Projective Reconstruction from Matching Tensors

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MATCHING TENSORS + CLOSURE RELATIONS $\longrightarrow$ PROJECTION MATRICES $\longrightarrow$ SCENE STRUCTURE

Joint Image Closure Relations

- Bilinear relations between ***projection matrices*** & ***matching tensors***.

Closure-based Multi-image Projective Reconstruction

⊕ Works directly from matching tensors

⊕ No need to track tokens through all images

⊕ All images equal, built in redundancy

⊕ Linear – no initialization required

⊖ Coherent scaling required for matching tensors

Stability: factorization $>$ closure $>$ "2 image" method

Previous Work

"2 image" methods

- Reconstruct from 2 images, reproject for others
- Coordinate frame $+$ algebra $\to$ closed form solution
- Hartley: $\mathbf{P}_1 \to (\mathbf{I}|0), \ldots$
- Faugeras: choose 5 points as projective basis,...
- Simple, but ***not very stable***

Projective factorization

- ***Depth coherence relations*** $\Rightarrow$ ***projective depths***
- Depths give ***implicit projective reconstruction***

— concretize by ***matrix factorization*** (SVD,...)

- Simple & very stable
- BUT ***all points must be visible in all images!***

Joint Image Picture

Basic Reconstruction System

- Image projection: $\lambda_i \mathbf{x}_i = \mathbf{P}_i \mathbf{X} \;\Rightarrow$

$$\begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} \mathbf{X} = \lambda_1 \begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{0} \end{pmatrix} + \ldots + \lambda_m \begin{pmatrix} \mathbf{0} \\ \vdots \\ \mathbf{x}_m \end{pmatrix}$$

Joint Image

- Span of columns of ***Joint Projection***
- Faithful replica of world in image coordinates
- Trivial image projections
- ***Projective depths*** λ_i $\approx$ Reconstruction
- ***Joint projection*** columns $\approx$ Basis

Matching constraints

- Conditions for solution of ***basic reconstruction system***
- ***3 types*** for 2,3,4 images
- Bi-, tri-, quadrilinear in ***image points***

Matching tensors

- Coefficient tensors of ***matching constraints***
- Epipole, fundamental matrix, tri- & quadrivalent tensors
- Algebraic encoding of ***location of joint image***
- Components are 4×4 ***minors*** of joint projection

