The Advantage of Mounting a Camera onto a Robot Arm

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Abstract – *In this paper we describe a method for recovering Euclidean structure of an unknown scene with an uncalibrated camera mounted onto a robot arm. More precisely the method achieves the following tasks: the projective structure of the scene obtained with a projective reconstruction algorithm is converted into an Euclidean structure and the intrinsic camera parameters are estimated as well as the hand/eye calibration parameters. We cast the problem into a simple set of matrix equations and we determine under which motions the solution is unique. Then we provide a simple algorithm for implementing the method in practice. Finally the experimental results obtained with our method are compared with classical camera and hand/eye calibration techniques.*

1 Introduction

One of the most challenging tasks in Computer Vision is the task of recovering the 3-D Euclidean structure of a scene with a moving camera. Traditionally there are two possible approaches to this problem. The first approach uses a calibrated camera while the second approach attempts to build a projective reconstruction using an uncalibrated camera and then to convert this projective structure into Euclidean structure. The quality of the results associated with the former approach is tightly related to the quality of camera calibration and to the assumption that this calibration is stable over time. While it is possible, using robust numerical methods, to perform an off-line precise calibration it is not realistic to suppose that the parameters thus computed are valid on line.

As already mentioned above, an alternative approach that has been thoroughly investigated for the past 5 years, consists of performing projective reconstruction first and of computing Euclidean reconstruction second, by looking for a transformation that maps the 3-D projective space onto its Euclidean subspace. This is also equivalent to camera self calibration – the camera is calibrated on line, in the same time as the reconstruction itself, and no special "calibration grid" is necessary. Nevertheless, while projective reconstruction may be, more or less, considered as solved, the problem of converting projective reconstruction into Euclidean reconstruction has not been solved. Numerical implementations based on the theoretical work of Maybank & Faugeras [9] give partial satisfaction [4]. Alternatively, brute force non-linear least squares approaches have problems with initialization.

In this paper we assume that the camera is mounted onto a robot arm and that the robot gripper (or the hand) undergoes a known rigid motion. However, neither the camera calibration parameters nor the relationship between the camera and the robot hand are known. We farther assume that the camera and the hand rigidly move together. We attempt to solve simultaneously the following problems:

(i) Perform a projective reconstruction of an unknown 3-D scene and convert it into an Euclidean one;

(ii) Estimate the intrinsic parameters associated with camera calibration, and

(iii) Determine the rigid transformation between the robot hand and the camera – this is the so called hand/eye calibration problem.

In the past, the hand-eye calibration problem has been associated with the extrinsic camera calibration parameters [11]. In [8] an approach is suggested in order to perform hand/eye calibration in conjunction with Euclidean reconstruction. In this paper we extend the work presented in [8] and we show how hand/eye calibration can be obtained solely from projective reconstruction provided that the hand motion is known.

More generally we establish a mathematical formulation that links camera calibration, hand/eye calibration, projective reconstruction and robot motion. Moreover, we determine a necessary condition for insuring the uniqueness of the solution. We show that the formulation that we propose leads to a set of quadratic equations that are solved using a classical non-linear least-square minimization technique. We show how this non linear process can be properly initialized if some prior Euclidean knowledge is available – such as off line calibration, for example. Preliminary results obtained both with simulated and experimental data are promising.

2 Preliminaries

First we consider the camera calibration problem where the camera is described by a standard pin-hole model. It is well known that calibrating such a camera is equivalent to estimating a projective transformation between a 3-D Euclidean frame and the 2-D image frame. Let M be a 3×4 matrix describing such a projective transformation. We have:

$$p = \lambda\, MP \tag{1}$$

where $p = (u\ v\ w)^T$ is an image point and $P = (x\ y\ z\ s)^T$ is a 3-D point with Euclidean coordinates x/s, y/s, and z/s, and λ is an arbitrary scale factor.

