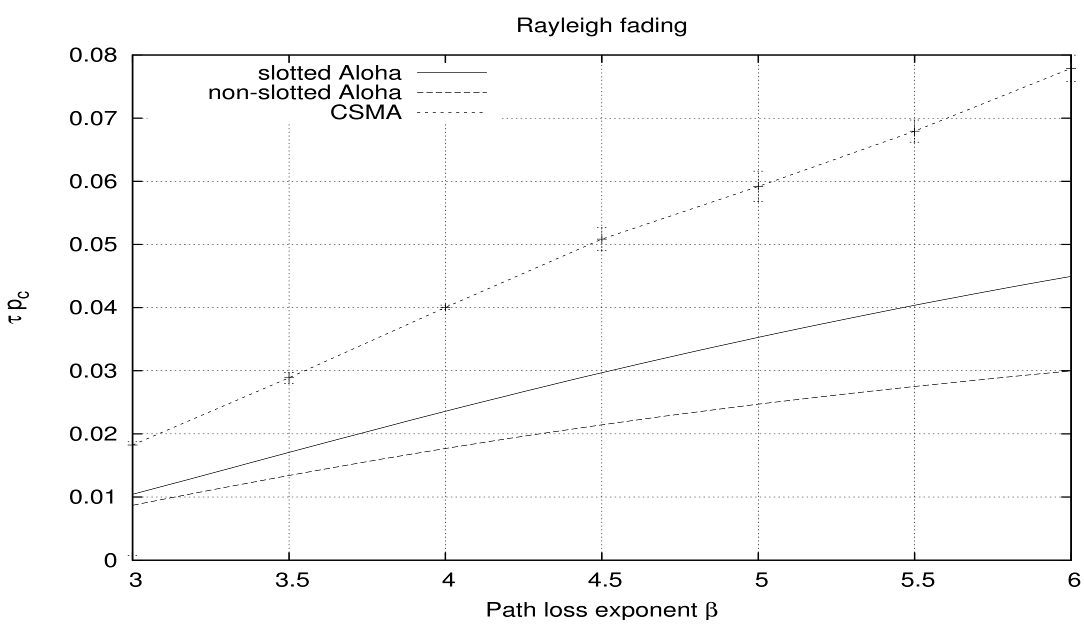

Fig. 5. Maximal achievable mean throughput per node τp_c versus path-loss exponent β with Rayleigh fading.

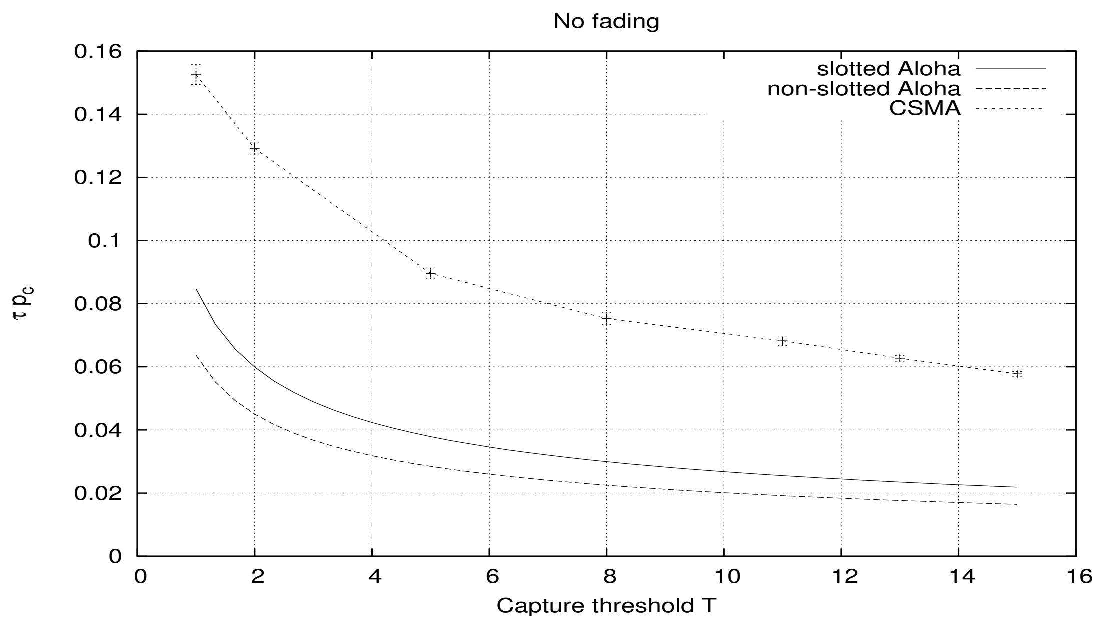

Fig. 6. Maximal achievable mean throughput per node τp_c versus SIR threshold T in the absence of fading.