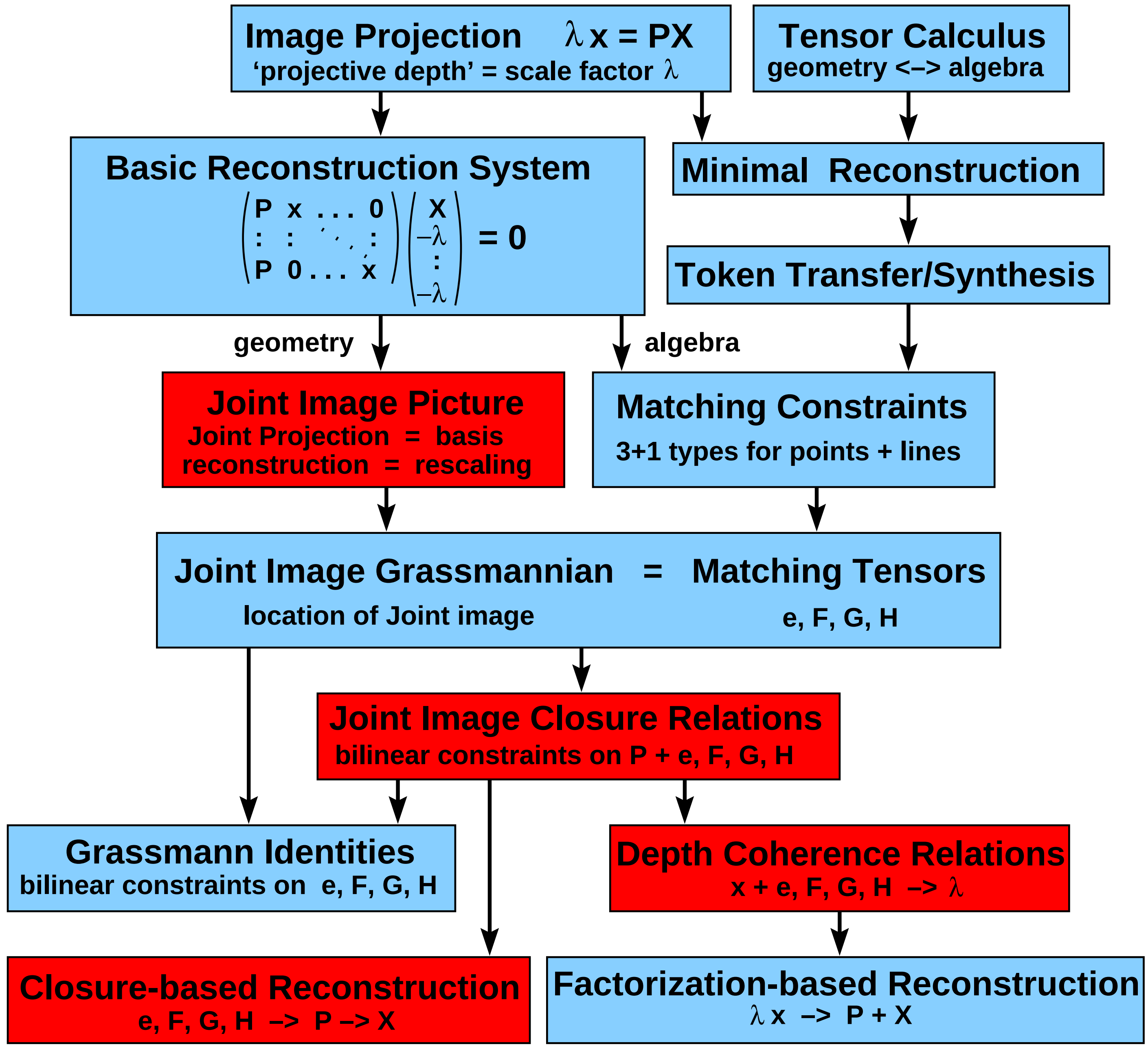


Image Projection λx = PX
'projective depth' = scale factor λ
Tensor Calculus
geometry <–> algebra
Basic Reconstruction System
= 0
Minimal Reconstruction
Token Transfer/Synthesis
geometry
algebra
Joint Image Picture
Joint Projection = basis
reconstruction = rescaling
Matching Constraints
3+1 types for points + lines
Joint Image Grassmannian = Matching Tensors
location of Joint image
e, F, G, H
Joint Image Closure Relations
bilinear constraints on P + e, F, G, H
Grassmann Identities
bilinear constraints on e, F, G, H
Depth Coherence Relations
x + e, F, G, H –> λ
Closure-based Reconstruction
e, F, G, H –> P –> X
Factorization-based Reconstruction
λx –> P + X

Joint Image Closure Relations

- Bilinear ***consistency conditions*** between ***projection matrices*** & ***matching tensors***.

- ***Five types*** in 2–6 images — *e.g.*:

$$\mathbf{F}_{21}\ \mathbf{P}_1 + [\mathbf{e}_{12}]_\times \mathbf{P}_2 = \mathbf{0}$$
$$\mathbf{G}_{B_2}{}^{A_1 C_3}\ \mathbf{P}_a^{B_2} + \mathbf{e}_2^{A_1}\ \mathbf{P}_a^{C_3} - \mathbf{P}_a^{A_1}\ \mathbf{e}_2^{C_3} = \mathbf{0}$$

- Express ***four dimensionality*** (“closure”) of joint image

- Sensitive to ***relative scaling*** of tensors

USES

- ***Linear recovery*** of projections from matching tensors

- ***Depth coherence relations***

  $\Rightarrow$ factorization-based reconstruction

- Bilinear ***Grassmann identities*** on matching tensors

Derivation

- Joint projection has rank 4
- 5×5 minors of 5 of its columns vanish
- Expand & identify 4×4 minors as matching tensors

Closure-Based Reconstruction

MATCHING TENSORS + CLOSURE RELATIONS $\Rightarrow$ PROJECTION MATRICES

