Adaptive Design of Sampling Directions in Diffusion Tensor MRI and Validation on Human Brain Images

Emmanuel Caruyer, Rachid Deriche

Adaptive Design of Sampling Directions in Diffusion Tensor MRI and Validation on Human Brain Images

Emmanuel Caruyer[1,2] and Rachid Deriche[1]

[1] Odyssée Project-Team, INRIA Sophia Antipolis - Méditerranée, France.
[2] ENS Cachan - Britanny extension, Rennes, France.
{Emmanuel.Caruyer,Rachid.Deriche}@sophia.inria.fr,

Abstract. Diffusion tensor reconstruction is made possible through the acquisition of several diffusion weighted images, each corresponding to a given sampling direction in the **q**-space. In this study, we address the question of sampling efficiency, and show that in case we have some prior knowledge on the diffusion characteristics, we may be able to adapt the sampling directions for better reconstruction of the diffusion tensor. The prior is a tensor distribution function, estimated over a given region of interest, possibly on several subjects. We formulate an energy related to error on tensor reconstruction, and calculate analytical gradient expression for efficient minimization. We validate our approach on a set of 5199 tensors taken within the corpus callosum of the human brain, and show improvement by an order of 10% on the MSE of the reconstructed tensor.

1 Introduction

With the discovery of diffusion tensor MRI, Basser and colleagues [1] suggested to use at least 6 non-collinear diffusion gradients for an accurate reconstruction of the self-diffusion covariance matrix. Since then, several authors have been working on the optimal choice of sampling directions. A family of methods aim at reducing error propagation from diffusion measurements to the reconstructed tensors. Papadakis and colleagues [2] propose the minimization of an index κ, calculated as the mean squared error on the tensor. Equivalently, Skare and colleagues [3] proposed to minimize the condition number of the measurement matrix. Other approaches aim at isotropically spread the directions on the unit sphere. Such diffusion gradient sets may be constructed either using an electrostatic repulsion energy minimization, as proposed by Jones and colleagues [4]. Papadakis and colleauges also proposed a variant [5], recursively minimizing an r^{-n} repulsion energy for growing n, up to convergence. Interestingly, they report it to also optimize the index κ they introduced previously [2]. A new trend tend to also optimize the arrangement of these directions along the sequence, so that any subset of the scan may correspond to a valid scheme, in oreder to recover information even if the patient has moved at a given moment of the scan [6–8].

Finally, geometric construction is an alternative, cite for instance the tesselation of the sphere proposed by Tuch [9].

The convergence of the community towards direction sets isotropically spread on the sphere is motivated by the need for rotational independence of the reconstruction accuracy [10],[11]. Indeed, there may be a huge variability (intra and inter subject) in diffusion characteristics, and without prior on this variability, we should be able to estimate with equal accuracy any tensor, with any orientation. For several applications however, such as spinal cord imaging, or study of region of interest within the brain white matter, the tensor distribution may be far from isotropic. Hence the efficiency of isotropic sampling scheme is questionable in such situation.

Recent works have proposed to include some prior on the tensor field under study in the design of optimal sampling scheme. Peng and Arfanakis [12] consider a collection of tensors of selected orientations, and optimize the sampling scheme for these tensors. They show significant improvement over conventional methods in the literature, however as their method require long computation to generate direction sets, they propose a workaround consisting in using for actual application a direction set previously designed for some similar situation, and their proposal ends up with a lack of generality. More recently, Gao and colleagues [13] proposed a framework to take into account some knowledge on the tensor field under study in order to design optimal diffusion imaging parameters, including directions and physical parameters involved in the b-value. However they did not address applicability in practical applications, and only performed their validation on synthetic tensor field.

In this article, we derive a cost function leading to an adaptive choice of direction set, develop our approach and clearly show the improvement brought by the prior. We show possible application on two selected ROIs in the brain white matter, namely the corpus callosum (CC) and cortico-spinal tract (CST), and provide validation through Monte-Carlo simulations. For a given number of unique directions ($N = 12$ in our experiment), we show significant improvement in using an adaped set of gradient directions over an isotropic sampling scheme.

2 Reducing error propagation for a given tensor distribution

The focus ring of a camera allows one to take a picture making the edges of an item or a person appear sharper, while the remaining area of the photograph remains blurred. There may be a similar trade-off in diffusion MRI, in case one is only interested in a region of interest (ROI) within the field of view of the scan. In this section, we explain how it is possible, with some knowledge on the tensor characteristics in the ROI, to design an acquisition scheme that may be optimal for this ROI.

