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To cite this version:

Rinaldo M. Colombo, Paola Goatin, Giulio Maternini, Massimiliano D Rosini. Macroscopic Models for Pedestrian Flows. Big events and transport : the transportation requirements for the management of large scale events, Oct 2010, Venice, Italy. 2010. <inria-00534882>

HAL Id: inria-00534882

https://hal.inria.fr/inria-00534882

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Macroscopic Models for Pedestrian Flows

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Abstract

In this paper we present macroscopic models for pedestrian flows that recently appeared in the literature. The first one was proposed by Colombo, Rosini, 2005. In a 1D setting, this model properly describes the movements of pedestrians, the onset of panic and the dynamics of a panicking crowd. Furthermore, its assumptions were experimentally confirmed by an empirical study of a crowd crush, see Helbing, Johansson, Al-Abideen, 2007. Then, we consider a 2D model that aims at describing similar phenomena while taking care of more complex geometries. Numerical integrations show that some realistic features are captured.

Keywords: Crowd Modeling, Macroscopic Pedestrian Flow models, Conservation Laws

Introduction

The modeling and analysis of traffic phenomena can be performed at different scales. We can distinguish between the following main approaches: continuum (or macroscopic), kinetic, microscopic and cellular automata models. Differently from microscopic or cellular automata models, continuum models aim at minimizing the number of equations and parameters, while capturing the essence of various features of traffic evolution.

Since the introduction of the classical LWR model, see Lighthill, Whitham, 1955, and Richards, 1956, continuum modeling of traffic phenomena has captured the attention of engineers, architects, physicists and mathematicians. *"Traffic"* is here understood to comprehend both vehicular and pedestrian traffic. In particular, we present a nonclassical conservation law describing people escaping from a room. This model, first presented in Colombo, Rosini, 2005, relies on a particular fundamental diagram (i.e. the flow-density relation) and a suitable nonclassical evolution. The former was recently experimentally observed by Helbing, Johansson, Al-Abideen, 2007. The latter contributes to a more realistic description of near-to-panic situations.

This framework is able to capture relevant features of the outflow of people thorough an exit in standard and up to emergency situations. In fact, pedestrians evacuating a closed space accumulate near to door exits. The rise of panic may create a dramatic fall in the overall people outflow. Effective methods often adopted to speed up the evacuation of a large room are in some senses nonintuitive. For instance, the evacuation time may be reduced by carefully inserting a well designed *"obstacle"*, such as a second door or a tall column, at a suitable location before the exit. This obstacle reduces the interpedestrian pressure in front of the door exit, decreases the magnitude of clogging, makes the overall outflow higher and more regular, see Colombo, Rosini, 2009 and Colombo, Facchi, Maternini, Rosini, 2009. This can be interpreted as a sort of Braess' paradox for pedestrian flows, see Braess, 1968. Indeed, at very small crowd densities, these *"obstacles"* do hinder the crowd outflow. Nevertheless, at high densities, their role may turn out to be essential in lowering the evacuation time.

The optimal management problem concerning the choice of the best position of such an obstacle is a crucial issue. Numerical integrations of the present models, both in the 1D and in the 2D setting, allow to compute the evacuation time in various conditions and to compare the results obtained in the two cases: with an obstacle or without it. At the designing level, these tools may thus help in choosing among various architectural solutions. Moreover, macroscopic models also allow for a rigorous analytical treatment of the optimal positioning of these obstacles.

Hopefully, modern developments may help prevent some of the approximately two thousand deaths that annually occur in accidents owing to, or related to, crowding. In Table 1 we give a non-exhaustive survey of severe crowd accidents. Often, lower evacuation times cold have helped preventing or, at last diminishing the effects of these accidents.

A possible benchmarking of crowd dynamics models may be based on the answers that different model give to the following questions:

Panic: When, where, how and why does panic arise?

Clog doors: When, how and why does the efficiency of the exit fall down?
Braess' paradox: When, how and why is the obstacle helpful in the evacuation?