- Accumulate constraints from several images.
- Homogeneous linear system in ***Joint Projection***, *e.g.*:

$$\begin{pmatrix} & \vdots & & \vdots & \\ \cdots & \mathbf{F}_{ij}^\top & \cdots & [\mathbf{e}_{ij}]_\times & \cdots \\ & \vdots & & \vdots & \end{pmatrix} \begin{pmatrix} \vdots \\ \mathbf{P}_i \\ \vdots \\ \mathbf{P}_j \\ \vdots \end{pmatrix} = \mathbf{0}$$

- ***Null Space*** $=$ Joint Image (span of Joint Projection)
- Find ***null space basis*** by SVD/iterative methods
- ***Any*** basis gives possible joint projection

  — freedom is projective deformation of scene
- Find ***3D structure*** by ***back-projection***: $\mathbf{x} \wedge \mathbf{P}\mathbf{X} = 0$

Depth Coherence Relations

- Bilinear ***consistency conditions*** between ***rescaled image points*** & ***matching tensors***
- Specializations of ***closure relations***: $\mathbf{P} \to \mathbf{PX}$
- ***Five types*** in 2–6 images — *e.g.*:

$$\mathbf{F}_{21}\,(\lambda_1 \mathbf{x}_1) + \mathbf{e}_{12} \wedge (\lambda_2 \mathbf{x}_2) = \mathbf{0}$$

$$\mathbf{G}_{B_2}{}^{A_1 C_3}\,(\lambda_2 \mathbf{x}^{B_2}) - (\lambda_1 \mathbf{x}^{A_1})\,\mathbf{e}_2^{C_3} + \mathbf{e}_2^{A_1}\,(\lambda_3 \mathbf{x}^{C_3}) = \mathbf{0}$$

- $\approx$ ***"Factorization"*** or ***"radical"*** of matching constraints

— linear in $\mathbf{x}_i$, but unknown scale factors (depths)

— eliminate scales $\Rightarrow$ matching constraints

Coherence-Based Reconstruction

IMAGE POINTS +
MATCHING TENSORS +
COHERENCE RELATIONS $\Rightarrow$ PROJECTIVE DEPTHS

- Accumulate constraints from several images.
- Homogeneous linear system in ***projective depths*** λ_{ip}

— solve recursively/iteratively/SVD...

