Fuel-Optimal Trajectories for Continuous-Thrust Orbital Rendezvous with Path Constraints

Richard Epenoy

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Outline

Problem statement

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Dynamical equations

Tschauner-Hempel equations in Hill's frame

- Keplerian motions - Small intersatellite distance
- (a, e, v): semi-major axis, eccentricity and true anomaly of the target satellite

- $X(v)$, $Y(v)$, $Z(v)$: relative coordinates of the chaser

$$\begin{bmatrix} x_1(v) \\ x_2(v) \\ x_3(v) \end{bmatrix} = (1 + ecos(v)) \begin{bmatrix} X(v) \\ Y(v) \\ Z(v) \end{bmatrix}$$

- $x_4(v)$, $x_5(v)$, $x_6(v)$: derivatives of $x_i(v), (i = 1, ..., 3)$ w.r.t. v
- $m(v)$: mass of the chaser at true anomaly v
- $u(v)$: normalized thrust vector of the chaser at true anomaly v

Problem statement
Optimal control formulation

State-constrained minimum-fuel rendezvous (1/2)

The problem to solve

$$
(P)\begin{cases}
\text{Find} \quad \overline{u} = \underset{u}{\operatorname{argmin}}\; J(u) = -m(v_f) \\
\text{s.t.} \\
\dot{x}(v) = A(v)x(v) + B(v)\,\dfrac{u(v)}{m(v)} \\
\dot{m}(v) = -c(v)\,\|u(v)\| \\
\|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\
g\,(v, x(v)) \leq 0 \quad v \in [v_0, v_f] \\
x(v_0) = x_0 \qquad h\,(x(v_f)) = 0 \\
m(v_0) = m_0
\end{cases}
$$

State-constrained minimum-fuel rendezvous (2/2)

Collision avoidance constraint

$$g\left(v, x(v)\right) = 1 - \frac{\sqrt{x_1(v)^2 + x_2(v)^2 + x_3(v)^2}}{d_{min}(1 + e\cos(v))} \leq 0 \quad v \in [v_0, v_f]$$

Key parameter

d_{min}: minimum safety distance between the chaser and the target

Main issues for shooting methods

- The control is bang-off-bang $\Longrightarrow$ numerical difficulties
- State constraint $\Longrightarrow$ the number and location of constrained arcs must be defined beforehand in order to build the MPBVP

Solving the path-constrained rendezvous problem

Solving the path-constrained rendezvous problem

Smoothing the bang-off-bang control

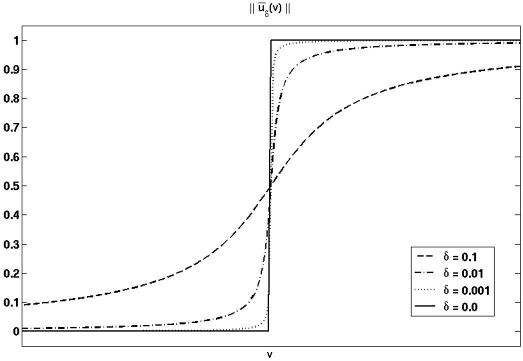
Problem regularization (1/2)

A logarithmic barrier approach

$$
(P)_\delta \begin{cases}
\text{Find} \quad \overline{u}_\delta = \underset{u}{\operatorname{argmin}}\, J_\delta(u) = J(u) - \delta \int_{v_0}^{v_f} F\left(v, u(v)\right) dv \\
\text{s.t.} \\
\dot{x}(v) = A(v)x(v) + B(v)\,\dfrac{u(v)}{m(v)} \\
\dot{m}(v) = -c(v)\,\|u(v)\| \\
\|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\
g\left(v, x(v)\right) \leq 0 \quad v \in [v_0, v_f] \\
x(v_0) = x_0 \qquad h\left(x(v_f)\right) = 0 \\
m(v_0) = m_0
\end{cases}
$$

$$F(v, u(v)) = c(v)\left(log(\|u(v)\|) + log(1 - \|u(v)\|)\right)$$

Problem regularization (2/2)

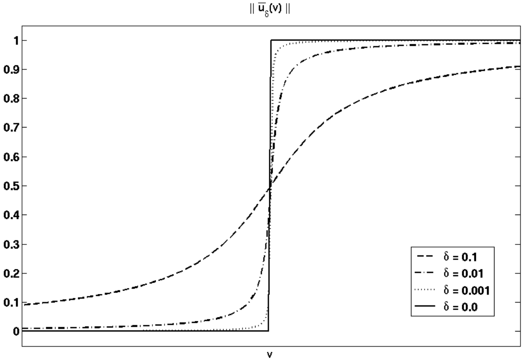


Convergence results - No state constraint

$J_\delta\left(\overline{u}_\delta\right) \to J(\overline{u})$ as $\delta \to 0$

$\overline{u}_\delta \to \overline{u}$ for the weak-* topology on $L^\infty\left(\left[v_0, v_f\right], \mathbb{R}^3\right)$ as $\delta \to 0$

Problem regularization (2/2)



Reference

R. Epenoy and R. Bertrand: New Smoothing Techniques for Solving Bang-Bang Optimal Control Problems - Numerical Results and Statistical Interpretation, *Optimal Control Applications and Methods*, Vol. 23, No. 4, 2002, pp. 171-197.