Matrix M can be linearly estimated if at least 6 image-point-to-scene-point correspondences are provided [3]. This matrix can be further decomposed into intrinsic parameters (matrix C) and extrinsic parameters (rotation matrix R and translation vector $\mathbf{t}$):

$$M = \underbrace{\begin{pmatrix} \alpha_u & r & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} \underbrace{R}_{3\times 3} & \underbrace{\mathbf{t}}_{3\times 1} \end{pmatrix} \tag{2}$$

Second we consider the projective reconstruction problem or, equivalently, the problem of scene reconstruction with an uncalibrated camera. Let the moving camera observe a scene with k points denoted by P_j, $j \in \{1, ..., k\}$ and consider n positions of the camera where each position is indexed by i, $i \in \{1, ..., n\}$. Eq. (1) can be therefore written for each i and for each j as:

$$p_{ij} = \lambda_i\ M_i P_j \tag{3}$$

It is important to notice that the equation above defines M_i and P_j only up to a 4×4 invertible matrix, i.e., up to a projective transformation of the 3-D projective space. Indeed we have:

$$p_{ij} = \lambda_i\ M_i W W^{-1} P_j$$

In order to avoid confusion we will designate by S_j points in the 3-D projective space, by P_j points in the 3-D Euclidean space, by N_i projective matrices that map points from the 3-D projective space to the 2-D projective images, and by M_i matrices that map points from the 3-D Euclidean space to 2-D images.

The problem of projective reconstruction is the problem of simultaneously estimating S_j and N_i given p_{ij}, for all i and for all j. Projective reconstruction can be cast into a non-linear least-square optimization problem. The error function to be minimized is the following:

$$f(N_1, \ldots N_i, \ldots N_n, S_1, \ldots S_j, \ldots S_k) =$$
$$\sum_{ij} \left(\left(\frac{u_{ij}}{w_{ij}} - \frac{n_{11}^{(i)} x_j + n_{12}^{(i)} y_j + n_{13}^{(i)} z_j + n_{14}^{(i)} s_j}{n_{31}^{(i)} x_j + n_{32}^{(i)} y_j + n_{33}^{(i)} z_j + n_{34}^{(i)} s_j} \right)^2 + \left(\frac{v_{ij}}{w_{ij}} - \frac{n_{21}^{(i)} x_j + n_{22}^{(i)} y_j + n_{23}^{(i)} z_j + n_{24}^{(i)} s_j}{n_{31}^{(i)} x_j + n_{32}^{(i)} y_j + n_{33}^{(i)} z_j + n_{34}^{(i)} s_j} \right)^2 \right)$$

Each projection matrix and each 3-D point is defined up to a scale factor. Therefore there are 11 unknowns associated with a projection matrix and 3 unknowns associated with each point: a total of $11 \times n + 3 \times k$ unknowns. In spite of this large number of variables, the minimization of such an error function can be easily carried out by such methods as Levenberg-Marquardt [1], [10], [6] [5]. A good initial solution can be provided by using just two of the initial set of images (for example the first one and the last one) and by applying the epipolar constraint with its associated linear projective reconstruction [2]. Such an initialization procedure is described in detail in [6].

3 Problem formulation

We consider now a camera that is rigidly mounted onto a robot arm and we associate an Euclidean frame with the robot hand – this frame is also known in Robotics as the tool frame. The position and the orientation of the tool frame is related to the robot base frame by the direct kinematic model of the robot. Let $X = (R\ \mathbf{t})$ be the rigid transformation (rotation and translation) between the hand frame and the camera standard Cartesian frame. The parameters of X are the parameters associated with hand/eye calibration. If we combine X with the intrinsic camera parameters – a 3×3 matrix C – we obtain a projective matrix:

$$M = CX \tag{4}$$

It is worthwhile to notice that the matrix M above encapsulates both the camera calibration problem and the hand/eye calibration problem. Of course, M cannot be directly estimated. Instead, M will be indirectly estimated as follows.