- Depths give ***implicit projective reconstruction***

— concretize by ***matrix factorization***:

$$\begin{pmatrix} \lambda_{11}\mathbf{x}_{11} & \cdots & \lambda_{1n}\mathbf{x}_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{m1}\mathbf{x}_{m1} & \cdots & \lambda_{mn}\mathbf{x}_{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_m \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \cdots \mathbf{X}_n \end{pmatrix}$$

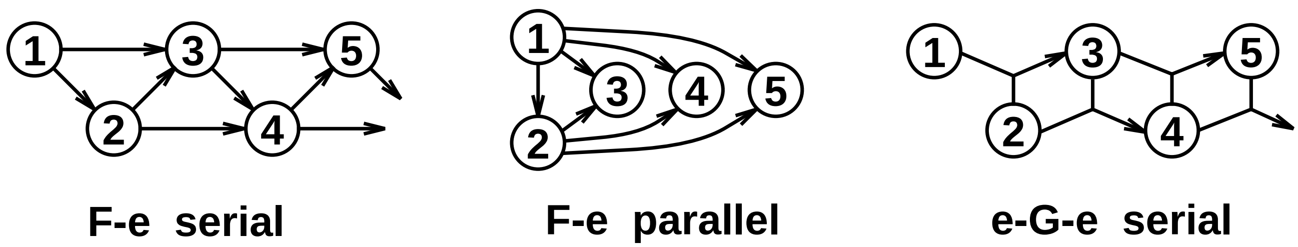
— factorize by ***SVD*** or iterative methods

BUT All points must be visible in all images!

Image Chains

Which images to connect with ***which constraints?***

- Wide choice
- Must connect each image to ≥ 2 others
- Experiments use nonredundant ***serial*** or ***parallel chains***



- $\mathbf{F}$-$\mathbf{e}$ constraints ***fail for straight line motion!***

SCALE COHERENCE

- Need ***coherent scale factors*** for matching tensors
- Use ***depth coherence relations***
- 'Unit gain' around ***each closed loop*** of chain

Algorithm

0 Extract & match features between images

1 Standardize image coordinates to unit box

2 Estimate chains of matching tensors linking images

3 Choose consistent scales for matching tensors

4 Build matrix of closure constraints

5 Find null space (*e.g.* by SVD...)

6 Read off projections from null space basis

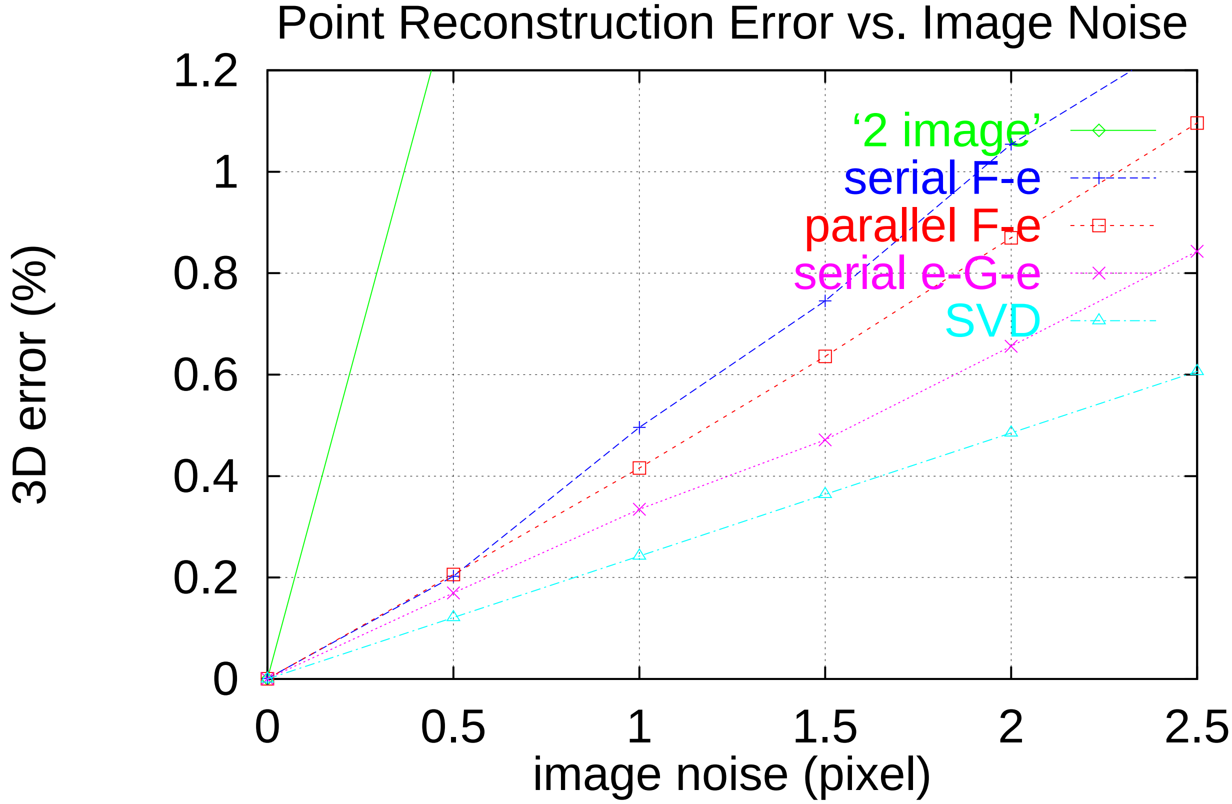
7 Back-project for 3D structure: $\mathbf{x} \wedge \mathbf{P}\mathbf{X} = 0$

