



Incorporating low-thrust propulsion into the invariant manifolds technique for space trajectories design

Franco Bernelli-Zazzera

► To cite this version:

Franco Bernelli-Zazzera. Incorporating low-thrust propulsion into the invariant manifolds technique for space trajectories design. SADCO A2CO, Mar 2011, Paris, France. inria-00582991

HAL Id: inria-00582991

<https://inria.hal.science/inria-00582991>

Submitted on 4 Apr 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



POLITECNICO
DI MILANO



Incorporating low-thrust propulsion into the invariant manifolds technique for space trajectories design

F. Bernelli-Zazzera, F. Topputo

Politecnico di Milano, Dipartimento di Ingegneria Aerospaziale

G. Mingotti

Universität Paderborn, Institut für Mathematik



Context

- Space trajectory design using non-Keplerian orbits
 - Transfers to libration point orbits

Aims

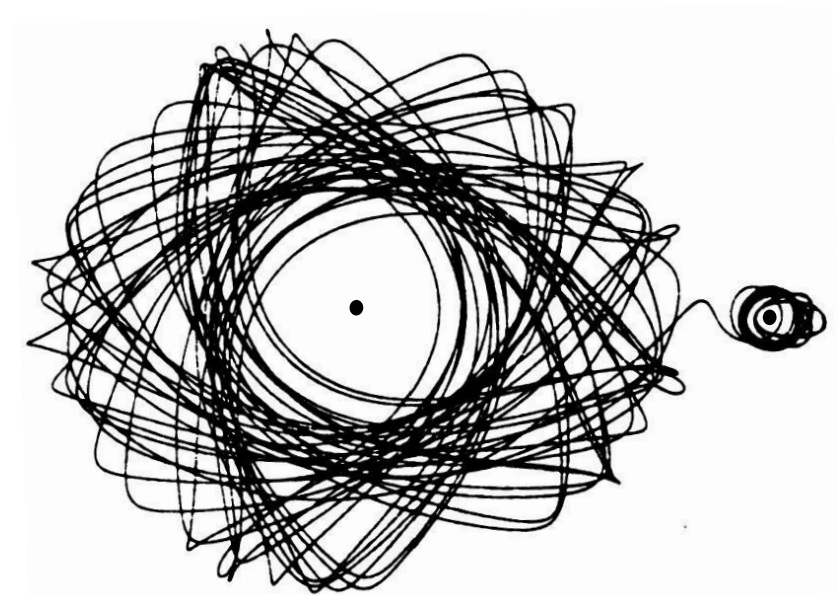
- Improve existing methods
 - Modify “invariant manifolds technique” to include low-thrust propulsion



1. Introduction to **non-Keplerian** orbits
2. Introduction to the **restricted three body problem**
3. Introduction to **equilibrium points**, **Halo orbits** and **invariant manifolds**
4. Dynamical system theory in space trajectory design
 - transfers to **Lagrange point orbits**
5. Introduction of **low-thrust** propulsion: controlled RTBP
6. Definition and manipulation of **attainable sets**
7. End-to-end trajectory **optimization**: controlled RFBP
8. Results and comparisons
 - **low-thrust, stable-manifold** transfers to halo orbits
9. Final remarks



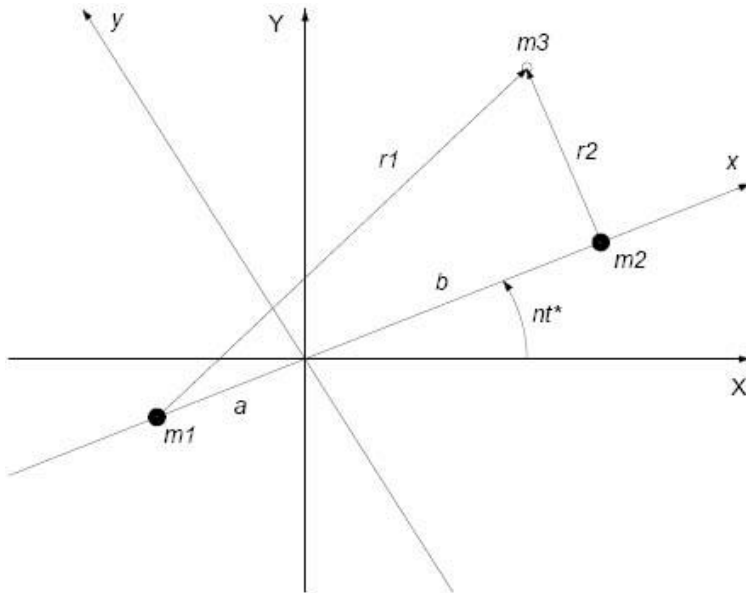
- Solution of n -body problems, $n \geq 3$
 - Unique features
 - Less propellant
 - Accurate solutions
 - Non-integrable systems
 - Chaotic dynamics
 - Long transfer times
 - No systematic approach
-
- How can we design non-Keplerian orbits for space missions?
-
- ➔ Exploit knowledge of the **phase space** (e.g., equilibrium points, periodic orbits, invariant manifolds, etc.)