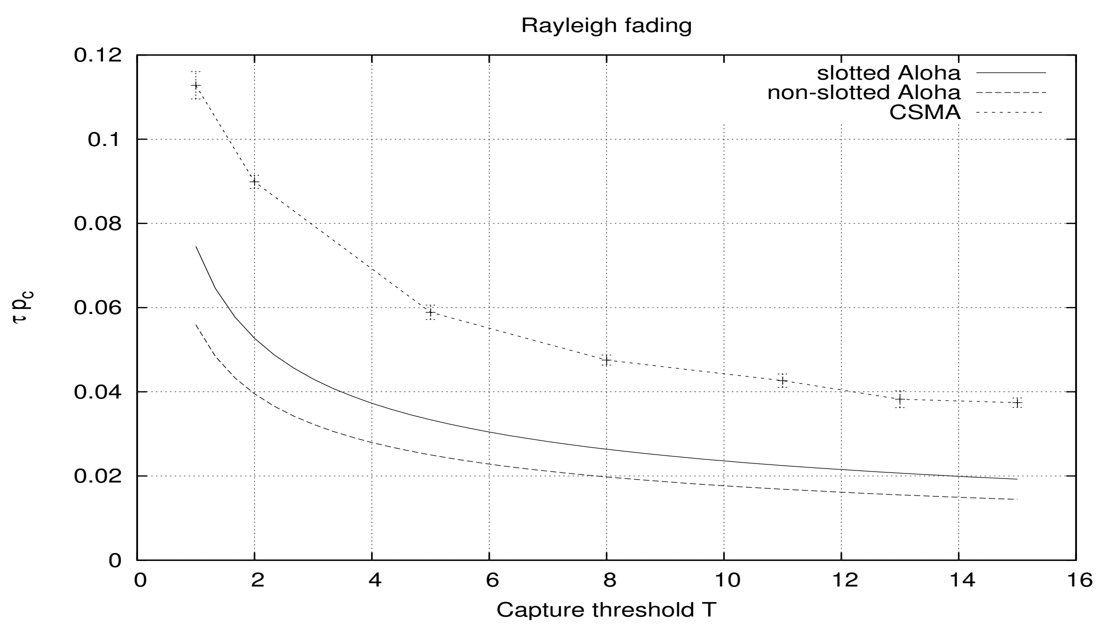

Fig. 7. Maximal achievable mean throughput per node τp_c versus SIR threshold T. Rayleigh fading.

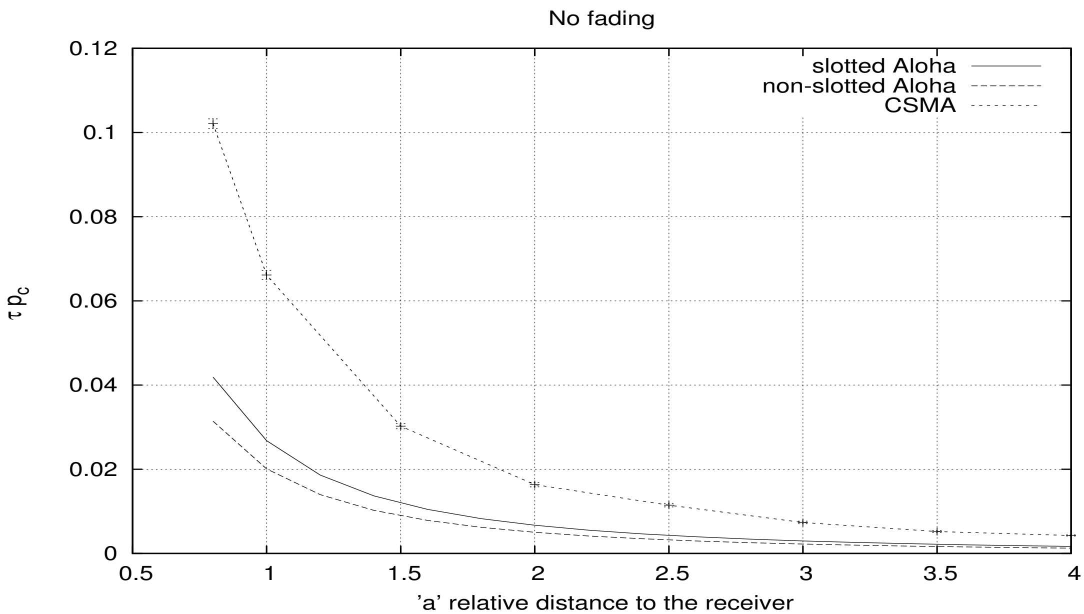

Fig. 8. Maximal achievable mean throughput per node τp_c versus a the distance transmitter receiver. No fading.

