

## **An incomplete multifrontal method for element-structured linear systems**

Nick Vannieuwenhoven, Karl Meerbergen

► **To cite this version:**

Nick Vannieuwenhoven, Karl Meerbergen. An incomplete multifrontal method for element-structured linear systems. International Conference On Preconditioning Techniques For Scientific And Industrial Applications, Preconditioning 2011, May 2011, Bordeaux, France. inria-00581592

**HAL Id: inria-00581592**

**<https://hal.inria.fr/inria-00581592>**

Submitted on 31 Mar 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# An incomplete multifrontal method for element-structured linear systems

*List of authors:*

N. Vannieuwenhoven <sup>1</sup>

K. Meerbergen <sup>2</sup>

In this talk, we will propose an incomplete multifrontal  $LU$ -factorization (IMF) that extends supernodal multifrontal methods to incomplete factorizations [3]. IMF can be used as a preconditioner in a Krylov-subspace method to solve large-scale sparse linear systems with an underlying element structure. In these applications, the system's coefficient matrix is a sum of small dense element matrices, i.e.

$$A = \sum_{e \in \mathcal{E}} P_e A_e P_e^T.$$

Herein,  $\mathcal{E}$  is a set of *elements* and  $P_e$  a mapping from the local system dofs to the global dofs. Such systems arise e.g. from a finite element discretization of a partial differential equation. The fact that the element matrices are dense is exploited to increase the computational performance and the robustness of the factorization (through partial pivoting within the dense matrices). We compare IMF with the, similar, multilevel ARMS [1], the level of fill-in ILU and the threshold-based ILUT [2] preconditioners. We demonstrate that IMF can attain a higher throughput than the aforementioned preconditioners, by exploiting the dense block structure in element-structured linear systems. IMF is demonstrated to be effective on linear systems derived from some incompressible flow simulation model problems, outperforming the aforementioned preconditioners by one order-of-magnitude in one instance. The preconditioner was also applied to solve general sparse systems, without an underlying element structure. It is shown to be effective and robust on some matrices from the University of Florida sparse matrix collection and the Matrix Market, provided that an artificial element structure can be extracted that is similar to a finite element discretization.

## References

- [1] Y. Saad and B. Suchomel, *ARMS: An algebraic recursive multilevel solver for general sparse linear systems*, Numerical Linear Algebra with Applications, 9 (2002), pp. 359–378.
- [2] Y. Saad, *ILUT: a dual threshold incomplete ILU factorization*, Numerical Linear Algebra with Applications, 1 (1994), pp. 387–402.
- [3] N. Vannieuwenhoven and K. Meerbergen, *IMF: An incomplete multifrontal LU-factorization for element-structured sparse linear systems*, tech. rep. TW581, KULeuven, December 2010. <http://www.cs.kuleuven.be/publicaties/rapporten/tw/TW581.abs.html>

---

<sup>1</sup>Department of Computer Science, KULeuven

<sup>2</sup>Department of Computer Science, KULeuven