Table 1: The main crowd accidents occurred in the recent years in the world

YEAR	Casualties	CITY	NATION	YEAR	Casualties	CITY	NATION
1872	19	Ostrów	Poland	1999	53	Minsk	Belarus
1876	278	Brooklyn	USA	2001	43	Henderson	USA
1883	12	Brooklyn	USA	2001	126	Accra	Ghana
1883	180	Sunderland	England	2001	7	Sofia	Bulgaria
1896	1, 389	Moscow	Russia	2003	21	Chicago	USA
1908	16	Barnsley	England	2003	100	West Warwick	USA
1913	73	Michigan	USA	2004	37	Beijing	China
1943	173	London	England	2004	251	Mecca	Saudi Arabia
1956	124	Yahiko	Japan	2005	265	Maharashtra	India
1971	66	Glasgow	England	2005	1,000	Baghdad	Iraq
1979	11	Cincinnati	USA	2006	345	Mecca	Saudi Arabia
1982	66	Luzhniki	Russia	2006	74	Pasig City	Philippines
1985	39	Brussels	Belgium	2006	51	Ibb	Yemen
1989	96	Sheffield	England	2007	12	Chililabombwe	Zambia
1991	42	Chalma	Mexico	2008	12	Mexico City	Mexico
1991	40	Orkney	South Africa	2008	162	Himachal Pradesh	India
1993	21	Hong Kong	Cina	2008	147	Jodhpur	India
1993	73	Madison	USA	2008	8	Karila	India
1994	270	Mecca	Saudi Arabia	2008	20	Tabora	Tanzania
1996	82	Guatemala City	Guatemala	2009	19	Abidjan	Côte d'Ivoire
1998	70	Kathmandu	Nepal	2010	71	Kunda	India
1998	119	Mecca	Saudi Arabia	2010	21	Duisburg	Germany

Source: http://en.wikipedia.org/wiki/Stampede

The models presented below, in particular the 1D one, do provide answers to the questions above. We stress that these results are obtained in a continuum setting. This means that the key quantity describing the crowd is a density, i.e. a function assigning to time t and to a space coordinate x the quantity $\rho(t,x)$ such that the total number of pedestrians that at time t are in a region A is $\int_A \rho(t,x)\,dx$, see Cristiani et al., 2010. In particular, no proper volume for pedestrians is introduced.

A 1D macroscopic model describing pedestrian flow

We want to describe the evacuation of pedestrians from a narrow corridor or a bridge, mathematically represented by the interval [0,D], see Fig. 1. We assume that the escaping pedestrians have to pass through an "exit door" sited at D. Before reaching it, they have to go through an "obstacle" at d, whose role is to regulate the evacuation process in the sense that will be explained later.

Obviously, the total number of pedestrians is conserved. We also assume that the average velocity v of the pedestrians at time t and location x is a function of the crowd density $\rho(t, x)$, namely $v = v(\rho)$, so that the crowd flow is $f(\rho) = \rho\ v(\rho)$. Then, we are led to the conservation law

$$\partial_t \rho + \partial_x f(\rho) = 0 \qquad (1)$$

analogous to the classical LWR model.

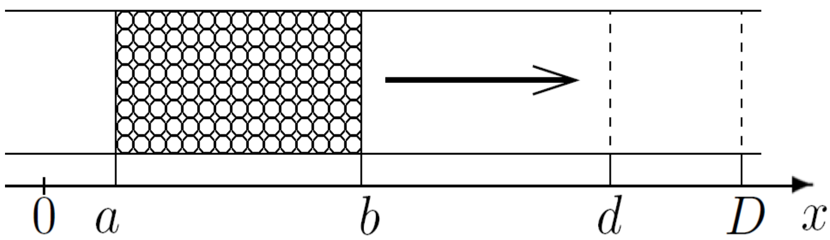


Fig. 1: Evacuation of a corridor [0,D] through the exit door at D. An obstacle at d regulates the crowd flow. At the initial time pedestrians are assumed to be uniformly distributed along the segment [a,b].

One might be now lead to force pedestrian flow to follow the same description provided in the case of vehicular traffic by the classical LWR model. This would amount first to introduce also for pedestrians a speed law and a fundamental diagram, roughly speaking, such as those in Fig. 2. Then, the standard classical definitions of entropy solutions could be applied, see Bressan, 2000. However, the resulting model would not be able to capture relevant patterns that are typical of crowd dynamics and that are not present in vehicular traffic. In particular, the resulting description of the behavior of pedestrians in panic situations would be hardly acceptable. More than that, the very definition of panic would be difficult.