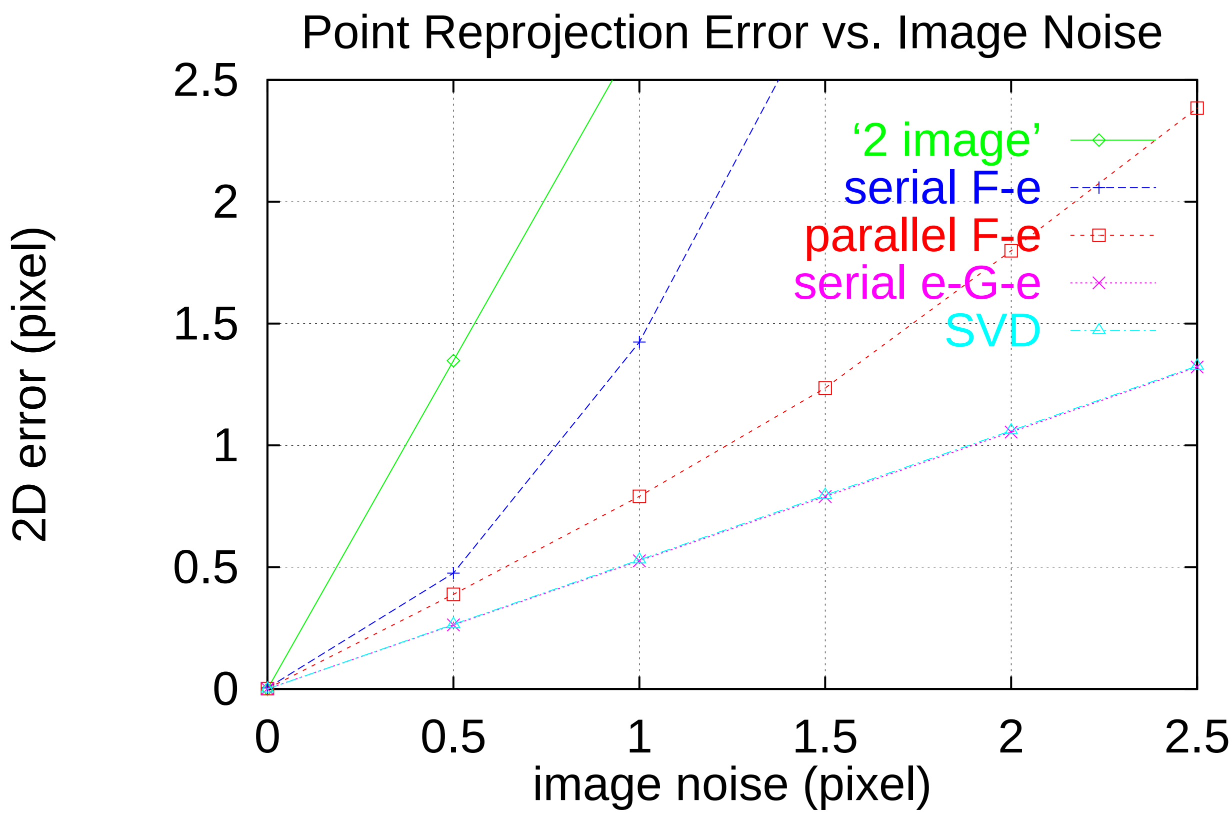
OPTIONAL

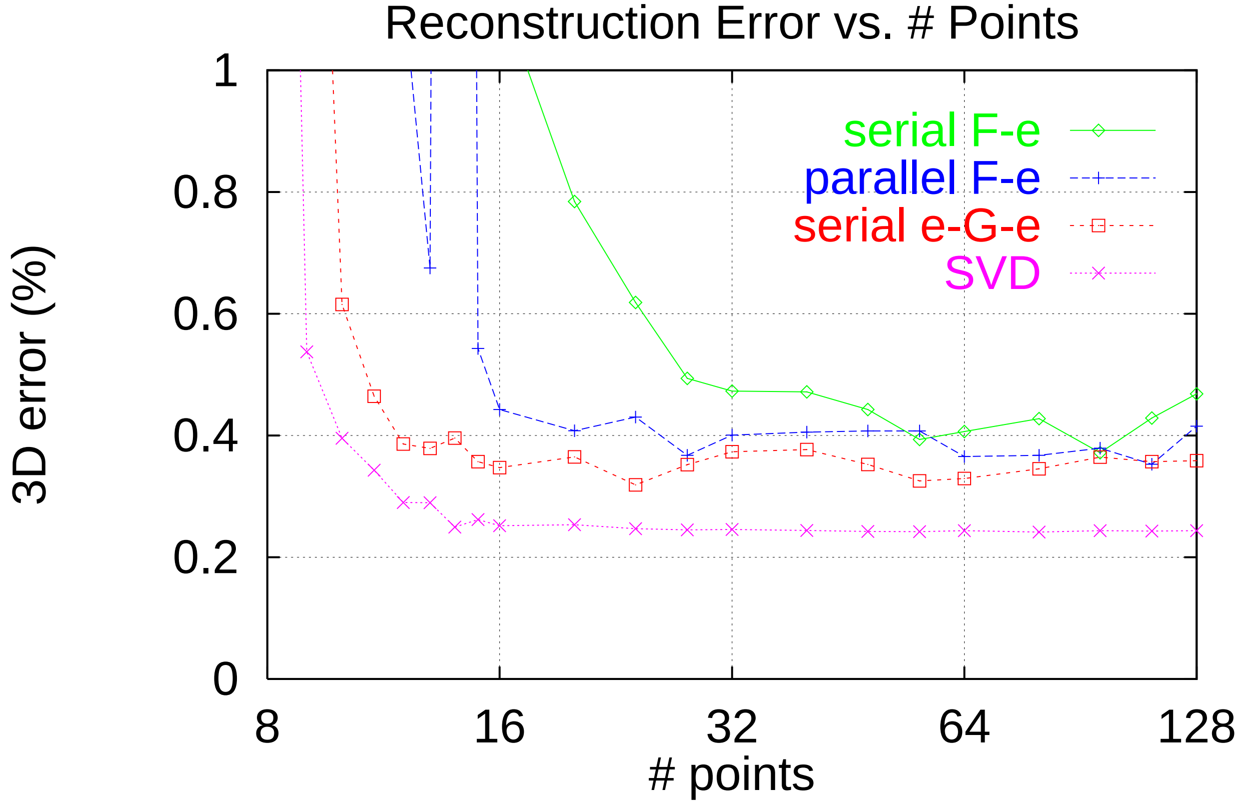
Use ***redundant*** constraints for improved stability