Solving the path-constrained rendezvous problem
A new approach to deal with the state constraint

The smoothed exact penalty method

The penalized problem $(P)^2_{\sigma,\alpha,\epsilon}$

$$
\begin{cases}
\text{Find } u^2_{\sigma,\alpha,\epsilon} = \underset{u}{\text{argmin}}\ J^2_{\sigma,\alpha,\epsilon}(u) = J_\delta(u) + \int_{v_0}^{v_f} \psi_{\sigma,\alpha,\epsilon}\left(g(v, x(v))\right) dv \\
\text{s.t.} \\
\dot{x}(v) = A(v)x(v) + B(v)\,\dfrac{u(v)}{m(v)} \\
\dot{m}(v) = -c(v)\,\|u(v)\| \\
\|u(v)\| \leq 1 \quad v \in [v_0, v_f] \\
x(v_0) = x_0 \qquad h\left(x(v_f)\right) = 0 \\
m(v_0) = m_0
\end{cases}
$$

$$
\psi_{\sigma,\alpha,\epsilon}(z) = \sigma \, log\left(1 + exp\left[\frac{z}{\sigma\epsilon}\left(1 + \frac{\epsilon}{\alpha - z}\right)\right]\right) \underset{\sigma\to 0}{\longrightarrow} Max\left\{0, \frac{z}{\epsilon}\left(1 + \frac{\epsilon}{\alpha - z}\right)\right\}
$$

The smoothed exact penalty method

The penalized problem $(P)^2_{\sigma,\alpha,\epsilon}$

$$\begin{cases} \text{Find } u^2_{\sigma,\alpha,\epsilon} = \underset{u}{\operatorname{argmin}}\ J^2_{\sigma,\alpha,\epsilon}(u) = J_\delta(u) + \int_{v_0}^{v_f} \psi_{\sigma,\alpha,\epsilon}\left(g(v, x(v))\right) dv \\ \text{s.t.} \\ \dot{x}(v) = A(v)x(v) + B(v)\,\dfrac{u(v)}{m(v)} \\ \dot{m}(v) = -c(v)\left\|u(v)\right\| \\ \left\|u(v)\right\| \leq 1 \quad v \in [v_0, v_f] \\ x(v_0) = x_0 \qquad h\left(x(v_f)\right) = 0 \\ m(v_0) = m_0 \end{cases}$$

G. Liuzzi and S. Lucidi: A Derivative-Free Algorithm for Inequality Constrained Nonlinear Programming via Smoothing of an l_∞ Penalty Function, *SIAM Journal on Optimization*, Vol. 20, No. 1, 2009, pp. 1-29.

Algorithm

Let $0 < q_1 < q_2 < 1, \quad 0 < \alpha_{lim} < \alpha_0, \quad \epsilon_0 > 0, \quad \sigma_0 > \alpha_{lim}^{q_1}, \quad 0 < \theta < 1, \quad 0 < \tau < 1$
Let $k = 0$, end = false

WHILE (end = false)
  Solve problem $(P)^2_{\sigma_k,\alpha_k,\epsilon_k} \to \left(x^2_{\sigma_k,\alpha_k,\epsilon_k}, m^2_{\sigma_k,\alpha_k,\epsilon_k}, u^2_{\sigma_k,\alpha_k,\epsilon_k}\right)$
  IF $(\alpha_k \leq \alpha_{lim})$ THEN
    end = true
    $(\overline{x}_\delta, \overline{m}_\delta, \overline{u}_\delta) \leftarrow \left(x^2_{\sigma_k,\alpha_k,\epsilon_k}, m^2_{\sigma_k,\alpha_k,\epsilon_k}, u^2_{\sigma_k,\alpha_k,\epsilon_k}\right)$
  ELSE
    IF $\text{Min}\left\{\epsilon_k, \int_{v_0}^{v_f} \text{Max}\left(0, g(v, x^2_{\sigma_k,\alpha_k,\epsilon_k}(v))\right) dv\right\} > \dfrac{\alpha_k^{q_2}}{\sigma_k}$ THEN
      $\epsilon_{k+1} = \tau \dfrac{\alpha_k^{q_2}}{\sigma_k}$
    ELSE
      $\epsilon_{k+1} = \epsilon_k$
    ENDIF
    $\alpha_{k+1} = \theta \alpha_k$
    $\sigma_{k+1} = \text{Min}\left\{\sigma_k, \alpha_{k+1}^{q_1}\right\}$
    $k = k + 1$
  ENDIF
END WHILE

