The PSOR-like preconditioner for CGS method

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Let us consider a preconditioning for the iterative method for solving a nonsymmetric sparse linear system with the approximate inverse preconditioner P,

$$PA\boldsymbol{x} = P\boldsymbol{b},$$

where A is a given $n \times n$ real matrix, $\boldsymbol{b}$ is a given real vector, and $\boldsymbol{x}$ is the solution vector which is to be determined. In 1991, Gunawardena et al. [1]proposed the modified Gauss-Seidel method in which $P = I + S$, with

$$S = (s_{ij}) = \begin{cases} -a_{ii+1} & \text{for} i = 1, 2, \cdots, n-1, j = i+1 \\ 0 & \text{otherwise} \end{cases}$$

In 2002, Kotakemori et al. [2] proposed to use $P_m = I + S_m$, where S_m is defined by,

$$S_m = (s_{ij}^m) = \begin{cases} -a_{ik_i} & \text{for} i = 1, 2, \cdots, n-1, j \geq i+1 \\ 0 & \text{otherwise} \end{cases}$$

$k_i = \min_j \{j | \max_{j>i+1} |a_{ij}\}$, for $i = 1, 2, \cdots, n-1$. In addition, the relating preconditioners are discussed [3, 4]. As you known, the combination of preconditioning and iterative method is effectiveness.

The Krylov subspace methods include popular methods such as Conjugate Gradients(CG), Bi-Conjugate Gradients(Bi-CG), Bi-CGstab, CGS and GMRES, etc. And there are the classical iterative method such as Jacobi, Gauss-Seidel and SOR. The preconditioner K approximates the coefficient matrix A under the assumption that $K^{-1}\boldsymbol{v}$ is solved more easily and faster than $A^{-1}\boldsymbol{v}$, where $\boldsymbol{v}$ is a vector which use in Krylov subspace algorithm. The Incomplete LU decomposition (ILU) and Incomplete Cholesky decomposition (IC) are more widely and frequently used method for designing the preconditioner. Recently, the methods, which compute the linear system $K\boldsymbol{z} = \boldsymbol{v}$ by a classical iterative method at each iteration of the Krylov subspace method, have proposed by many researchers. These methods are so-called Hybrid algorithm. In this paper, we propose the Hybrid algorithm which use the Preconditioned SOR-like method and a Krylov subspace methods.

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References

[1] A.D.Gunawardena, S.K.Jain and L.Snyder, Modified Iterative Methods for Consistent Linear Systems, *Linear algebra Appl.* 154-156:123-143,1991.

[2] H. Kotakemori, H.Niki, N.Okamoto, Accelerated iterative method for Z-matrices, *JCAM*,75(1996)87-97.

[3] T. Kohno, H. Niki, A note on the preconditioner $P_m = (I + S_m)$, Journal of Computational and Applied Mathematics,225:316-319, 2009.

[4] T. Kohno, H.Kotakemori, H.Niki and M.Usui, Improving Modified Gauss-Seidel Method for Z-matrices, *Linear Algebra and its Applications*,267:113-123,1997.