From the analytical point of view, we stress that classical solutions to (1) satisfy the maximum principle, see Lefloch, 2002, Chapter IV, Theorem 2.1(a). This elementary analytical result prevents any increase in the maximal density, in contrast with a realistic description of panic, where a sort of overcompression arises in panic situation and is often a cause of major accidents.

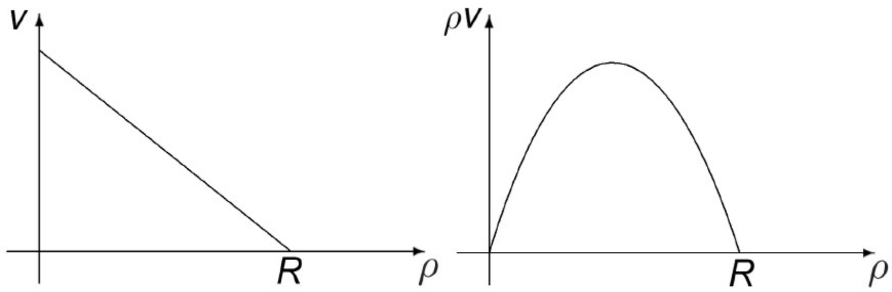


Fig. 2: Qualitative behaviour of, left, the speed law and, right, the fundamental diagram typically used in the classical LWR model for vehicular traffic.

The model proposed by Colombo and Rosini in 2005, relies on an extension of the interval of the possible crowd densities: beyond the interval [0,R] of the standard densities they introduced the panic states corresponding to densities in the interval $]R,R^*]$. Therefore, the speed law and the fundamental diagram proposed in Colombo, Rosini, 2005, are those here displayed in Fig. 3.

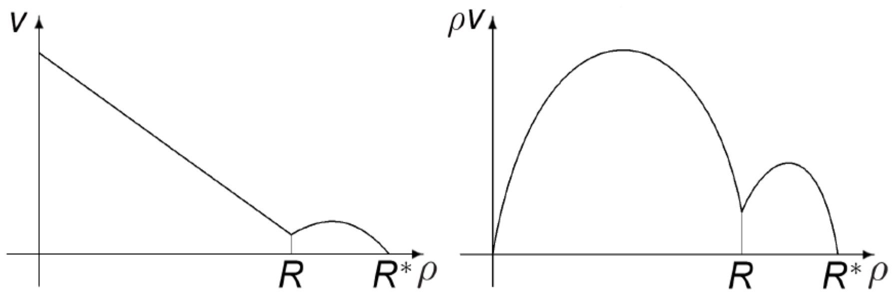


Fig. 3: Qualitative behaviour of, left, the speed law and, right, the fundamental diagram introduced in Colombo, Rosini, 2005, to describe pedestrian flows. Note the density interval $]R,R^*]$: it contains the panic states.

We point out that the shape of the fundamental diagram showed in Fig. 3 matches experimental measurements of a crowd crush, see Helbing, Johansson, Al-Abideen, 2007 and Fig. 4.

However, to avoid the implications of the maximum principle, which prevents standard solutions to grow above the maximal value of the initial datum, also the very definition of solution needs to be suitably modified. The introduction of nonclassical (or *non-entropic*) shocks allows the sharp increasing of the crowd density and the appearance of panic states. We refer to Colombo, Rosini, 2005 and Colombo, Rosini, 2009, for the detailed definitions and the analytical results. An example of how panic can arise is described in the following section. Roughly speaking, the presence of a sufficiently large density jump between a small density upstream and a density close to R downstream create the condition for panic emergence.