Convergence results

Lemma

- Let $\alpha > 0$, $z < \alpha$ and $\epsilon > 0$ be given. Then, $(\sigma \to \psi_{\sigma,\alpha,\epsilon}(z))$ is strictly increasing on $]0, +\infty[$
- Let $z < 0$, $\epsilon > 0$ and $\sigma > 0$ be given. Then, $(\alpha \to \psi_{\sigma,\alpha,\epsilon}(z))$ is strictly increasing on $]0, +\infty[$

Convergence theorem - See Epenoy (2011) in JGCD

- $J^2_{\sigma_k,\alpha_k,\epsilon_k}\left(u^2_{\sigma_k,\alpha_k,\epsilon_k}\right) \to J_\delta(\overline{u}_\delta)$ as $k \to \infty$
- $u^2_{\sigma_k,\alpha_k,\epsilon_k} \to \overline{u}_\delta$ according to the weak-* topology on $L^\infty\left([v_0, v_f], \mathbb{R}^3\right)$ as $k \to \infty$
- $x^2_{\sigma_k,\alpha_k,\epsilon_k} \to \overline{x}_\delta$ uniformly on $[v_0, v_f]$ as $k \to \infty$
- $m^2_{\sigma_k,\alpha_k,\epsilon_k} \to \overline{m}_\delta$ uniformly on $[v_0, v_f]$ as $k \to \infty$

Numerical results - A rendezvous in Highly Elliptical Orbit

Numerical results - A rendezvous in Highly Elliptical Orbit

Statement of the test case

Numerical data - SIMBOL-X project

$a = 106246.9753$ km $\quad F_{max} = 0.1$ N $\quad m_0 = 960.0$ kg

$e = 0.798788 \quad Isp = 220$ s

$v_0 = 3.317940017547$ rad $\quad t_0 = 0.0$ s

$v_f = 3.349161118514$ rad $\quad t_f = 8000.0$ s

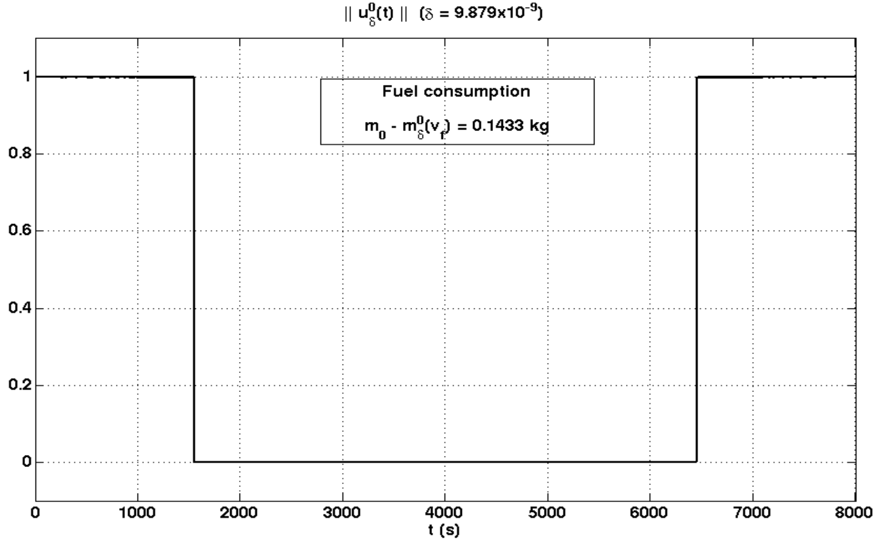
$$\begin{bmatrix} X(t_0) \\ Y(t_0) \\ Z(t_0) \end{bmatrix} = \begin{bmatrix} -100 \text{ m} \\ -100 \text{ m} \\ -100 \text{ m} \end{bmatrix} \qquad \begin{bmatrix} X(t_f) \\ Y(t_f) \\ Z(t_f) \end{bmatrix} = \begin{bmatrix} 500 \text{ m} \\ 500 \text{ m} \\ 500 \text{ m} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dX}{dt}(t_0) \\ \frac{dY}{dt}(t_0) \\ \frac{dZ}{dt}(t_0) \end{bmatrix} = \begin{bmatrix} 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \end{bmatrix} \qquad \begin{bmatrix} \frac{dX}{dt}(t_f) \\ \frac{dY}{dt}(t_f) \\ \frac{dZ}{dt}(t_f) \end{bmatrix} = \begin{bmatrix} 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \\ 0.0 \text{ m/s} \end{bmatrix}$$