2.1 Error propagation to tensor estimate

Under the assumption of Gaussian diffusion of water molecules, the attenuated diffusion signal is given by the well known Stejskal-Tanner equation:

$$s_i = s_0 \exp(-b\mathbf{g}_i^T \mathbf{D} \mathbf{g}_i) \tag{1}$$

where s_0 is the signal without diffusion gradient, s_i is the attenuated signal corresponding to the gradient direction $\mathbf{g}_i$, with the corresponding diffusion weighting factor given by the b-value, and $\mathbf{D}$ is the diffusion tensor, representing the covariance matrix of displacement of water molecules during the effective diffusion time.

If we rewrite the tensor $\mathbf{D}$ in a vector form (note the $\sqrt{2}$ factor in front of the off-diagonal elements, ensuring isometric properties of the transform, as reminded by Basser and Pajevic [14]):

$$\mathbf{d} = [D_{xx}\ D_{yy}\ D_{zz}\ \sqrt{2}D_{xy}\ \sqrt{2}D_{xz}\ \sqrt{2}D_{yz}]^T, \tag{2}$$

then the model can be written in a linear form as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{d} \quad \text{where } \forall i \in \{1..N\}\ y_i = \frac{1}{b} \ln\left(\frac{s_0}{s_i}\right), \tag{3}$$

where i-th row of the measurement matrix $\mathbf{H}$ is given by:

$$\mathbf{h}_i = [g_{ix}^2\ g_{iy}^2\ g_{iz}^2\ \sqrt{2}g_{ix}g_{iy}\ \sqrt{2}g_{ix}g_{iz}\ \sqrt{2}g_{iy}g_{iz}]. \tag{4}$$

In case the tensor is reconstructed using the classical linear least-squares technique, the tensor estimate has a simple expression as a function of the log-transformed signal. Indeed if we note $\hat{\mathbf{d}}$ the estimate of the tensor, we have $\hat{\mathbf{d}} = \mathbf{H}^+\mathbf{y}$, where $\mathbf{H}^+ = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$ is the pseudo-inverse of the observation matrix, and the error on the estimate of the tensor is $\delta\hat{\mathbf{d}} = \mathbf{H}^+\delta\mathbf{y}$. From there, we can estimate the mean squared error on the estimate of the tensor (where the *mean* should be understood on the outcomes of the signal, as well as over the space of symmetric positive-definite tensors S^+):

$$\mathrm{MSE}(\hat{\mathbf{d}}) = \int_{S^+} \mathrm{E}[\delta\hat{\mathbf{d}}^T \delta\hat{\mathbf{d}}]\mathrm{p}(\mathbf{d})\mathrm{d}\mathbf{d} \tag{5}$$

where p is a probability density function on S^+, representing the prior. From equation 5, if we consider $(\delta\hat{\mathbf{d}}^T\delta\hat{\mathbf{d}})$ as a matrix rather than as a scalar value and take its trace, using $\mathrm{Tr}(\mathbf{AB}) = \mathrm{Tr}(\mathbf{BA})$ we can write:

$$\mathrm{MSE}(\hat{\mathbf{d}}) = \mathrm{Tr}\left(\mathbf{H}^+ \left(\int_{S^+} \mathrm{E}[\delta\mathbf{y}\ \delta\mathbf{y}^T]\mathrm{p}(\mathbf{d})\mathrm{d}\mathbf{d}\right)\mathbf{H}^{+T}\right) = \kappa_{\mathrm{adapt}}(\mathbf{\Theta}). \tag{6}$$

The expression in equation 6 depends on the observation matrix (and consequently on the direction set $\mathbf{\Theta}$), as well as on the signal, which in turn depends on the diffusion characteristics. This equation opens a way to derive an optimal direction set $\mathbf{\Theta}_{\mathrm{opt}}$ for the given prior tensor distribution p. We call this cost function κ_{adapt}, with reference to the index κ defined by Papadakis [2].