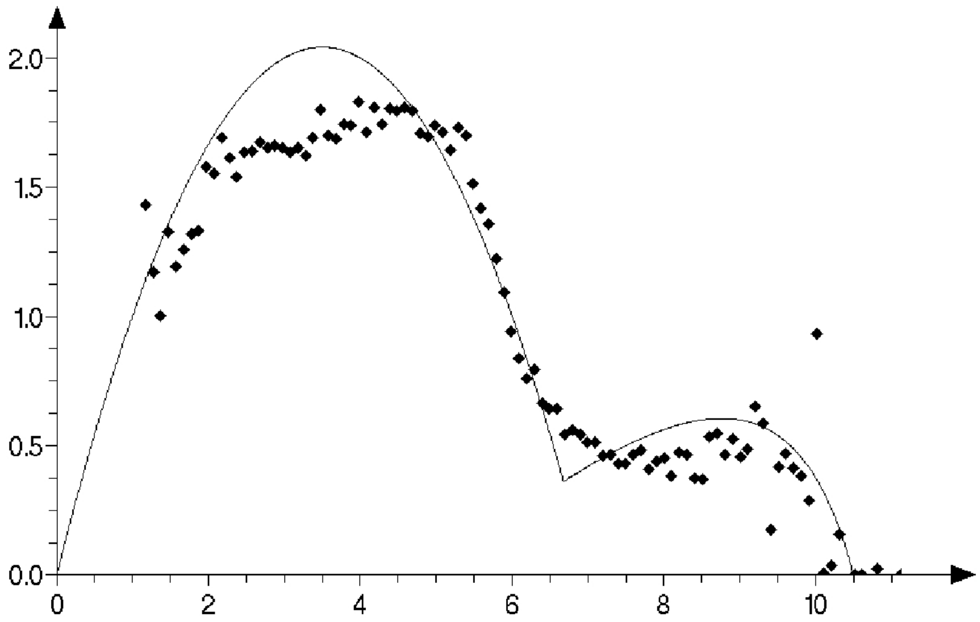


Fig. 4: Fundamental diagram as postulated in Colombo, Rosini, 2005. The superimposed dots are the experimental measurements from Helbing, Johannson, Al-Abideen, 2007.

Assume that, at the initial time $t = 0$, the crowd is uniformly distributed in $[a,b] \subset [0,D]$, with a relatively high density $\rho \in [0,R]$. Consider two doors in d and D, $b < d \leq D$, with maximal loads p, P : $[0,R^*] \to [0,\max f]$, respectively. High densities at the doors affect on their efficiency, and therefore proper choices of the functions p and P are $p = p_1\, \chi_{[0,R]} + p_2\, \chi_{]R,R^*]}$ and $P = P_1\, \chi_{[0,R]} + P_2\, \chi_{]R,R^*]}$, with $P_2 < p_2 < P_1 < p_1$. Roughly speaking, the first door is larger than the second one, $p > P$, and the high densities of people close to the doors clog them, $p_1 > p_2$ and $P_1 > P_2$. Mathematically, this situation is described by the following Cauchy problem with constraints.

$$\begin{aligned} &\partial_t\rho + \partial_x f(\rho) = 0 && \text{for } t\geq 0,\ x\in[0,D] \\ &\rho(0,x) = \rho\,\chi_{[a,b]}(x) && \text{for } x\in[0,D] \\ &f(\rho(t,d))\leq p(\rho(t,d)) && \text{for } t>0 \\ &f(\rho(t,D))\leq P(\rho(t,D)) && \text{for } t>0 \end{aligned} \qquad (2)$$

By applying the wave front tracking method, and solving a certain number of Riemann problems, it is possible to construct a solution to (2), see Fig. 5.

If there is no door at x=d, the situation is represented in Fig. 5, right. At time t=0, the crowd starts moving towards the exit sited at D. At t=A the first pedestrians reach the door. The outflow at D increases up to C, when the maximal outflow through the door is reached. Then, a queue is formed, corresponding to the (classical) shock S_2. Between t=C and t=F the exit works at its maximum efficiency. At t=K, the backward moving shock S_2 causes panic to arise. Indeed, at that point, pedestrians behind the shock meet a sharp sudden increase in the density. The nonclassical shock N_1 arises and all the dark shaded area above N_1 is filled with crowd in panic states. At t=F the overcompressed states reach the exit and the efficiency of the door is reduced.