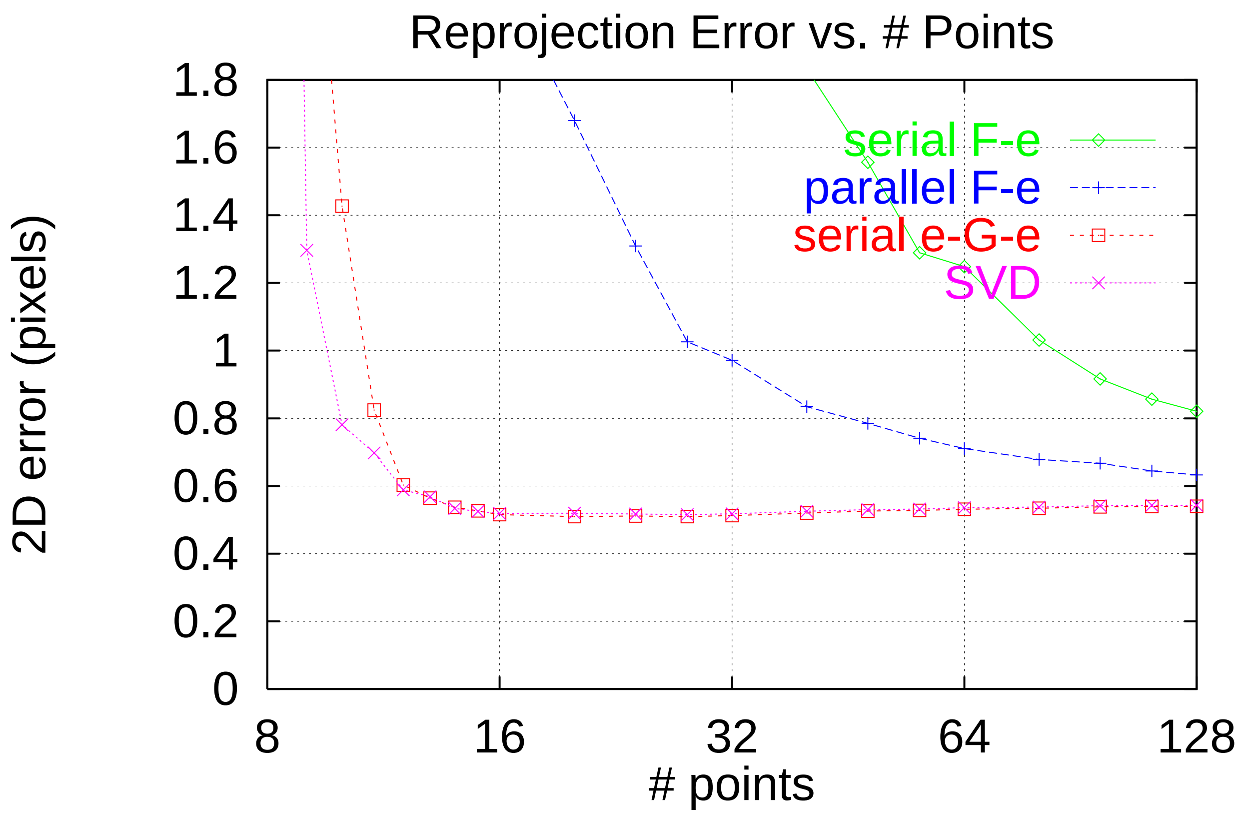
Experiments

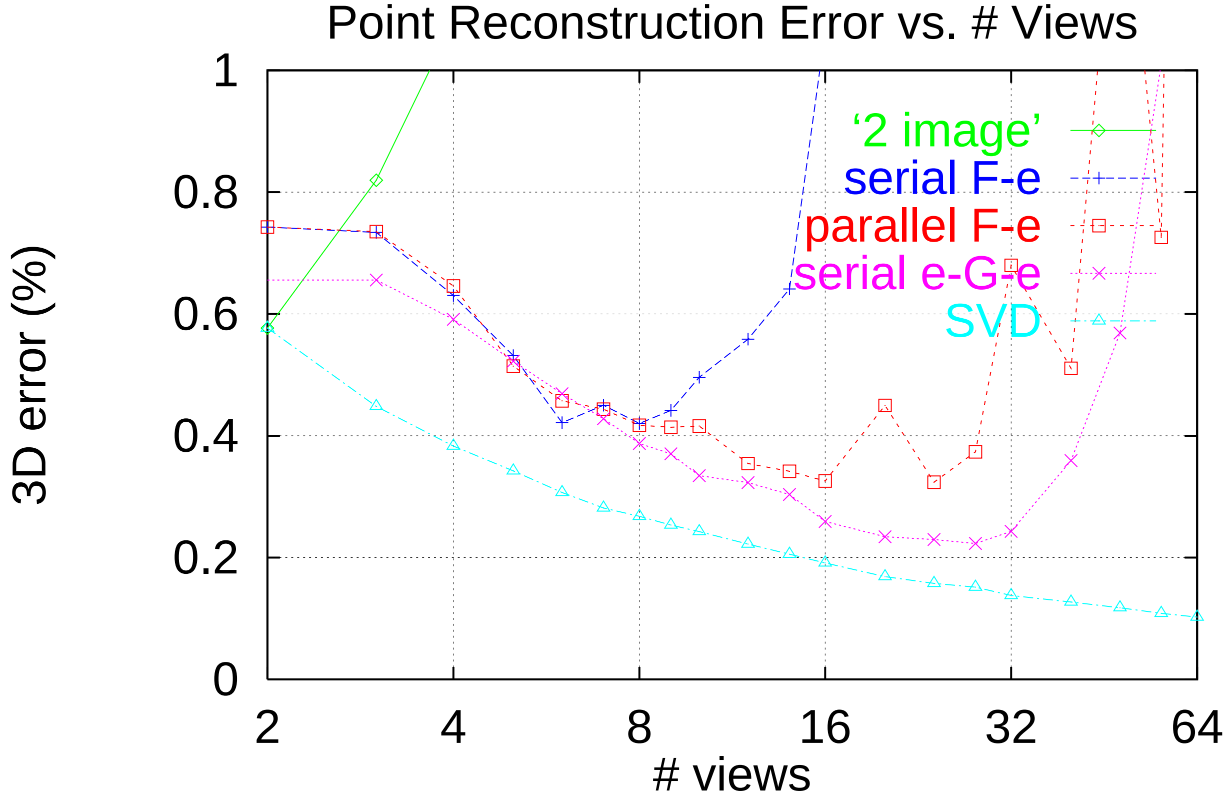
- Synthetic data
- Wide angle lens, near perspective
- Cameras in arc around scene
- Uniform noise
- Also tested on hand-matched images
- Real images soon!