The restricted three-body problem (1)

- Two *primaries* act on a third body (of negligible mass)
- The third body does not affect the motion of the primaries
- The two primaries move in circular orbits around their center of mass (circular restricted three-body problem)



The motion of the primaries is *given*:

$$\begin{aligned} X_1 &= -a \cos nt^*, & X_2 &= b \cos nt^*, \\ Y_1 &= -a \sin nt^*, & Y_2 &= b \sin nt^*. \end{aligned}$$



The restricted three-body problem (2)

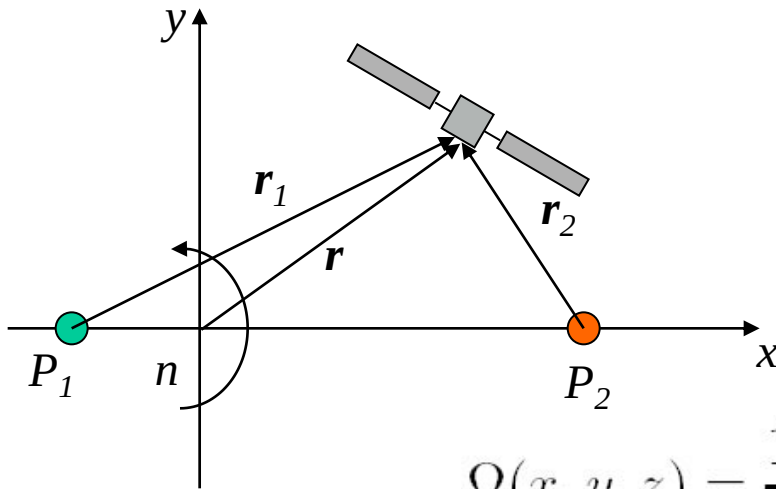
- the problem is *restricted* to the motion of the third body
- Newton equation written in *inertial* frame:

$$\left\{ \begin{array}{l} \frac{d^2 X}{dt^{*2}} = -G \left[\frac{m_1 (X + a \cos nt^*)}{R_1^3} + \frac{m_2 (X - b \cos nt^*)}{R_2^3} \right], \\ \frac{d^2 Y}{dt^{*2}} = -G \left[\frac{m_1 (Y + a \sin nt^*)}{R_1^3} + \frac{m_2 (Y - b \sin nt^*)}{R_2^3} \right], \\ \frac{d^2 Z}{dt^{*2}} = -G \left[\frac{m_1 Z}{R_1^3} + \frac{m_2 Z}{R_2^3} \right], \end{array} \right.$$



The restricted three-body problem (3)

- Equations written in a *dimensionless* rotating (*synodic*) frame make the dynamical system autonomous



$$\ddot{x} - 2\dot{y} = \Omega_x,$$

$$\ddot{y} + 2\dot{x} = \Omega_y,$$

$$\ddot{z} = \Omega_z,$$

$$\Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu)$$

$$r_1^2 = (x + \mu)^2 + y^2 + z^2, \quad r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2$$