The effect of the second door in x=d is represented in Fig. 5, left. As before, at time t=0, pedestrians start moving rightwards and the first one exit the door D at t=A. At t=C, a backward moving shock S_2 appears. However, now, at t=L the maximal through flow of the door at d is reached. Therefore, a second classical backward moving shock S_5 is formed at t=L. As a consequence, S_2 may not cause the insurgence of panic states. The exit at D works at its maximum efficiency all along the segment between t=C and t=R, resulting in a lower evacuation time.

The detailed construction of these solutions can be found in Colombo, Rosini, 2009, section 4.2.

We observe that, in this particular situation, the evacuation time without the first door is larger than the evacuation time with the first door. The presence of the obstacle avoids the density to reach these high values, thus allowing for a faster evacuation of the corridor.
Moreover, changing the position of the first door, namely, letting varying d in [b,D], we obtain the graph for the evacuation time represented in Fig. 6. Remarkably, there is an interval of values of d such that the presence of the first door helps for the evacuation time, showing that the model properly describe the Braess' paradox for pedestrian flows. Furthermore, it is also clear that the presence of a first door too close to the second one does not have any effect on the evacuation time.

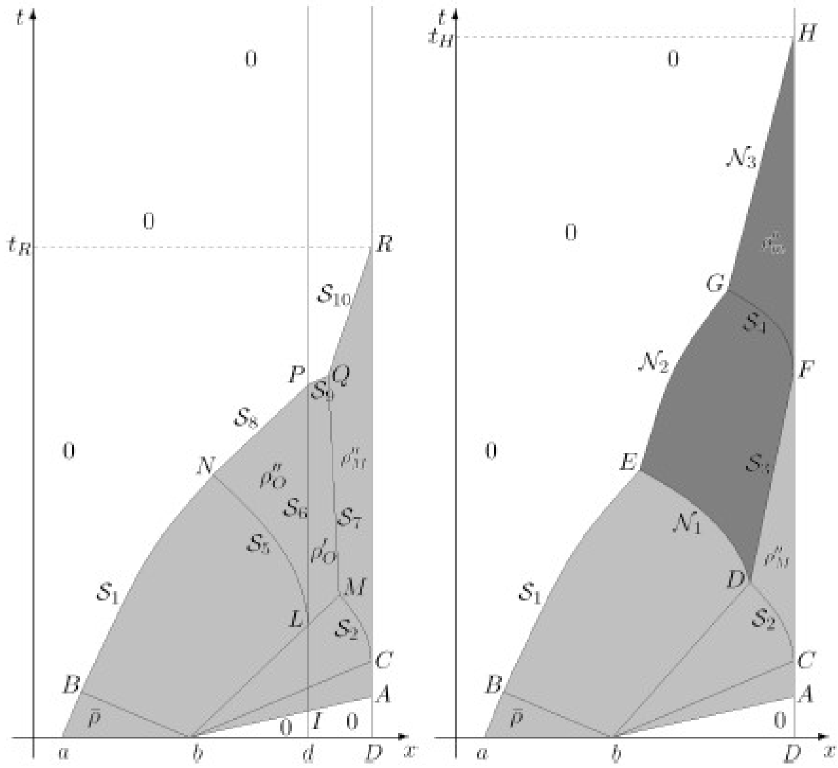


Fig. 5: The solution to (9): left, of the whole system and, right, neglecting the third equation, i.e. without the obstacle at *d*. Note that the evacuation time t_R on the left is smaller than the analogous time t_H on the right.

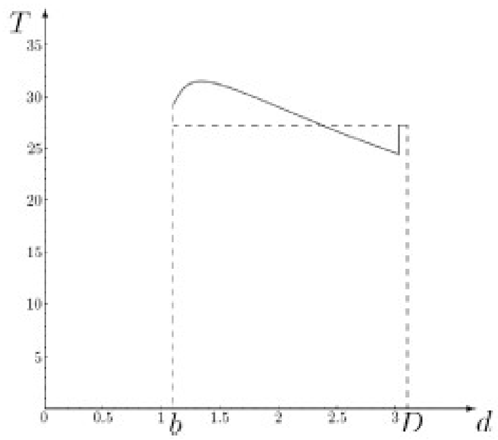


Fig. 6: The horizontal dotted line is the evacuation time without the obstacle at *d*, see Figure 5, right. The solid line is the evacuation time as a function of the position of the first door, *d*, see Figure 5, left.