2.2 Taking into account the prior

Given a prior tensor distribution, we would like to estimate the mean covariance matrix on the log-measurement. The magnitude MR signal is known to be Rician distributed, however if the SNR is sufficiently high (which is a reasonable assumption for clinical DTI sequence, where moderate b-value are involved), it is a good approximation to consider the noise on s_i as Gaussian (see Sijbers' Ph.D dissertation [15] for reference). The measurement y_i is obtained by a non-linear transform of the signal, however a first-order approximation can accurately describe the noise on $\mathbf{y}$ as Gaussian, as initially proposed by Basser and colleagues [1], and its covariance matrix is, for a single tensor:

$$\mathrm{E}[\delta\mathbf{y}\ \delta\mathbf{y}^T] = \mathrm{diag}\left(\frac{\sigma^2}{\tilde{s}_i^2}\right), \tag{7}$$

where $\tilde{s}_i$ is the expected value of the signal. This expression can be integrated over the space of tensors, to evaluate the mean covariance matrix $\mathbf{\Sigma}$. In this equation, σ represents the noise standard deviation of the signal, and is supposed equal for all diffusion images.

2.3 Energy minimization

The problem of finding a suitable direction set then comes to minimizing a cost function, over the space of direction sets $(\mathcal{S}^2)^N$, where the unit half-sphere $\mathcal{S}^2$ can be parameterized by angles $(\vartheta, \varphi) \in [0, \pi/2] \times [0, 2\pi]$. The relation between this parameterization with angles and the diffusion gradients is simply:

$$\mathbf{g}_i = [\sin(\vartheta_i)\cos(\varphi_i)\ \ \sin(\vartheta_i)\sin(\varphi_i)\ \ \cos(\vartheta_i)]^T \tag{8}$$

We recall the cost function hereafter:

$$\kappa_{\mathrm{adapt}}(\mathbf{\Theta}) = \mathrm{Tr}\left(\mathbf{H}(\mathbf{\Theta})^{+}\mathbf{\Sigma}(\mathbf{\Theta})\mathbf{H}(\mathbf{\Theta})^{+T}\right) \tag{9}$$

for which we calculate an analytical expression of the partial derivatives, so as to fasten the gradient computation used in the minimization procedure.

The partial derivative of κ_{adapt} with respect to the i-th angle ϑ_i is (similar expression is obtained for $\partial\kappa_{\mathrm{adapt}}/\partial\varphi_i$):

$$\frac{\partial\kappa_{\mathrm{adapt}}}{\partial\vartheta_i} = \mathrm{Tr}\left(2\mathbf{\Sigma}\mathbf{H}^{+T}\frac{\partial\mathbf{H}^{+}}{\partial\vartheta_i}\right) + \mathrm{Tr}\left(\mathbf{H}^{+}\frac{\partial\mathbf{\Sigma}}{\partial\vartheta_i}\mathbf{H}^{+T}\right) \tag{10}$$

where the derivative of the pseudoinverse matrix can be calculated as:

$$\frac{\partial\mathbf{H}^{+}}{\partial\vartheta_i} = (\mathbf{H}^T\mathbf{H})^{-1}\left(\frac{\partial\mathbf{H}}{\partial\vartheta_i} - \left(\frac{\partial\mathbf{H}}{\partial\vartheta_i}^T\mathbf{H} + \mathbf{H}^T\frac{\partial\mathbf{H}}{\partial\vartheta_i}\right)\mathbf{H}^{+}\right) \tag{11}$$

As the matrix $\mathbf{H}$ is calculated as a function of the angles ϑ_i and φ_i (involving sine, cosine and square functions, see equations 4 and 8), the partial derivative $\partial\mathbf{H}/\partial\vartheta_i$ has a straightforward expression.

3 Validation

We validated our method on a scan of a human brain, acquired on a 3T scanner with 41 gradient directions and a baseline image, under a b-value of $700\text{mm}^2\text{s}^{-1}$. Once reconstructed from these images, the tensor field was considered as our gold standard.

3.1 Some sample tensor distribution on real dataset

As explained in the previous section, our method may find an interest in situations our prior on tensor field significantly deviates from the isotropic distribution. To show that such ROI exist in practice, we selected two different structures in the white matter of the brain, namely the corpus callosum (CC) and the cortico-spinal tract (CST). We estimated the tensor orientation distribution function over the set of voxels within these ROI. The construction of these ROI we performed by segmentation on the previously reconstructed orientation distribution function (ODF) image, using Descoteaux and colleagues' algorithm [16].

As in these regions, tensors have similar anisotropy values, we only focused on the distribution of the principal eigenvector of tensors. We estimated this distribution function through kernel density estimation, using a symmetrized Von Mises-Fischer kernel. The results are plotted in figures 1 and 2. As expected, the tensors within the CC have a preferred orientation along the left-right axis, as this region of white matter connects the two brain hemisphere together. More surprising is that within the CST, although this structure is more complex, we also report a distribution which is far from being uniform.