Conventions:

$$G = 1$$

$$m_1 = 1 - \mu$$

$$m_2 = \mu$$

$$n = 1$$

$$x_1 = -\mu$$

$$x_2 = 1 - \mu$$

$$\mu = m_2 / (m_1 + m_2)$$

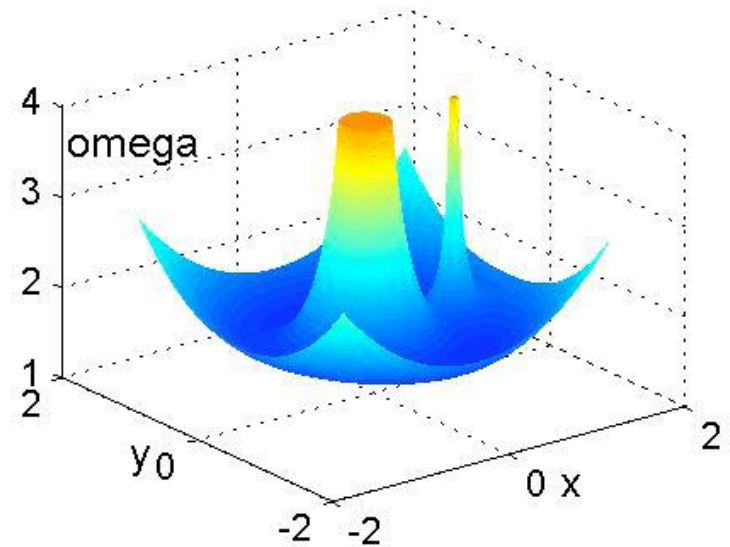
Mass parameter of the R3BP



The restricted three-body problem (4)

- The R3BP has *only one* integral of motion: the Jacobi integral
- The problem is not integrable! Qualitative studies only
- Allowed and forbidden regions are established (Hill's curves)

$$C = 2\Omega(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

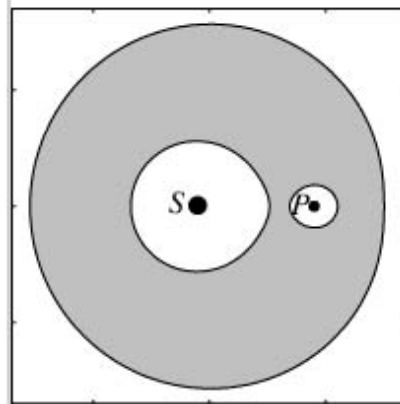


- Given x_0 , then $C_0 = C(x_0)$ and the allowed region will be (x, y, z) such that $\Omega(x, y, z) \geq C_0/2$

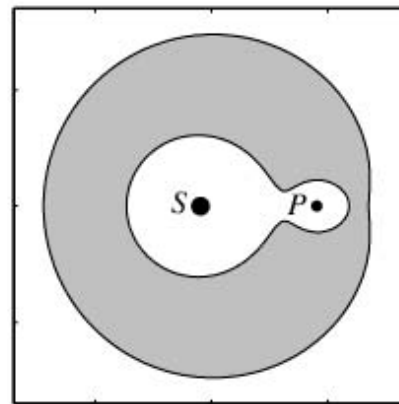


The restricted three-body problem (5)

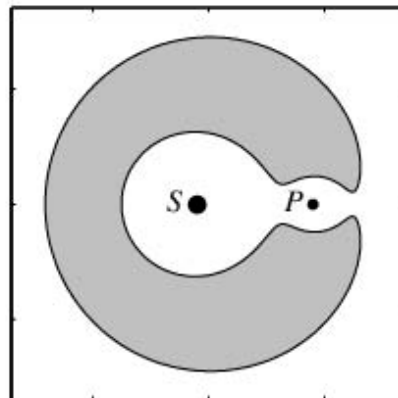
- $C = -2E$
- The higher C , the lower energy!
- For $C < 3$, the S/C is allowed to move in the whole configuration space



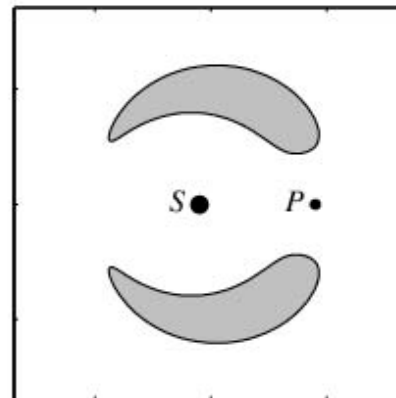
(a) Case 1 : $E < E_1$



(b) Case 2 : $E_1 < E < E_2$



(c) Case 3 : $E_2 < E < E_3$



(d) Case 4 : $E_3 < E < E_4 = E_5$



Equilibrium points (1)