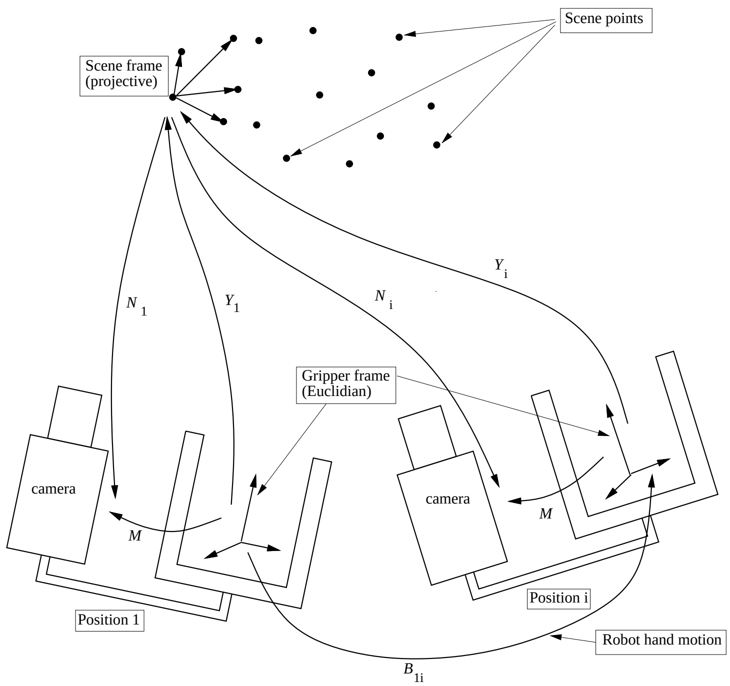

Figure 1: This figure shows the camera rigidly mounted onto a robot hand. The hand undergoes a rigid motion (and so does the camera) and the camera observes a 3-D scene materialized by a number of points.

We assume now that both the camera and the robot hand undergo a sequence of rigid motions and that C and X do not vary during these motions. The camera observes a 3-D scene and hence image-to-image correspondences can be established and a projective scene reconstruction can be performed as described in the previous section. Hence there is a projective frame associated with the 3-D scene. Let S_j be the scene points expressed in this frame and let N_i be the transformation from this frame to the frame associated with the i^{th} image, e.g., Figure 1.

Let Y_i be a 4×4 invertible matrix associated with a transformation from the hand Euclidean frame to the scene projective frame. Therefore, the matrix M can be written as a combination of two projective transformations (the symbol "$\cong$" indicates the the equalities are defined up to a scale factor):

$$\begin{aligned} M &\cong N_1 Y_1 \\ &\cong \ldots \\ &\cong N_i Y_i \\ &\cong \ldots \\ &\cong N_n Y_n \end{aligned} \tag{5}$$

Moreover, let B_{1i} be a 4×4 matrix that expresses the rigid hand motion between the first position – 1 – and any other position – i. The relationship between Y_1, Y_i, and B_{1i} is simply:

$$Y_i \cong Y_1 B_{1i}^{-1}$$

By substituting the Y_i's in eq. (5) we obtain the following set of $n-1$ matrix equations:

$$\left\{ \begin{array}{rcl} N_1 Y_1 B_{12} & = & \lambda_2 N_2 Y_1 \\ & \vdots & \\ N_1 Y_1 B_{1n} & = & \lambda_n N_n Y_1 \end{array} \right. \tag{6}$$

In these equations the unknowns are the scalars $\lambda_2, \ldots, \lambda_n$, and the 4×4 matrix Y_1 which represents the relationship between the projective structure described in a scene centered projective frame and the Euclidean structure described in the hand frame. Therefore, if one is able to solve for Y_1 then the problem of converting projective reconstruction into Euclidean reconstruction is solved.

Each matrix equality in eq. (6) provides 12 constraints. The unknown scalars are easily eliminated by dividing, for example, each one of the first 11 constraints with the 12^{th} one. Therefore, we are left with $11\times(n-1)$ quadratic constraints in 15 unknowns (the matrix Y_1 is defined up to a scale factor) and at least 3 robot positions (or 2 rigid motions) are necessary in order to be able to find a solution for Y_1.

Let us suppose that a solution is found for Y_1. We immediately obtain:

- *Euclidean structure*, by pre multiplying the projective coordinates of the scene points with the inverse of Y_1,

$$\forall j \;\; P_j = Y_1^{-1} S_j$$

- *Camera calibration*, by computing the matrix M using one of the expressions in eq. (5). Alternatively, another solution consists of linearly estimating the matrix M using the Euclidean coordinates of the scene points and their projections in the first image.

- *Hand/eye calibration* is obtained by simply extracting the extrinsic parameters of the matrix M just computed.