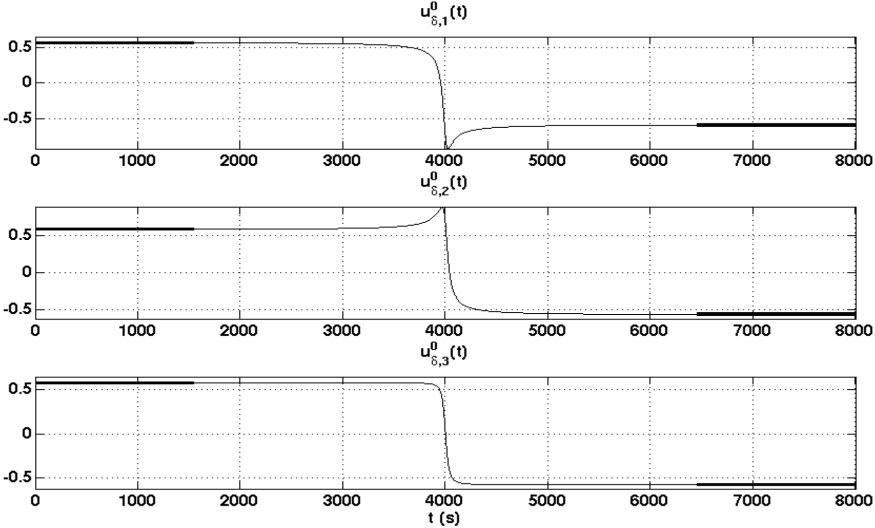
Numerical results - A rendezvous in Highly Elliptical Orbit

Unconstrained rendezvous

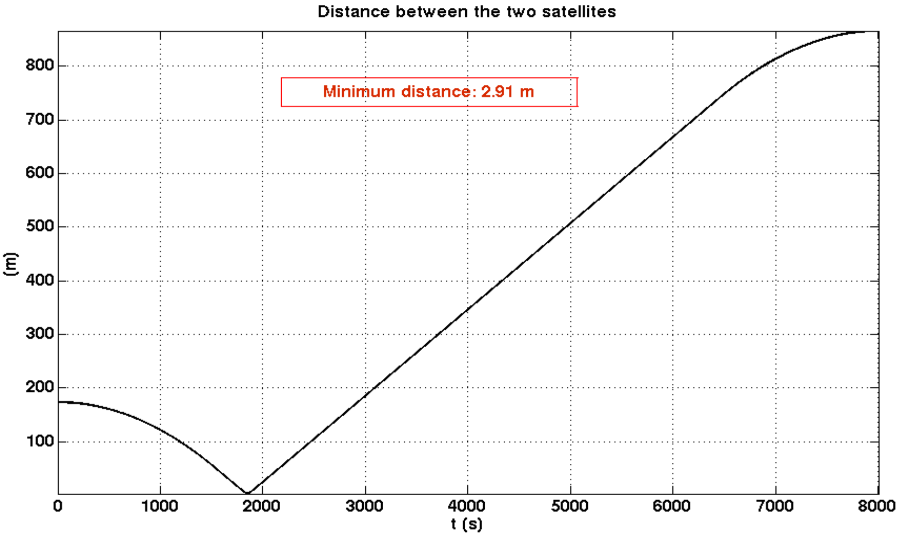
Norm of the normalized thrust vector



Normalized thrust vector



Intersatellite distance



Numerical results - A rendezvous in Highly Elliptical Orbit
Rendezvous under collision avoidance constraint

Parameters of the algorithm

Smoothing parameter: The same as for the unconstrained problem

$\delta = 9.879x10^{-9}$

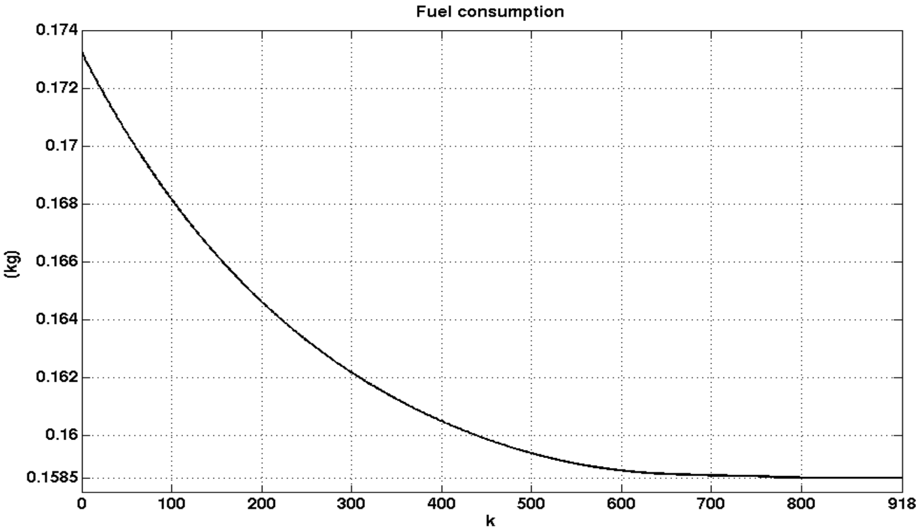
Case 1: $d_{min} = 50.0$ m

$q_1 = 0.3$	$q_2 = 0.6$	$\alpha_{lim} = 4.73x10^{-4}$	$\alpha_0 = 0.98$
$\epsilon_0 = 0.0999$	$\sigma_0 = 1.0$	$\theta = 0.99$	$\tau = 0.9$