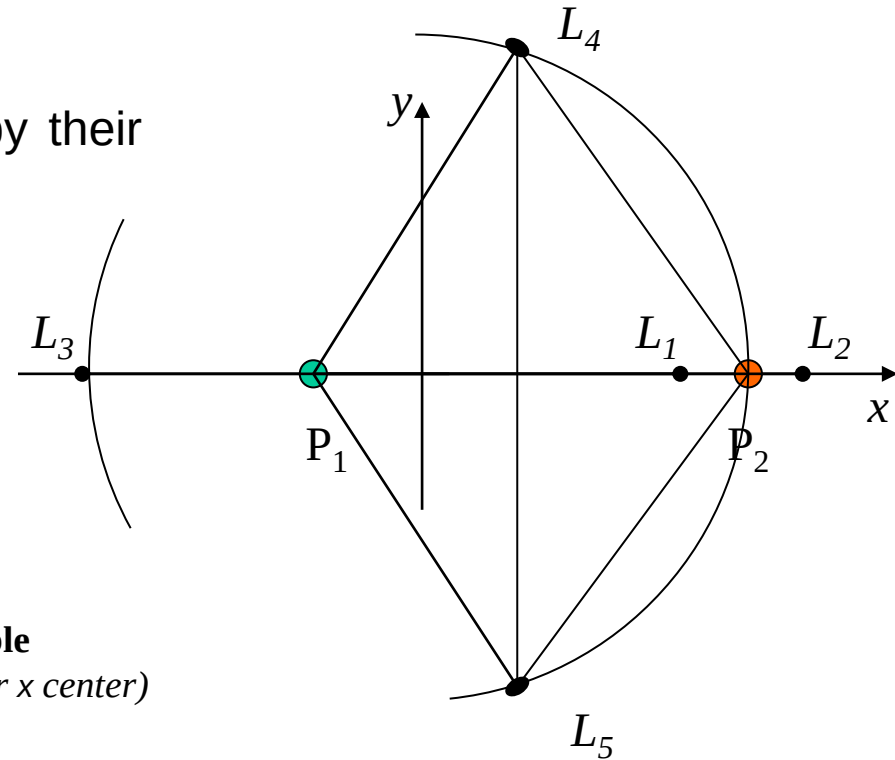
- The differential system has 5 equilibrium points (libration or Lagrangian points).
- The “nature” of the points is given by their linear behaviour.

$$\dot{x} = f(x) \longrightarrow \dot{x} = Ax \longrightarrow \lambda_i$$

$$L1, L2, L3: \begin{cases} \lambda_1 = -\lambda_2 \\ \lambda_{3/4} = \pm i\alpha_1 \longrightarrow \text{Unstable} \\ \lambda_{5/6} = \pm i\alpha_2 \quad (\text{saddle} \times \text{center} \times \text{center}) \end{cases}$$

$$L4, L5: \begin{cases} \lambda_{1/2} = \pm i\beta_1 \\ \lambda_{3/4} = \pm i\beta_2 \longrightarrow \text{Stable} \\ \lambda_{5/6} = \pm i\gamma \end{cases} \quad (\text{center} \times \text{center} \times \text{center})$$

if $\mu < \mu_0 = 0.0385$



Note: equilibrium is relative to the synodic system!



Equilibrium points (2)

- Collinear points $L1$ and $L2$ are very important for space applications
 - associated to *low energy* values (high values of C)
 - have “fast” stable and unstable dynamics (strong instability)
- *Saddle* part (stable and unstable eigenvalues)
 - Hyperbolic orbit near $L1$ and $L2$
- *Center* part (complex conjugate eigenvalues)
 - Periodic orbits around $L1$ and $L2$
- *Invariant manifolds* of the points
 - W^s_{Li} , stable manifold
 - W^u_{Li} , unstable manifold
- The invariant manifolds of the points have interesting dynamical properties that can be exploited for space applications



Invariant manifolds of equilibrium points (1)