Before we proceed farther, it is interesting to notice that eq. (6) is a generalization of the classical hand/eye calibration formulation [11], [7]. Indeed, if an Euclidean rather than a projective reconstruction is available then the matrix Y_1 describes a rigid displacement and the matrices N_i must be replaced by M_i which can be written as:

$$M_i = C A_i$$

where C contains the intrinsic camera parameters and A_i describes the rigid displacement between the scene frame and the camera frame in position i. By substitution in eq. (6) we obtain:

$$C A_1 Y_1 B_{1i} = C A_i Y_1$$

Because C has full rank we obtain:

$$A_1 Y_1 B_{1i} = A_i Y_1$$

The relationship between X (the hand/eye displacement), A_1, and Y_1 is:

$$X = A_1 Y_1$$

Finally we obtain the classical hand/eye formulation ($A_{1i} = A_i A_1^{-1}$):

$$A_{1i} X = X B_{1i} \tag{7}$$

4 Problem solution

As explained in the previous section, the key of the approach advocated herein is the resolution of a system of matrix equalities defined up to a scale factor, i.e., eq. (6). One such equality can be written as:

$$N_1 = \lambda_i N_i Y_1 B_{1i}^{-1} Y_1^{-1}$$

Let U be a 4×4 invertible matrix that belongs to the subgroup of matrices of the projective group that commutes with B_{1i}^{-1}, or with B_{1i}, and let $\mathcal{C}(B_{1i})$ denote this subgroup. We have:

$$\begin{array}{rcl} N_1 & = & \lambda_i N_i Y_1 B_{1i}^{-1} Y_1^{-1} \\ & = & \lambda_i N_i Y_1 \; U U^{-1} \; B_{1i}^{-1} Y_1^{-1} \\ & = & \lambda_i N_i \; (Y_1 U) \; B_{1i}^{-1} \; (Y_1 U)^{-1} \end{array}$$

Therefore, if Y_1 is a solution, Y_1U is a solution as well and Y_1 will not be uniquely determined. As a consequence, one has to choose a sequence of hand motions B_{1i} such that the only possible choice for U is the identity matrix:

$$\bigcap_{i>1} \mathcal{C}(B_{1i}) = \{I\}$$

4.1 A sufficient condition for uniqueness

In what follows we derive a *sufficient* condition for obtaining the result above. We show that at least 2 motions (or 3 robot positions) are necessary, that there should be at least 2 distinct axes of rotation, and that one among the motions should be such that the translation vector is not orthogonal to the axis of rotation.

Indeed, let U and B_{1i} be written as:

$$U = \begin{pmatrix} W & \mathbf{u} \\ \mathbf{v}^T & w \end{pmatrix} \qquad B_{1i} = \begin{pmatrix} R_i & \mathbf{t}_i \\ 0\,0\,0 & 1 \end{pmatrix}$$

We seek a matrix U satisfying:

$$B_{1i}U \cong UB_{1i}$$

In order to get rid of the scale factor and without loss of generality we fix the determinant of U:

$$\det(U) = 1$$

By developing the matrix products on both sides of the equation above we obtain:

$$\begin{aligned} WR_i &= R_iW + \mathbf{t}_i\mathbf{v}^T && (8)\\ W\mathbf{t}_i + \mathbf{u} &= R_i\mathbf{u} + \mathbf{t}_i && (9)\\ \mathbf{v}^T R_i &= \mathbf{v}^T && (10)\\ \mathbf{v}^T\mathbf{t}_i &= 0 && (11) \end{aligned}$$

Let us start by analyzing eq. (10). This equation simply implies that $\mathbf{v}$ must be identical with the direction of the axis of rotation of R_i. Therefore by selecting two distinct robot motions such that their axes of rotation are not parallel to each other, we obtain:

$$\mathbf{v} = 0$$

Therefore we are left with:

$$\begin{aligned} WR_i &= R_iW && (12)\\ (W - I)\mathbf{t}_i &= (R_i - I)\mathbf{u} && (13) \end{aligned}$$

Let $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ be the eigen vectors of the rotation matrix R_i. It is well known that a rotation matrix has three distinct eigenvalues: $\mu_1 = 1$, $\mu_2 = e^{i\theta}$, and $\mu_3 = e^{-i\theta}$. From eq. (12) we easily obtain that W and R_i must have *the same eigenvectors* but they can have distinct eigenvalues. Indeed, with $R_i\mathbf{n}_j = \mu_j\mathbf{n}_j$ and by substitution in eq. (12):

$$WR_i\mathbf{n}_j = W\mu_j\mathbf{n}_j = \mu_jW\mathbf{n}_j = R_iW\mathbf{n}_j$$

Hence, $W\mathbf{n}_j = \nu_j\mathbf{n}_j$ with $j = 1, 2, 3$ and one may notice that the eigenvalues of W, ν_1, ν_2, and ν_3 cannot be null because W is not singular.