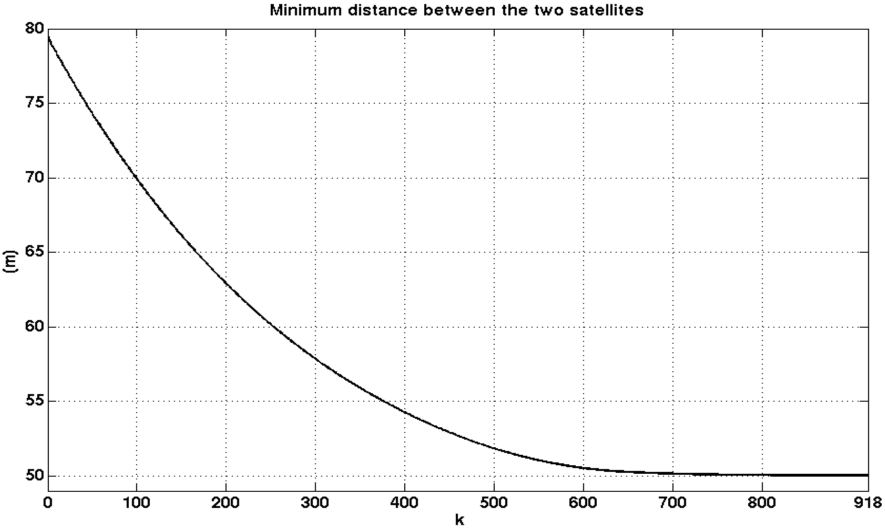
Case 2: $d_{min} = 140.0$ m

$q_1 = 0.3$	$q_2 = 0.6$	$\alpha_{lim} = 1.37x10^{-4}$	$\alpha_0 = 0.98$
$\epsilon_0 = 0.0157$	$\sigma_0 = 1.0$	$\theta = 0.99$	$\tau = 0.9$

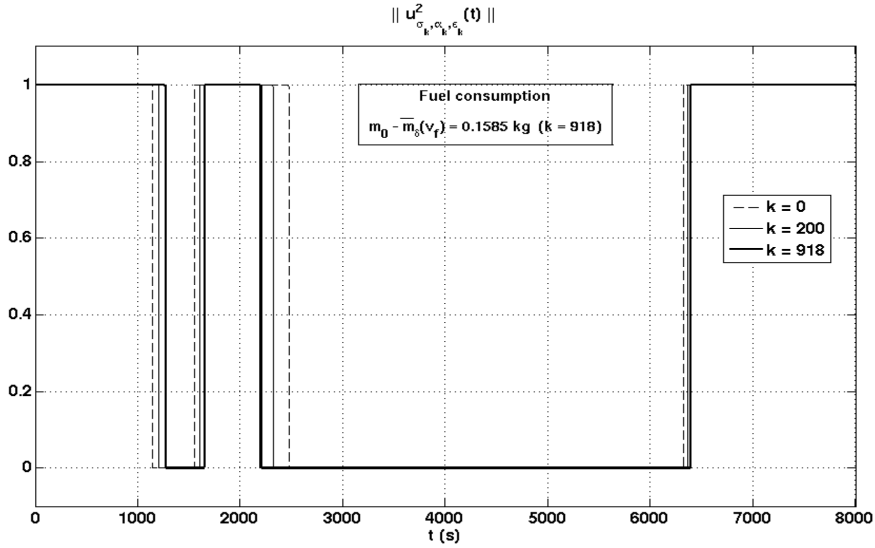
Case 1 - Fuel consumption vs. iteration index k



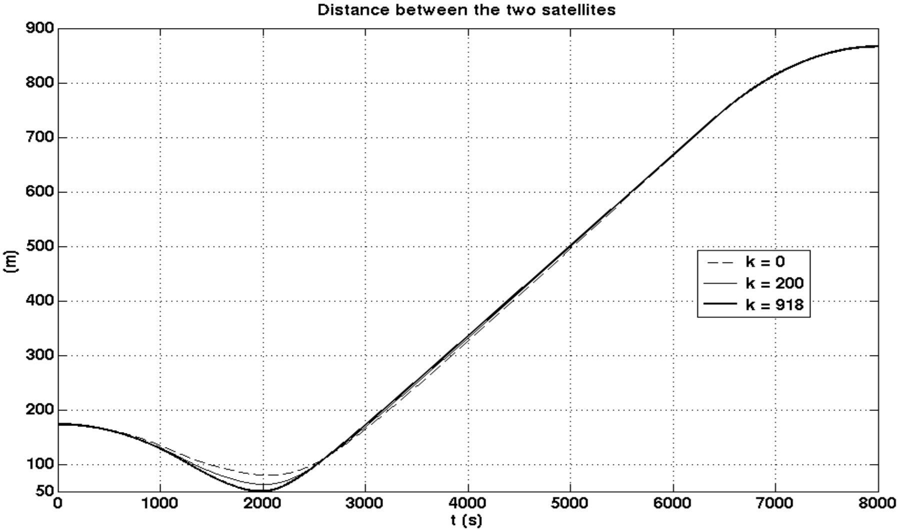
Case 1 - Minimum distance vs. iteration index k