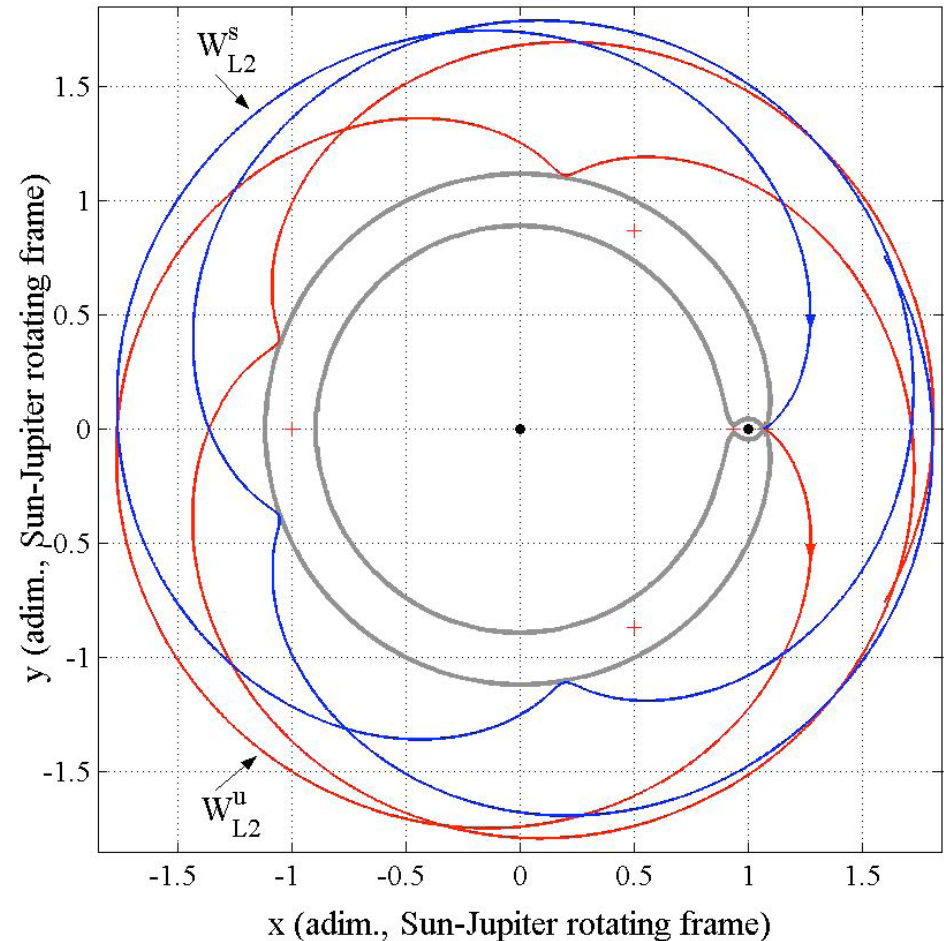
- *Nonlinear* invariant manifolds are defined in the phase space as *1D* invariant subsets such that

$$W_{L_i}^s \equiv \{x \in \mathbb{R}^6 | x(t) \rightarrow L_i, t \rightarrow +\infty\}$$

$$W_{L_i}^u \equiv \{x \in \mathbb{R}^6 | x(t) \rightarrow L_i, t \rightarrow -\infty\}$$

Sun-Jupiter system

- $W_{L_2}^s$ (blue)
- $W_{L_2}^u$ (red)
- The two *1D* manifolds extend for more than 3 *AU* far from the equilibrium point.

