Algebraic Multigrid using Energy-Minimization: a general framework to develop intergrid transfer operator

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The repeated solution of sparse linear systems is often a key computational bottleneck for large-scale computer simulations and development of highly scalable preconditioner must be addressed on the path of exascale computing. With this aim in view, we focus on multigrid methods which are among the most scalable algorithms in solving linear systems arising from the discretization of partial differential equations (PDEs). The basic idea of multigrid methods is to use coarse grid correction to handle the error which have not been eliminated by the smoothing process. Algebraic multigrid (AMG) methods construct their hierarchy of coarse systems directly from the initial matrix. The key to designing an effective AMG algorithm is to construct coarse grids and intergrid operators to obtain a good characterization in the coarse levels of error not eliminated by the smoother on the finest level. Our main intention is to extend the applicability of AMG methods to challenging problems like systems of PDEs by defining a general framework for the generation of intergrid transfer operator.

In [1], the authors propose a set of objectives to obtain an efficient prolongation operation motivated by both the convergence theory and their practical experience. They also present the Smoothed Aggregation (SA) method as an attempts to satisfy this requirements. In SA, the knowledge of near-kernel components are explicitly used to define an intergrid operator which allows an accurate representation of this specific modes on the coarser grid and the smoothing of the prolongator aims to reduce its energy. SA AMG has been applied successfully in many application domains. However, the cost to apply SA can grow unacceptably for systems of PDEs and for large number of near-kernel components. This issue becomes more pronounced for stretched meshes, large material variation, and higher dimensionality.

Considering the same set of objectives, we have developed algorithms for generating AMG grid transfers based on a more general interpolation strategy. In our energy-minimization algorithm, each column of the grid transfer operator P is minimized in an energy-based norm while enforcing two types of constraints: a defined sparsity pattern and preservation of specified modes in the range of P. The resulting constrained optimization problem can be solved by using Krylov based methods. While related to the method proposed in [2], our algorithm pays close attention to the compression of near-kernel information in order to maintain low preconditioner cost. One of the main advantages of the approach is that it is flexible, allowing for arbitrary coarsenings, unrestricted sparsity patterns, straightforward long distance interpolation, and general use of constraints, either user-defined or auto-generated.

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I will present a use case of the energy minimization algorithm on 3D linear elasticity. For this specific problem, we take advantage of the flexibility of energy-minimization algorithm to compress the information given by the near-kernel components and reduce the number of degree of freedom on coarse grids.

The algorithms presented here will be at the heart of the next multigrid solver of the Trilinos project [3, 4]. As future application will require increasingly complex and finely tuned preconditioning methods, I will discuss some aspects of the code design, which was carefully created to maximize the new algorithm's flexibility.

References

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