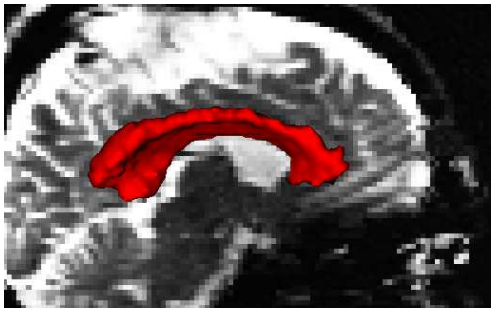
3.2 Adaptive choice of directions

In order to evaluate the gain of using adapted directions, we compare the reconstruction accuracy for isotropic directions [5] and adapted directions, constructed as minimizer of the index κ_{adapt}. Minimization was performed through steepest gradient descent, taking an isotropic direction set as an initial solution. This was calculated for the collection of 5199 tensors contained in the CC, as depicted on figure 1.

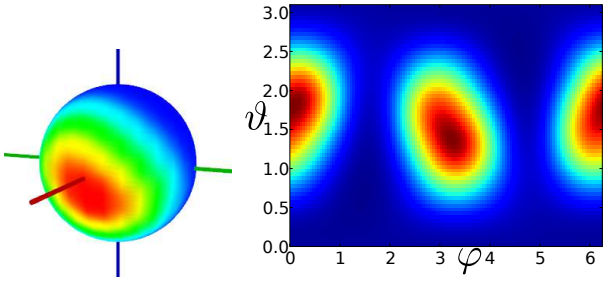
The optimal directions generated are plotted in figure 3, together with isotropic directions. We can see that the index κ_{adapt} tends to prefer directions oriented within the sagittal plane, which corresponds to the plane orthogonal to the preferred directions of fibres within the CC. It follows the intuition, in the sense that the signal we measure when gradient is applied orthogonal to the fibers is less attenuated than along the fibers, hence the SNR is higher.

3.3 Improvement on tensor reconstruction

To evaluate the accuracy of the reconstructed tensor, we performed Monte-Carlo simulations of DWI corrupted by Rician noise. The SNR, calculated on the baseline B0 image, ranged from 10 to 100. The tensor was reconstructed using linear



(a) Corpus Callosum

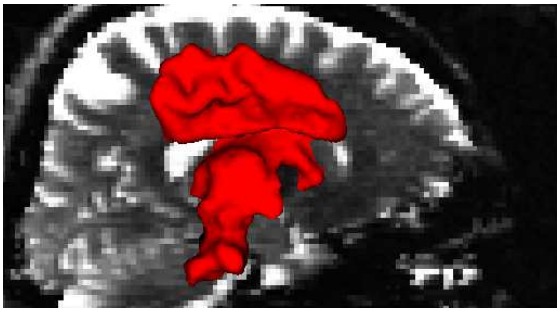


(b) Principal diffusion direction distribution

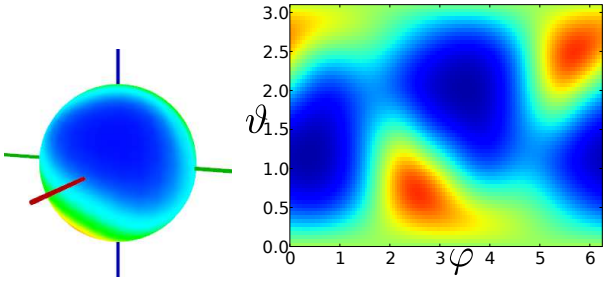
Fig. 1. Principal diffusion direction distribution within the CC. We represent here the principal eigenvector distribution function of the tensors, computed as a blurred indicator function on the sphere. Red, green and blue axes represent the left-right, dorso-ventral and anteroposterior axes, respectively. For clarity, we also projected the sphere on $[0, \pi] \times [0, 2\pi]$, using parameterization by azimut and elevation angles.

least-squares from these diffusion images. These simulation and reconstruction steps were repeated 3000 times, which seemed to be enough for stable computation of the bias and standard deviation of Trace and Fractional Anisotropy (FA), as well as Mean Squared Error. All these statistics were computed voxel-wise, then averaged over the ROI. We sum up the results on figure 4.