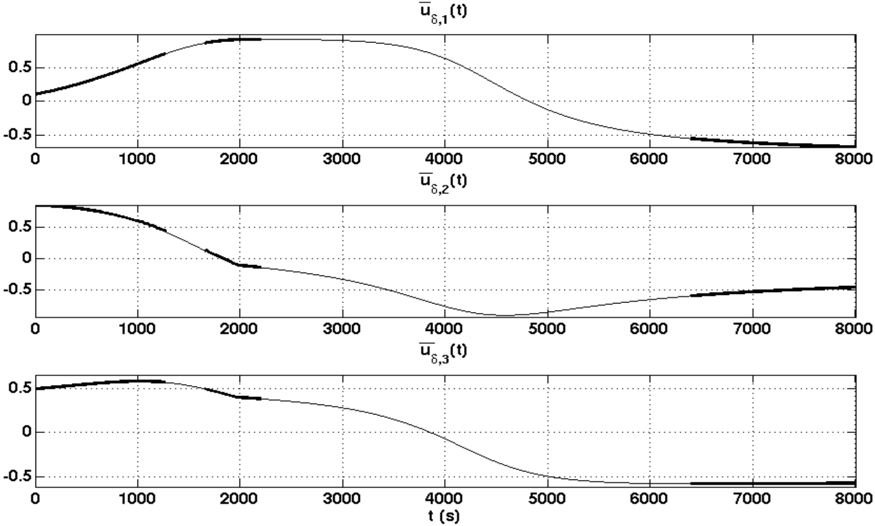
Case 1 - Norm of the normalized thrust vector



