Exploiting the Nested Block Structure of DG Matrices: a Block ILU Preconditioner with Deflation and a Spectral Two-Level Strategy

Paulien Van Slingerland, Kees Vuik

Exploiting the Nested Block Structure of DG Matrices: a Block ILU Preconditioner with Deflation and a Spectral Two-Level Strategy

List of authors:

P. van Slingerland [1]

C. Vuik [2]

The main advantages of Discontinuous Galerkin (DG) discretizations for elliptic problems are their flexibility in handling non-matching grids and in designing *hp*-refinement strategies. However, an important drawback is that the resulting linear systems are usually large (due to the large number of degrees of freedom) and ill-conditioned. As a consequence, standard Krylov methods typically require too much computational time to solve these systems.

To increase the practical applicability of DG discretizations, we propose a preconditioning technique that is based on a nested block incomplete LU factorization. Earlier, this type of preconditioner was applied successfully in the context of finite difference discretizations for oil reservoir simulation [1, 2]. Here, we combine their ideas with deflation [3] and a spectral two-level strategy to obtain a preconditioner that is suitable for our application, and that efficiently exploits the nested block structure of the coefficient matrix. Various symmetric test cases demonstrate that this technique can lead to uniform convergence of the conjugate gradient method.

References

[1] J. R. Appleyard and I. M. Cheshire. Nested factorisation. paper SPE 12264 presented at the SPE 1983 Reservoir Simulation Symposium, San Francisco, CA., 1983.

[2] J. A. Meijerink. Iterative methods for the solution of linear equations based on incomplete block factorization of the matrix. paper SPE 12262 presented at the SPE 1983 Reservoir Simulation Symposium, San Francisco, CA., 1983.

[3] C. Vuik, A. Segal, and J. A. Meijerink. An Efficient Preconditioned CG Method for the Solution of a Class of Layered Problems with Extreme Contrasts in the Coefficients. *Journal of Computational Physics*, 152:385–403, 1999.

[1]Delft University of Technology, Delft Institute of Applied Mathematics, Mekelweg 4, 2628 CD Delft, The Netherlands (p.vanslingerland@tudelft.nl)

[2]Delft University of Technology, Delft Institute of Applied Mathematics, Mekelweg 4, 2628 CD Delft, The Netherlands (c.vuik@tudelft.nl)