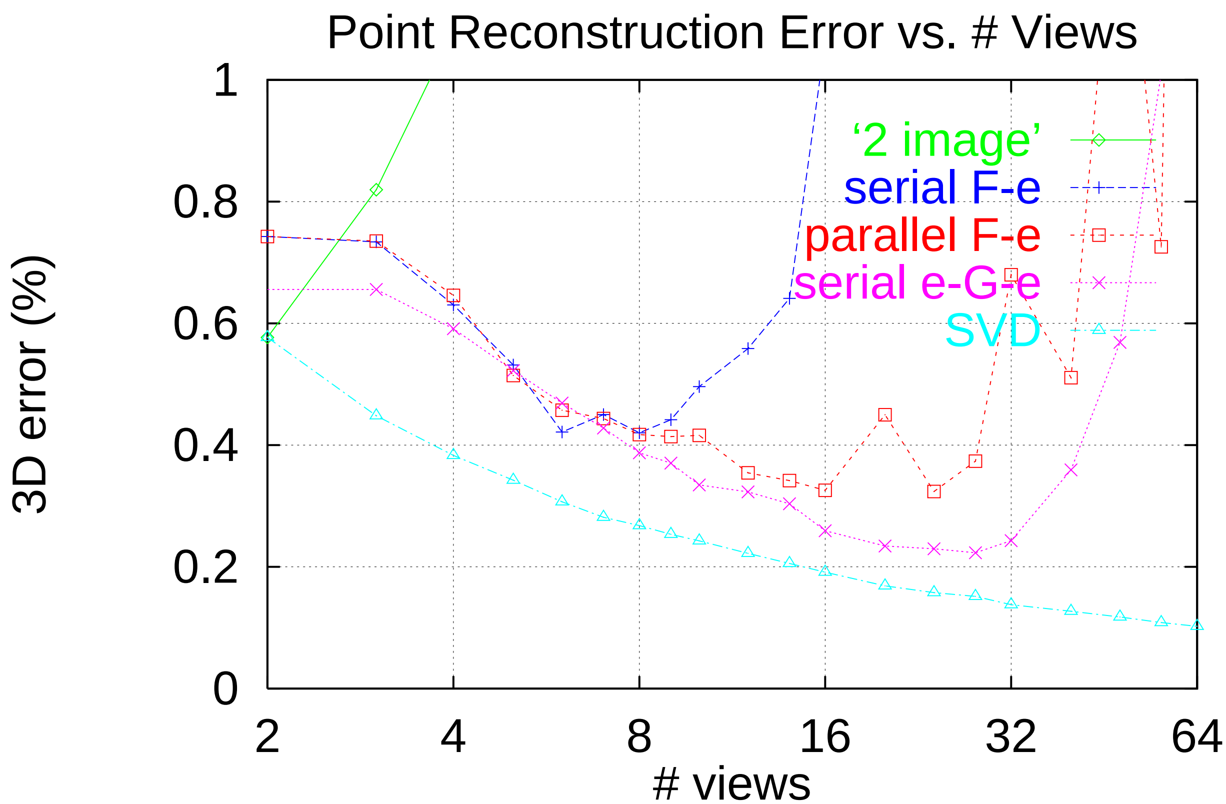

Point Reconstruction Error vs. Image Noise
3D error (%)
image noise (pixel)
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5

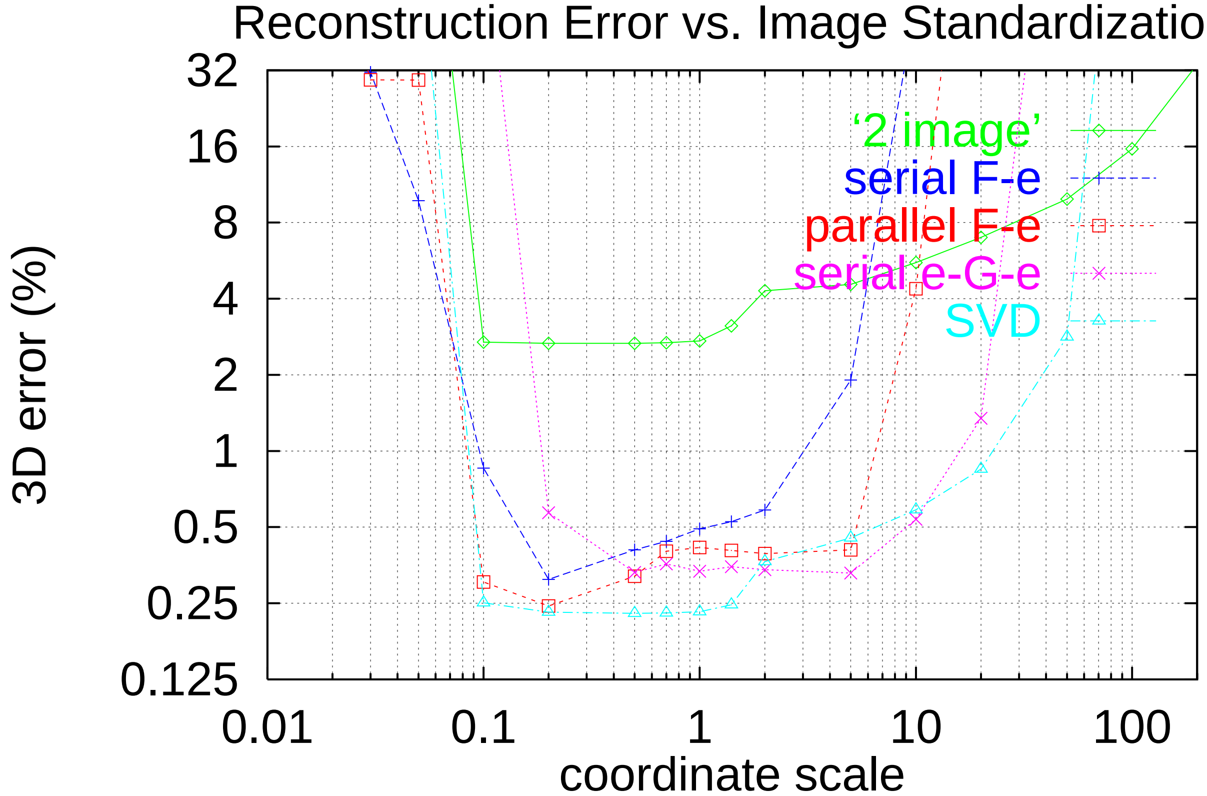

Point Reprojection Error vs. Image Noise
2D error (pixel)
image noise (pixel)
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.5
1
1.5
2
2.5
0
0.5
1
1.5
2
2.5

