A Multigrid Method for the Solution of Linear Systems with Multiple Right-Hand Sides

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In a wide number of applications in computational science and engineering the solution of linear systems of equations with several right-hand sides given at once is required. Direct methods based on Gaussian elimination are known to be popular in that setting. Nevertheless if the dimension of the problem is very large or if matrix-free algorithms are preferred, preconditioned block Krylov space solvers [2] are often considered as the method of choice.

For certain classes of problems related to elliptic or parabolic partial differential equations, geometric or algebraic multigrid methods [4] are known to be especially appealing. Indeed such methods lead to scalable algorithms since the total amount of operations required to solve a given linear system is directly proportional to the problem size. To the best of our knowledge the case of multiple right-hand sides in the framework of multigrid methods has been rarely addressed in the literature. Common approaches rely either on using a standard multigrid cycle with components operating on block vectors or solving an augmented system based on the general matrix form of multilevel projection methods initially developed in [1, 3]. Thus the purpose of this talk is to present a new variant suited for the solution of such systems with multiple right-hand sides, where multigrid could be applied as a solver or as a preconditioner.

First we introduce a two-grid cycle that includes a strategy for detecting when a linear combination of the linear systems has approximately converged. This explicit block size reduction leads to a decrease in terms of computational cost as will be shown on academic examples. Then we extend this idea to the case of a multilevel hierarchy and note that this variant can be applied to geometric and algebraic multigrid methods as well. Finally we illustrate the numerical behavior of these algorithms on both academic and industrial problems, where indefinite linear systems with multiple right-hand sides have been successfully solved in a parallel distributed memory environment.

References

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