We can improve the average MSE, by a factor of 5 to 10%; this was an expected result, as the index κ_{adapt} is directly related to the expected bayesian MSE. Regarding the scalar indices calculated on the tensor, our method also allows us a gain of about 2%. This shows that we can outperform isotropic sampling in terms of reconstruction accuracy, in this case the tensor distribution is not uniform and we have a prior on it.



(a) Cortico-Spinal Tract



(b) Principal diffusion direction distribution

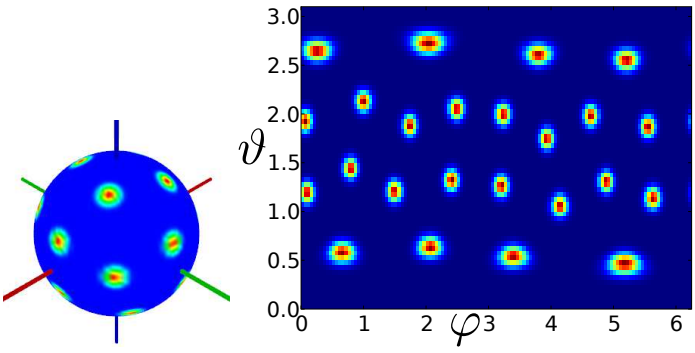
Fig. 2. Principal diffusion direction distribution within the CST.

4 Discussion and conclusion

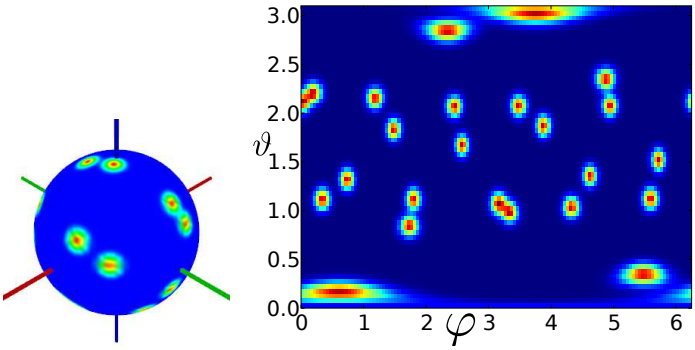
We have proposed an original and efficient method to calculate sampling gradient directions set in diffusion tensor MRI, using prior knowledge on diffusion characteristics. We show promising results on Monte-Carlo simulated data, for a tensor field within the human brain. Application of our technique have a great potential interest in diffusion imaging of the spinal cord, an important and hot topic of study (see the recent ISMRM conference). We are currently working on a validation of our technique for such region. Note that our method implies that we are able to define the sampling directions in a frame related to the patient. This is indeed made possible by most clinical scanners, which allow one to manually rotate a frame, using a fast sequence called localizer prior to acquisition.

References

1. Basser, P.J., Mattiello, J., LeBihan, D.: Estimation of the effective self-diffusion tensor from the nmr spin echo. J Magn Reson B **103**(3) (March 1994) 247–254
2. Papadakis, N.G., Xing, D., Huang, C.L.H., Hall, L.D., Carpenter, T.A.: A comparative study of acquisition schemes for diffusion tensor imaging using mri. Journal of Magnetic Resonance **137**(1) (March 1999) 67–82



(a) Isotropic directions

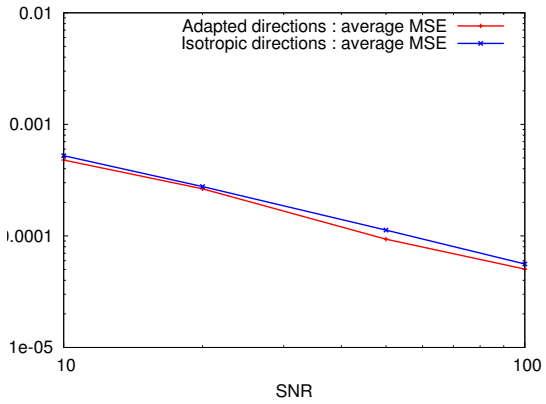


(b) Adapted directions for the CC

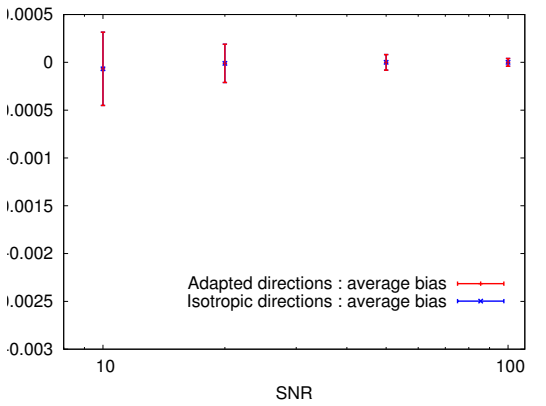
Fig. 3. Isotropic and adapted directions for the CC.