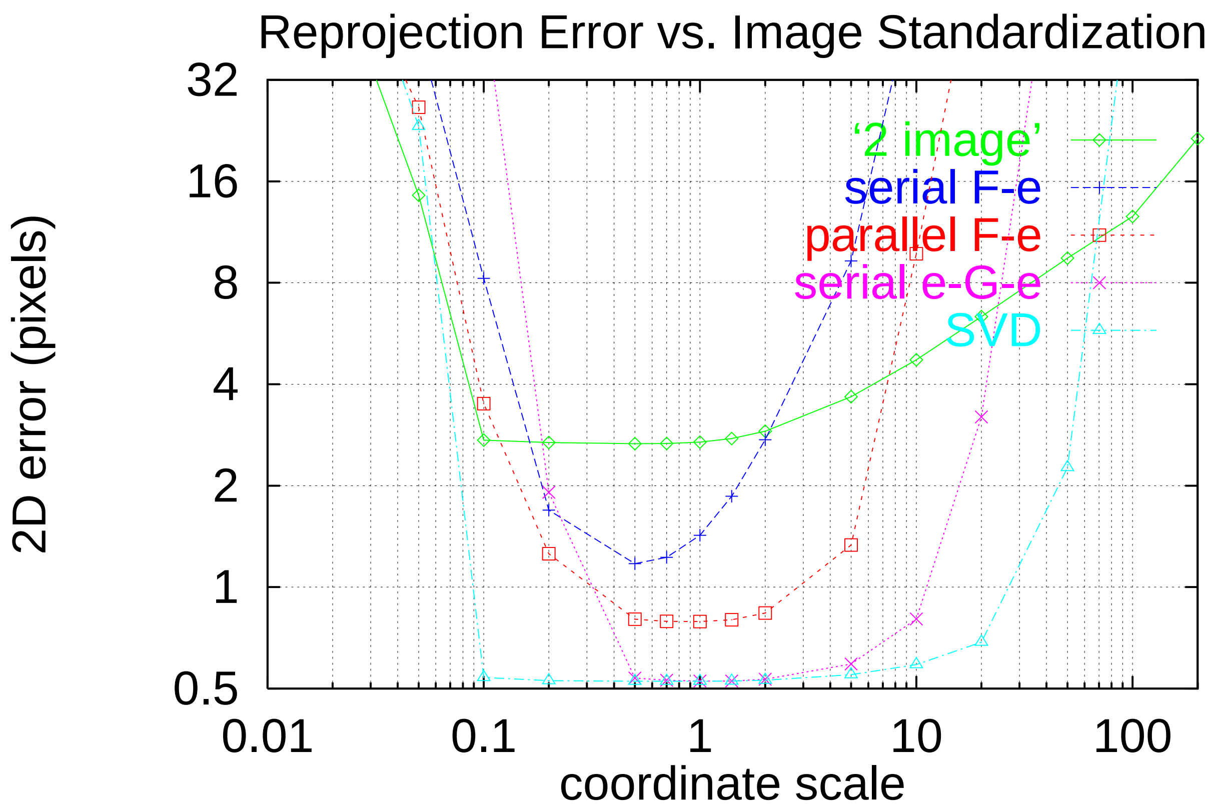

Reconstruction Error vs. # Points
3D error (%)
points
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.2
0.4
0.6
0.8
1
8
16
32
64
128


Reprojection Error vs. # Points
2D error (pixels)
points
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
8
16
32
64
128


Point Reconstruction Error vs. # Views
3D error (%)
views
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.2
0.4
0.6
0.8
1
2
4
8
16
32
64


Point Reconstruction Error vs. # Views
3D error (%)
views
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD
0
0.2
0.4
0.6
0.8
1
2
4
8
16
32
64


Reconstruction Error vs. Image Standardization
3D error (%)
32
16
8
4
2
1
0.5
0.25
0.125
0.01
0.1
1
10
100
coordinate scale
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD


Reprojection Error vs. Image Standardization
2D error (pixels)
32
16
8
4
2
1
0.5
0.01
0.1
1
10
100
coordinate scale
'2 image'
serial F-e
parallel F-e
serial e-G-e
SVD

Conclusions & Future Work

Closure Relations

- Bilinear relations on projections & matching tensors
- Useful intermediate form for vision calculations

Closure-Based Projective Reconstruction

- Recovers projections ***linearly*** from matching constraints
- Only needs token tracking across 2–3 images
- ***Short chains*** are best, even if ‘key’ image required
- ***Stability:*** factorization $>$ closure $>$ “2 image” method
  trivalent $>$ F-matrix (*e.g.* straight line motion)

Future Work

- ***Real images***
- Redundant constraints
- Recursive formulation
- Factorization-like methods with ***missing data***

http://www.inrialpes.fr/movi/Triggs