A 2D dynamic continuum pedestrian flow model

We consider here a 2D macroscopic model introduced by Xia, Wong, Shu, 2009. In this case, the pedestrian flow is governed by the conservation law for the mass variable

$$\partial_t \rho(\mathbf{x},t) + \nabla\cdot\mathbf{f}(\mathbf{x},t) = 0 \qquad \text{for } t\geq 0,\ \mathbf{x}\in\Omega, \tag{3}$$

where x=(x,y) is the space coordinate and $\nabla\cdot\mathbf{f}(\mathbf{x},t) = \partial_x f_1(\mathbf{x},t) + \partial_y f_2(\mathbf{x},t)$ is the usual divergence. Ω represents the two-dimensional walking facility whose boundary is given by $\partial\Omega=\Gamma_0\cup\Gamma_w$, Γ_0 denoting the exit and Γ_w the solid wall boundary.

The pedestrian flow is defined in its magnitude and direction by the following relations:

$$\|\mathbf{f}(\mathbf{x},t)\| = \rho(\mathbf{x},t)\ v(\rho(\mathbf{x},t)),$$
$$\mathbf{f}(\mathbf{x},t)\ //\ (\ -\nabla\phi(\mathbf{x})-\omega\nabla c(\rho(\mathbf{x},t))\ ),$$

where $v=v(\rho)$ denotes the magnitude of the mean pedestrian velocity. The former relation assigns the modulus of the flow, while the latter defines its direction. The speed $v=v(\rho)$ is assumed to be a function of the form

$$v(\rho) = v_{max}\,(1-\rho/\rho_{max}).$$

The flow direction is governed by the minimum travel cost ϕ from point $\mathbf{x}$ to the destination (exit) Γ_0 . This cost ϕ is computed as a solution to the Eikonal equation

$$\begin{aligned} \|\nabla\phi(\mathbf{x})\| &= 1/v_{max}, && \mathbf{x}\in\Omega, \\ \phi(\mathbf{x}) &= 0, && \mathbf{x}\in\Gamma_0\,. \end{aligned} \tag{4}$$

The function $c=c(\rho)= 1/v(\rho) + \beta\rho^2$, for a suitable $\beta>0$, defines the local travel cost due to pedestrian density. Essentially, the model assumes that pedestrians seek to minimize their travel cost based on their knowledge of the location of the exit, but are ready to modify their trajectories in order to avoid high densities.

The following numerical simulations have been carried out using the fast sweeping method (described in Zhao, 2004) to solve the Eikonal equation (4) and the classical Lax-Friedrichs scheme to integrate the conservation law (3). We considered a group of people exiting a room with and without an obstacle in front of the door (see Fig. 7).

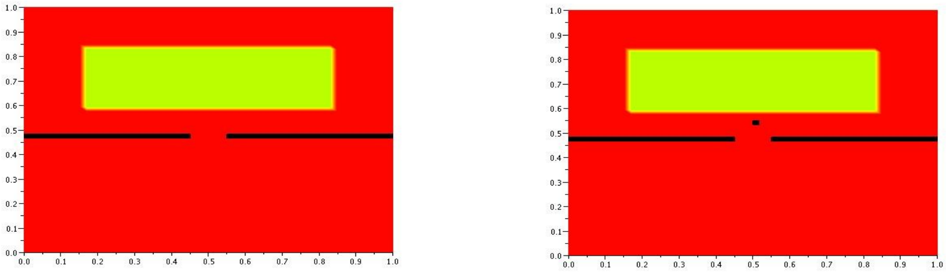
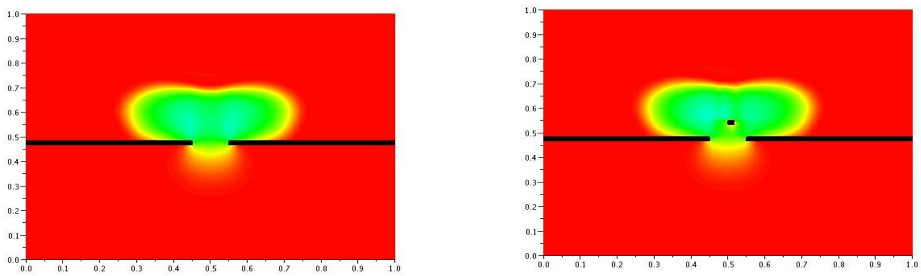
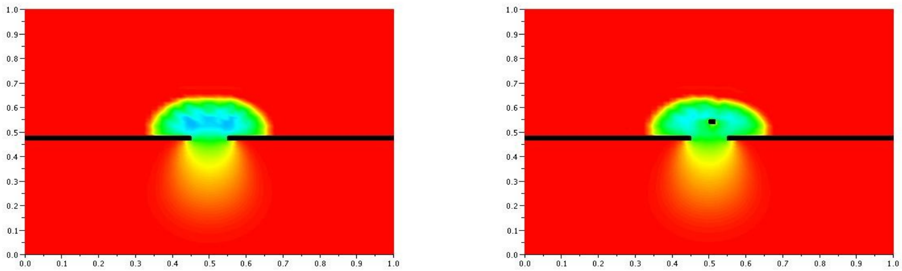