We consider now a second motion with its associated rotation matrix R_k whose eigen vectors are denoted by $\mathbf{m}_1$, $\mathbf{m}_2$, and $\mathbf{m}_3$. Let $\mathbf{m}_1$ be the axis of rotation of R_k and let us impose that this vector is neither parallel to $\mathbf{n}_1$ (the axis of rotation of R_i) nor in the plane defined by $\mathbf{n}_2$ and $\mathbf{n}_3$. $\mathbf{m}_1$ can therefore be written as a linear combination of the eigen vectors of R_i:

$$\mathbf{m}_1 = \sum_{j=1}^{3} \lambda_j\mathbf{n}_j \tag{14}$$

We obtain:

$$W\mathbf{m}_1 = W(\sum_{j=1}^{3} \lambda_j\mathbf{n}_j) = \sum_{j=1}^{3} \lambda_j\nu_j\mathbf{n}_j \tag{15}$$

However, the eigen vectors of W and R_k should also be the same:

$$W\mathbf{m}_1 = \nu_1 \mathbf{m}_1 = \nu_1 \sum_{j=1}^{3} \lambda_j \mathbf{n}_j = \sum_{j=1}^{3} \nu_1 \lambda_j \mathbf{n}_j \tag{16}$$

By identifying eq. (16) with eq. (15) we obtain:

$$\nu_1 = \nu_2 = \nu_3 = \nu$$

Since the matrix W has three identical eigen values it is necessarily of the form:

$$W = \nu I$$

As a consequence, eq. (13) becomes:

$$(\nu - 1)\mathbf{t}_i = (R_i - I)\mathbf{u}$$

In this expression, $(R_i - I)\mathbf{u}$ is a vector perpendicular to the axis of rotation of R_i – the eigen vector $\mathbf{n}_1$.[1] Therefore, unless the translation vector $\mathbf{t}_i$ lies in a plane perpendicular to $\mathbf{n}_1$ and if we eliminate the trivial solution $R_i = I$ and $\mathbf{t}_i = 0$, we have:

$$\nu = 1$$

$$\mathbf{u} = 0$$

Finally, since the determinant of U has been fixed to 1 we obtain that the unique solution for U is:

$$U = I$$

To conclude we proved the following sufficient condition which insures the uniqueness of the solution of eq. (6): *among all robot motions B_{1i}, (i) at least two motions must have distinct axes of rotation and (ii) at least one motion must have its translation vector non orthogonal to its rotation axis.*

4.2 Outline of the proposed solution

We are now ready to provide an overview of the method advocated in this paper in order to solve for Euclidean structure with an uncalibrated moving camera mounted onto a robot. The outline of the proposed solution is the following:

1. Perform a number of robot motions, at least two motions, and for each robot motion register its parameters using the kinematic model of the robot. Among these motions, at least two of them must have distinct axes of rotation, and at least one of them should not have its translation vector perpendicular to its axis of rotation.

2. Establish image-to-image correspondences between points in the image sequence and perform a projective reconstruction of the observed scene.

3. Solve eq. (6) in two steps:

   - *First step*: Use approximate values for the camera intrinsic parameters and for the position and orientation of the camera with respect to the hand in order to roughly estimate matrix M in eq. (4). Substitute the matrix M thus obtained into eq. (6) through eq. (5). This process allows a linear estimation for Y_1.
   - *Second step*: Solve for Y_1 in eq. (6) using a non-linear least-square minimization method that is initialized with the linear solution previously found.

4. Convert the projective structure of the scene into Euclidean structure.

5. Determine matrix M and extract its intrinsic parameters (camera calibration) and its extrinsic parameters (hand/eye calibration).