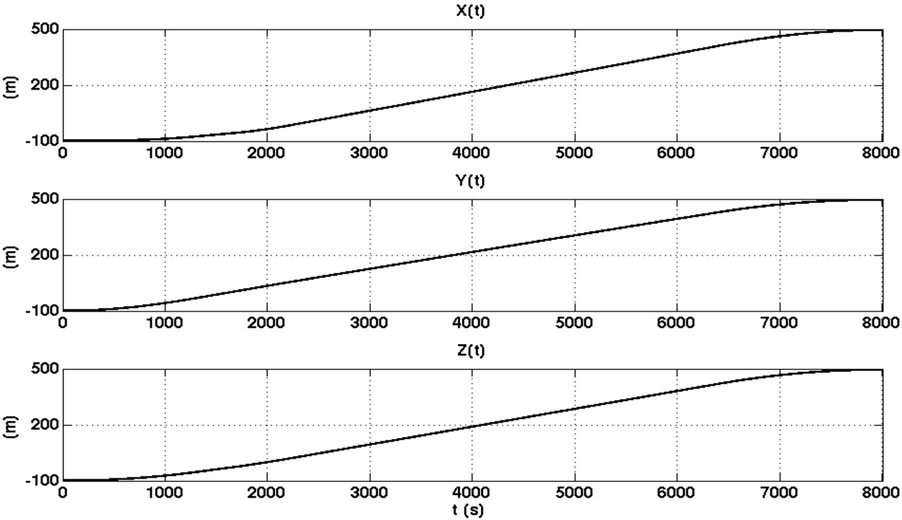
Case 1 - Intersatellite distance



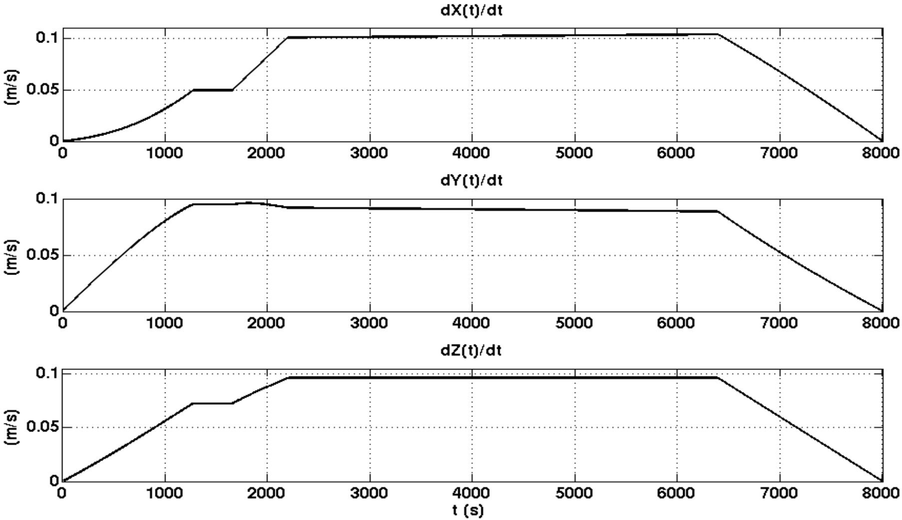
Case 1 - Normalized thrust vector



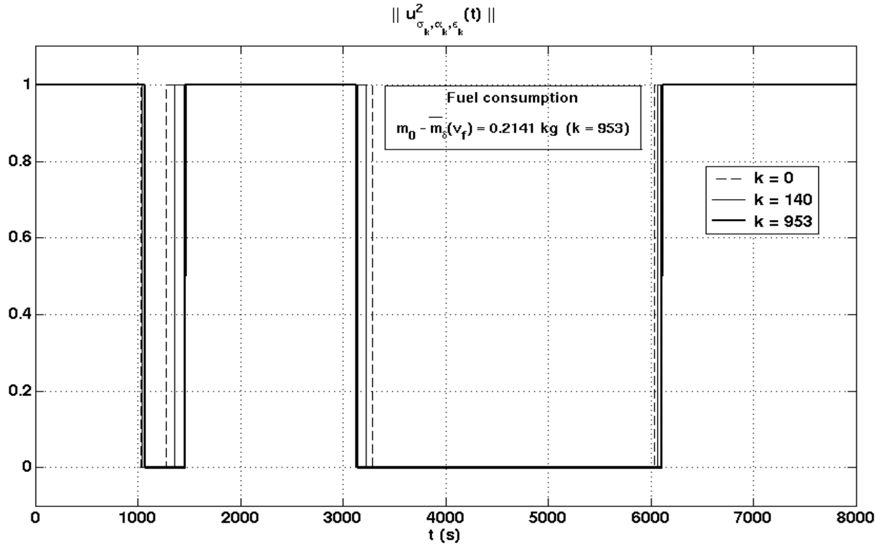
Case 1 - Relative position vector



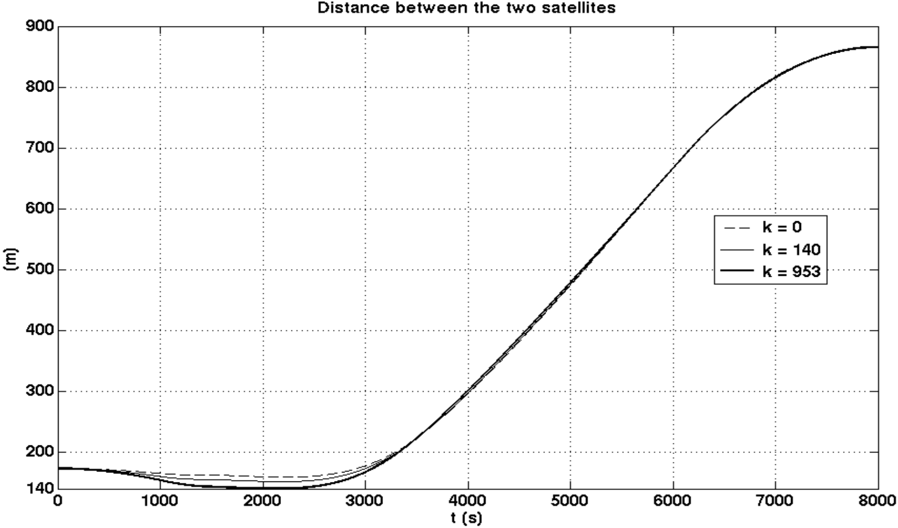
Case 1 - Relative velocity vector



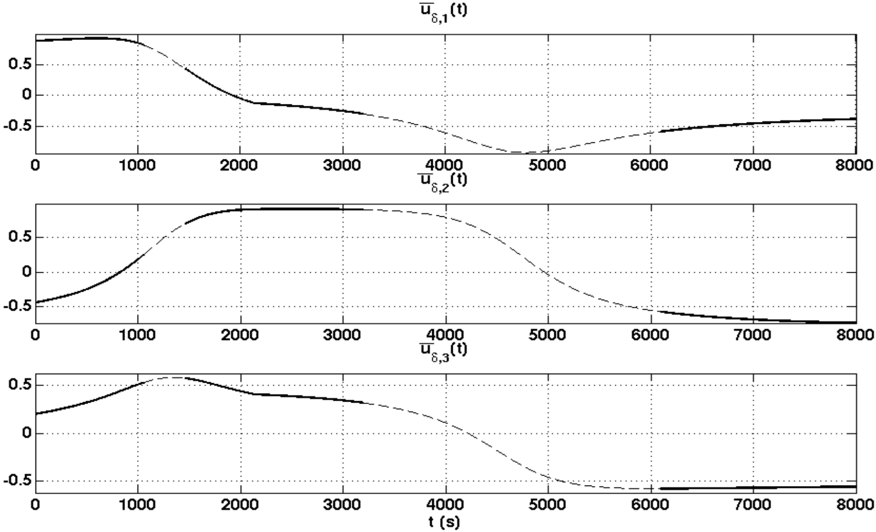
Case 2 - Norm of the normalized thrust vector