Fig. 7: Initial situation at time t=0 considered in the numerical simulations. Left, exit without obstacle and, right, exit with obstacle. The uniform initial crowd density of people is set to ρ_0=1.

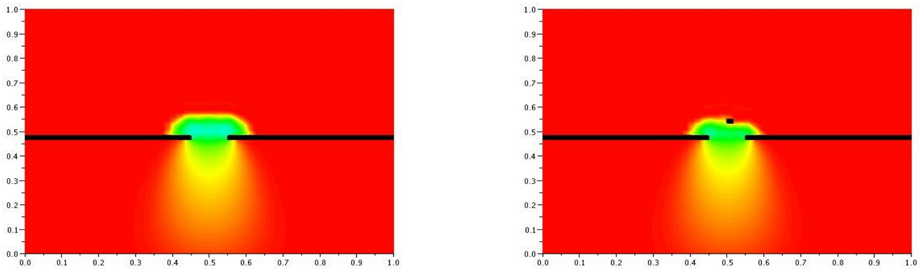
Again, without any assumption whatsoever on the proper volume of pedestrians, Fig. 8 shows that the presence of the obstacle avoids high concentrations and diminishes the exit time, thus confirming the Braess' paradox. The parameters chosen in this integration are v_{max}=0.5, ρ_{max}=4, ω=0.4 and β=0.2.



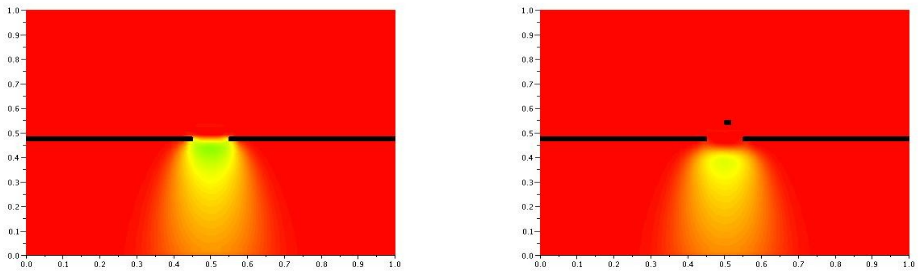
t=0.5



t=0.9



t=1.3



t=1.7

Fig. 8: Numerical simulations of a crowd exiting a room. Left, without obstacle and, right, with an obstacle in front of the door. This experiment confirms that there are situations in which the presence of the obstacle *diminishes* the evacuation time. This is obtained avoiding high inter-pedestrian pressure or, equivalently, high crowd densities.

Conclusions

We presented two macroscopic crowd dymanics models and some of their analytical properties. These models do capture some relevant features of pedestrian flows, in particular in the case of the evacuation of a closed space, e.g. a corridor or a room. Numerical integrations are possible, allowing a detailed description of the phenomena. Furthermore, the time necessary for people to exit the corridor can be computed.

Reasonable qualitative behaviors of the solutions are described. In particular, the models presented account for the possible decrease in the evacuation time thanks to the careful insertion of an obstacle at a well chosen position in front of the exit. This phenomenon, an analog of Braess' paradox, see Braess, 1968, is clearly non generic.

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