[1] In order to be convinced of this perpendicularity, consider the dot product between vector $\mathbf{u}$ and the axis of rotation of R_i, $\mathbf{n}_1$. This dot product is invariant with respect to rotation: $\mathbf{u} \cdot \mathbf{n}_1 = (R_i\mathbf{u}) \cdot (R_i\mathbf{n}_1) = (R_i\mathbf{u}) \cdot \mathbf{n}_1$. It follows that $(R_i\mathbf{u} - \mathbf{u}) \cdot \mathbf{n}_1 = 0$.

5 Experiments and discussion

In order to validate the method we performed the following experiment. A camera mounted onto a robot observes a calibration grid from 7 different positions (6 robot motions). The calibration grid consists of a planar pattern with 92 points that can move very precisely along its z-axis. This setup allows one to vary the total number of scene points by choosing a number of grid positions along its z-axis and to vary the depth of the scene by choosing an appropriate shift along the z-axis. The image points are located with sub-pixel precision – between 1/5 and 1/10 of a pixel.

The setup described above allows us (i) to calibrate the camera using a classical approach as many times as the number of positions (7 in this case) and (ii) to perform the self calibration process described in this paper. We therefore have an experimental and quantitative basis to compare our method with classical off-line calibration.

Both methods eventually estimate a 3×4 projection matrix M, as outlined in sections 2 and 3. Such a matrix must be decomposed into intrinsic and extrinsic parameters. This decomposition is performed using a variation of the well known QR decomposition of matrices, namely RQ decomposition [6]. Indeed matrix M can be written as, i.e., eq (2):

$$M = \begin{pmatrix} \underbrace{CR}_{3\times 3} & \underbrace{C\mathbf{t}}_{3\times 1} \end{pmatrix} \tag{17}$$

Matrices C and R can be recovered by decomposing the 3×3 sub-matrix of M into an upper triangular matrix and an orthogonal matrix. The results of applying the RQ matrix decomposition to off-line calibration and to self calibration are summarized on the table below. One may notice that the intrinsic parameters obtained by self calibration (the last row of the table) are within the same range of values as the intrinsic parameters obtained with a classical approach (the top seven rows). The result of self calibration reported in this table may also be compared with the results reported by Faugeras, Luong, & Maybank [4] and by Hartley [6]. All these authors noticed important variations in the estimation of the position of the optical center as well as a great sensitivity to noise.

	α_u	α_v	α_u/α_v	r	u_0	v_0
Position 1	-2084	1417	-1.4707	5	256	221
Position 2	-2055	1397	-1.4710	-3	257	223
Position 3	-2081	1415	-1.4706	0.6	265	230
Position 4	-2083	1416	-1.4710	0.45	261	213
Position 5	-2094	1422	-1.4726	3.3	270	222
Position 6	-2054	1426	-1.4404	2.1	260	217
Position 7	-2051	1420	-1.4444	2	251	230
Initialisation	-1700	1000	-1.7	0	256	256
Self Calibration	-1879	1310	-1.4343	46	278	199

Table 1: This table summarizes the results of camera calibration obtained with a classical off-line solution (the first seven rows) and with the method described in this paper (the last row).

The extrinsic parameters of the projection matrix M obtained by self calibration are associated with the hand/eye parameters. We compared these parameters with those obtained by calibrating the hand/eye relationship using the classical approach, i.e., solving an homogeneous matrix equation of the form $AX = XB$.

The rotational parameters obtained with our method are *exactly the same* as the parameters obtained with the classical hand/eye calibration method. We noticed a discrepancy between the two sets of translational parameters. This discrepancy is of the order of a few (3 to 4) centimeters, the distance between the camera origin and the hand origin being of approximatively 10 centimeters. We have not yet been able to properly analyse the source of this discrepancy. It is most probably due to errors in the robot's offsets which don't affect the rotational parameters but do affect the translational ones.

In the classical approach, Euclidean information is provided by a carefully machined calibration grid. In our approach the Euclidean information is provided by robot motions which are less precise. This loss of precision could be easily overcome with better robots[2] and it should not be interpreted as the price to be paid when one wants to recover Euclidean structure of an unknown scene with an uncalibrated camera.

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[2]the robot on which these experiments have been carried out is 10 years old!