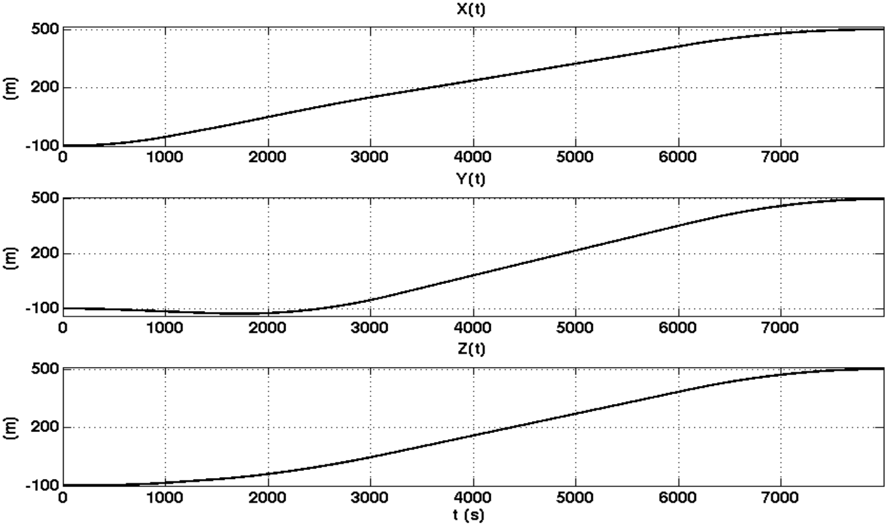
Case 2 - Intersatellite distance



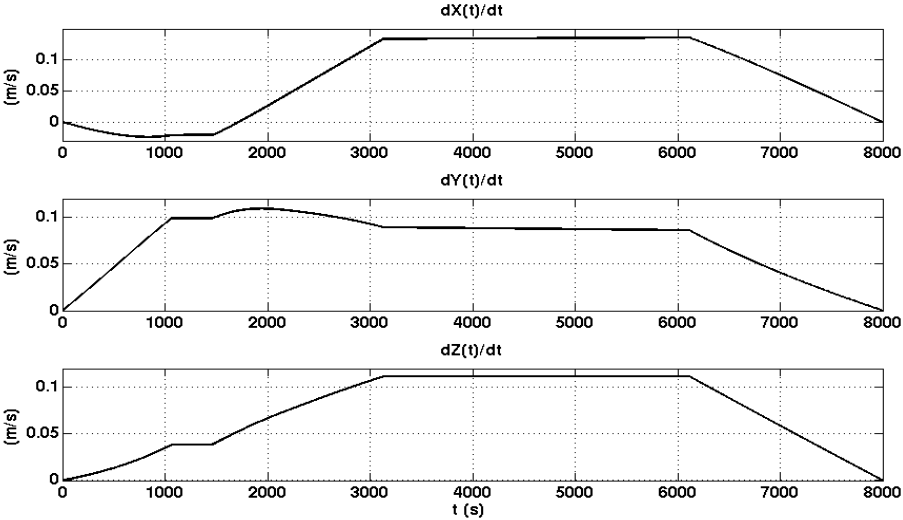
Case 2 - Normalized thrust vector



Case 2 - Relative position vector



Case 2 - Relative velocity vector



Conclusion and future prospects

Conclusion

The smoothed exact penalty approach

- Applied for the first time to an optimal control problem
- Efficient to deal with the collision avoidance constraint
- Implies solving a sequence of state constraint-free problems
- More theoretically based than other penalization techniques
- Can be used to solve a large class of problems

A just published paper

R. Epenoy: Fuel Optimization for Continuous-Thrust Orbital Rendezvous with Collision Avoidance Constraint, *Journal of Guidance, Control and Dynamics*, Vol. 34, No. 2, March-April 2011.

Future prospects

Minimum-fuel rendezvous in perturbed environment

Necessity to modify the dynamical equations

Low Earth Orbit applications $\Longrightarrow$ J_2 term of the Earth's potential

Application to other state-constrained problems

Reentry trajectories under thermal flux constraint

Interplanetary trajectories with minimum flyby altitude constraint

Toward closing the loop

Model Predictive Control methodology

Neighboring extremal paths in the presence of state constraints

Hamilton-Jacobi-Bellman equation for state-constrained problems

Thank you for your attention