3. Skare, S., Hedehus, M., Moseley, M.E., Li, T.Q.: Condition number as a measure of noise performance of diffusion tensor data acquisition schemes with mri. J Magn Reson **147**(2) (December 2000) 340–352
4. Jones, D.K., Horsfield, M.A., Simmons, A.: Optimal strategies for measuring diffusion in anisotropic systems by magnetic resonance imaging. Magn Reson Med **42**(3) (September 1999) 515–525
5. Papadakis, N.G., Murrills, C.D., Hall, L.D., Huang, C.L.H., Carpenter, T.A.: Minimal gradient encoding for robust estimation of diffusion anisotropy. Magnetic Resonance Imaging **18**(6) (2000) 671 – 679
6. Dubois, J., Poupon, C., Lethimonnier, F., Le Bihan, D.: Optimized diffusion gradient orientation schemes for corrupted clinical DTI data sets. MAGMA **19(3)** (2006) 134–43
7. Cook, P.A., Symms, M., Boulby, P.A., Alexander, D.C.: Optimal acquisition orders of diffusion-weighted mri measurements. Journal of magnetic resonance imaging : JMRI **25**(5) (May 2007) 1051–8

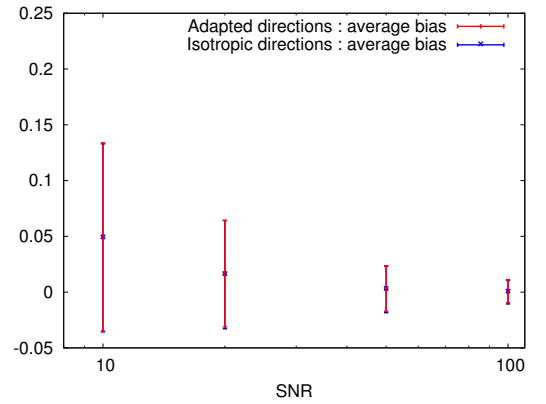
8. Deriche, R., Calder, J., Descoteaux, M.: Optimal real-time q-ball imaging using regularized kalman filtering with incremental orientation sets. Medical Image Analysis **13**(4) (August 2009) 564–579
9. Tuch, D.S.: High angular resolution diffusion imaging reveals intravoxel white matter fiber heterogeneity. Magn Reson Med **48** (2002) 577–582
10. Landman, B.A., Farrell, J.A., Jones, C.K., Smith, S.A., Prince, J.L., Mori, S.: Effects of diffusion weighting schemes on the reproducibility of dti-derived fractional anisotropy, mean diffusivity, and principal eigenvector measurements at 1.5t. NeuroImage **36**(4) (2007) 1123 – 1138
11. Batchelor, P.G., Atkinson, D., Hill, D.L., Calamante, F., Connelly, A.: Anisotropic noise propagation in diffusion tensor mri sampling schemes. Magn Reson Med **49**(6) (June 2003) 1143–1151
12. Peng, H., Arfanakis, K.: Diffusion tensor encoding schemes optimized for white matter fibers with selected orientations. Magn Reson Imaging **25** (Feb 2007) 147–153
13. Gao, W., Zhu, H., Lin, W.: A unified optimization approach for diffusion tensor imaging technique. NeuroImage **44**(3) (February 2009) 729–741
14. Basser, P.J., Pajevic, S.: Spectral decomposition of a 4th-order covariance tensor: Applications to diffusion tensor mri. Signal Process. **87**(2) (2007) 220–236
15. Sijbers, J.: Signal and noise estimation from magnetic resonance images. PhD thesis, Universiteit Antwerpen (1998)
16. Descoteaux, M., Deriche, R.: High angular resolution diffusion mri segmentation using region-based statistical surface evolution. Journal of Mathematical Imaging and Vision **33** (Feb 2009) 239–252



(a) Mean Squared Error on tensor



(b) Bias on Trace



(c) Bias on FA

Fig. 4. The MSE as well as bias on scalar parameters were estimated through Monte-Carlo simulations. We plot here these statistics averaged over the considered ROI.