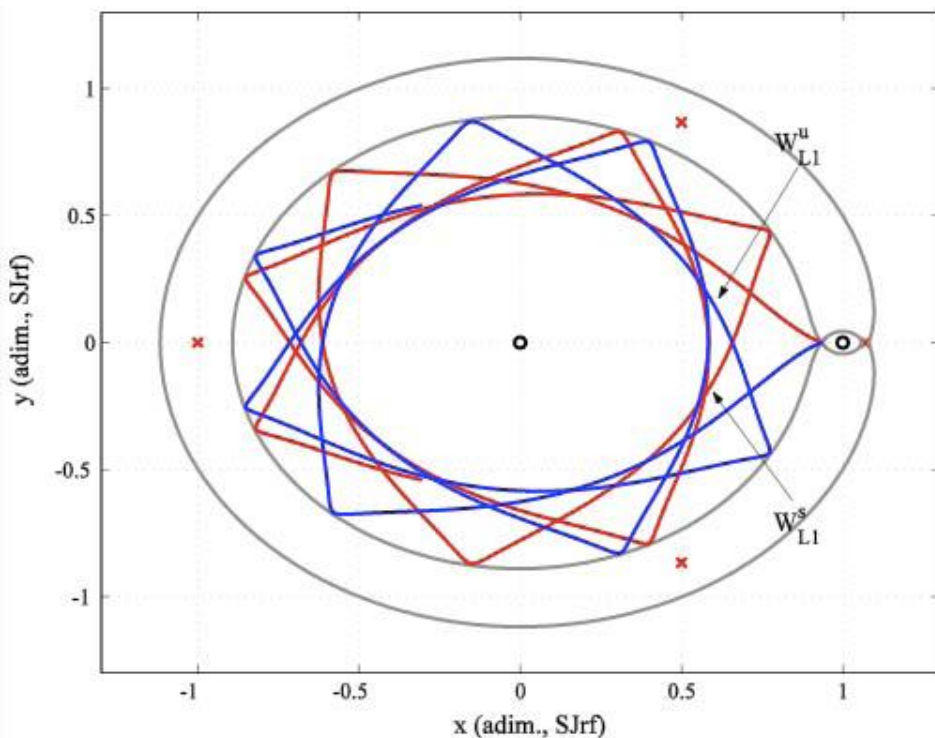




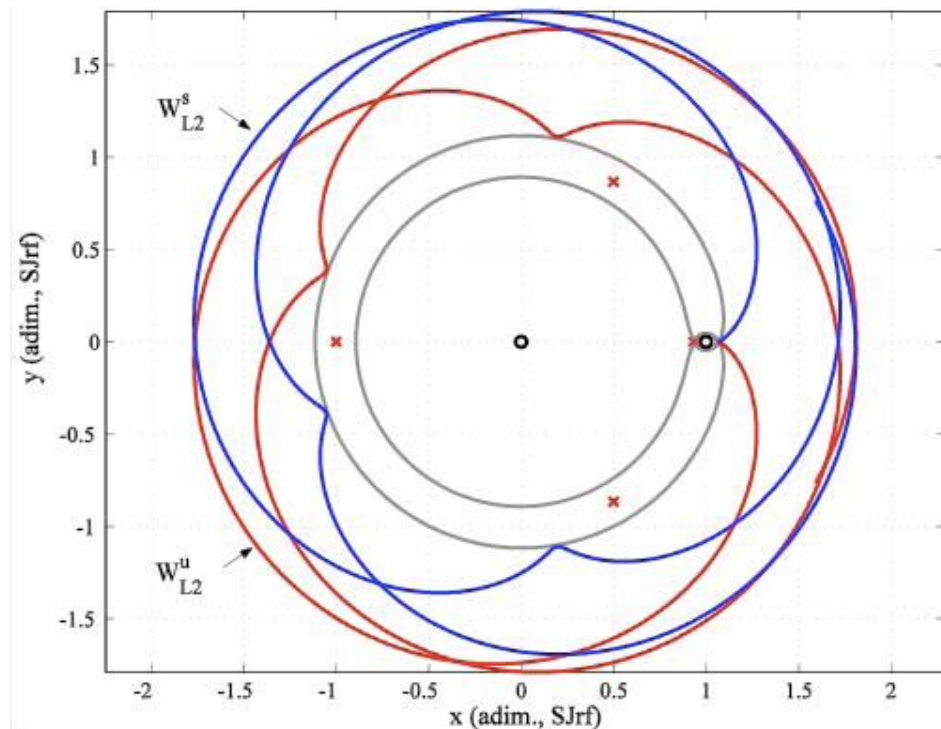
Invariant manifolds of equilibrium points (2)

- If a point lies on the stable (unstable) manifold, in the phase space sense, it will move toward (away) the libration point
- Possibility to have free transfers in the solar system