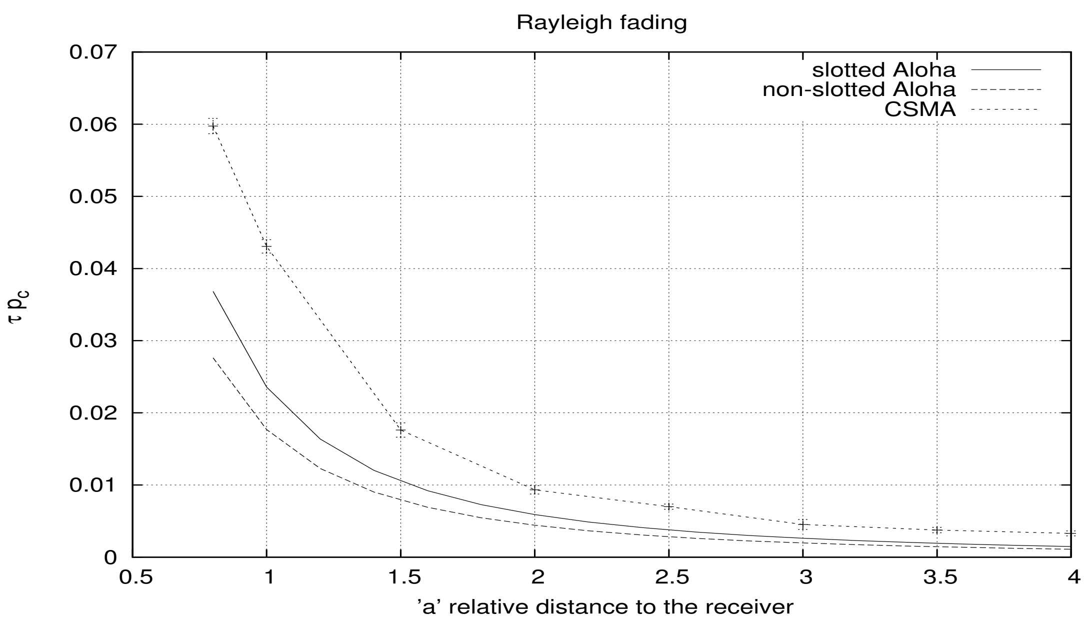

Fig. 9. Maximal achievable mean throughput per node τp_c versus a the distance transmitter receiver. Rayleigh fading.

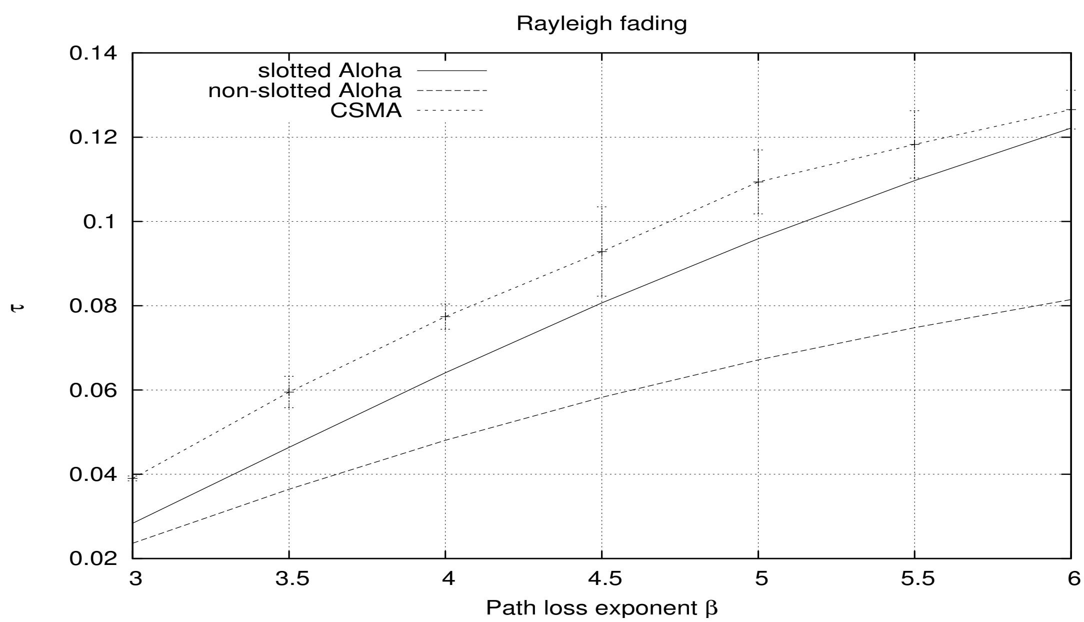

Fig. 10. Optimal value of τ for Aloha and CSMA versus β with Rayleigh fading