$W^{s,u}_{L1}$



$W^{s,u}_{L2}$

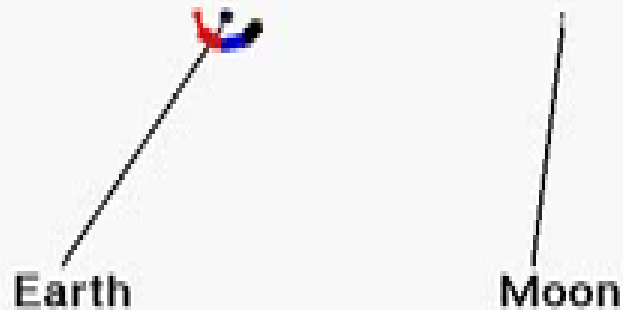




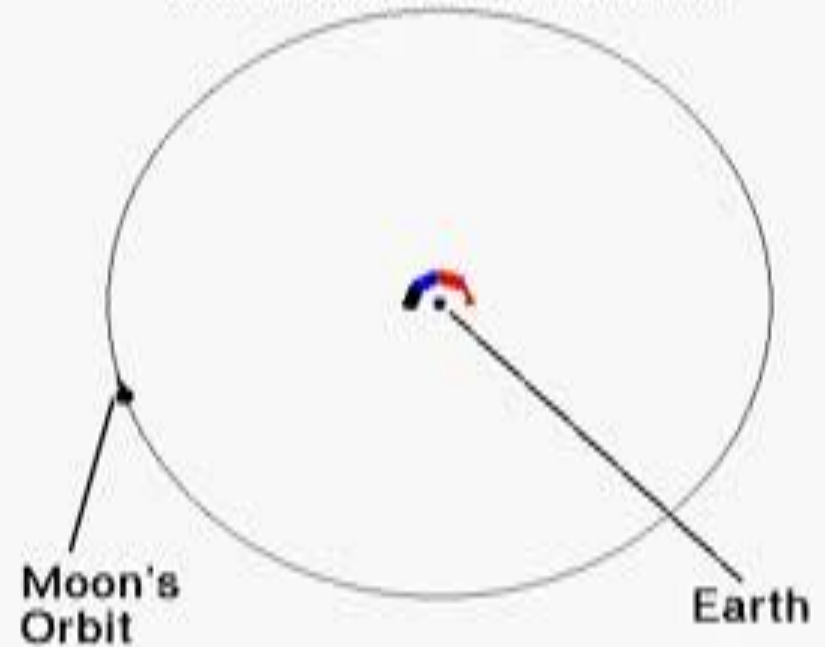
Example – video by Shane Ross, Virginia Tech



GEO to Moon Orbit Transfer
Seen in Lunar Rotating Frame



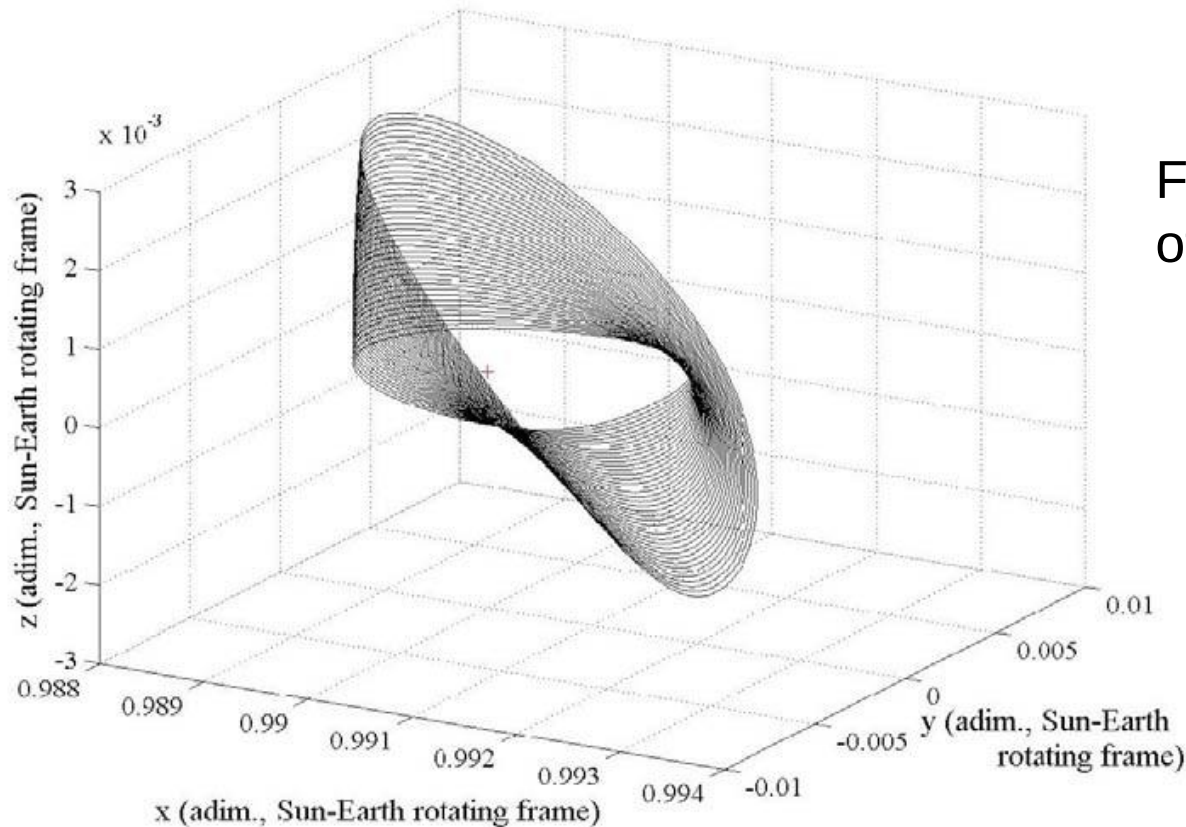
GEO to Moon Orbit Transfer
Seen in Geocentric Inertial Frame





Halo orbits (1)

- Large, 3D periodic orbits around collinear Lagrangian points
- $\omega = \nu$, *forced* by the higher order (nonlinear) terms
- Suitable for many missions (large out-of-plane excursions)
- Obtained through semi-analytical methods



Family of halos around $L1$
of the Sun-Earth system



Richardson's method to construct halo orbits:

1. Find a third-order *analytical* first guess solution
2. *Differential correction* under the nonlinear dynamics

The R3BP has the following symmetry

$$S : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \rightarrow (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t)$$

- a trajectory that crosses perpendicularly the $y=0$ plane twice is a periodic orbit

Given the analytic initial guess, it is corrected through

$$\Phi(\mathbf{x}_0 + \Delta\mathbf{x}, T_{1/2} + \Delta t) = \Phi(\mathbf{x}_0, T_{1/2}) + \left[\frac{\partial \Phi(\mathbf{x}_0, T_{1/2})}{\partial \mathbf{x}} \right] \cdot \Delta\mathbf{x} + \frac{\partial \Phi(\mathbf{x}_0, T_{1/2})}{\partial t} \cdot \Delta t$$



Invariant manifolds of Halo orbits (1)

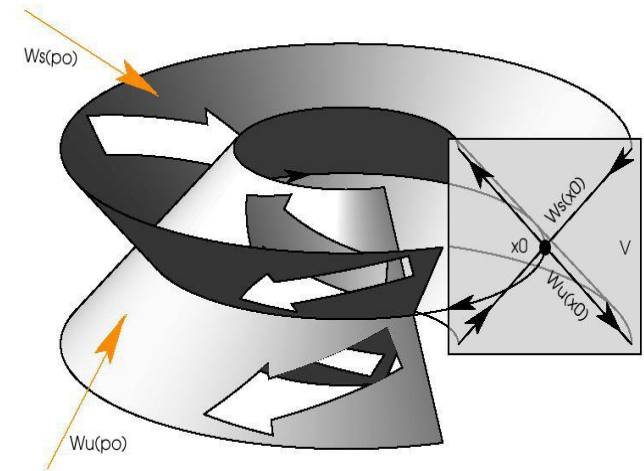
- In a neighborhood of the equilibrium points there are also *invariant manifolds* associated to the periodic orbits!
- These manifolds are more complicated “objects” than the simple manifolds associated to the equilibrium points
- They are due to the *saddle x center x center* part
- They are 2-dimensional subsets!
- Stable manifold (definition)
$$W^s_{Li,p.o.} \equiv \{x \in \mathbb{R}^6 | x(t) \rightarrow p.o. , t \rightarrow +\infty\}$$
- Unstable manifold (definition)
$$W^u_{Li,p.o.} \equiv \{x \in \mathbb{R}^6 | x(t) \rightarrow p.o. , t \rightarrow -\infty\}$$



Invariant manifolds of Halo orbits (2)

- Periodic orbits have stable and unstable manifolds too!
- Viewed as equilibrium points on a Poincaré section

$$\mathbf{x} \mapsto \mathbf{x}_0 + M(\mathbf{x} - \mathbf{x}_0)$$



- They are given by the stable and unstable eigenvalues and eigenvectors of the monodromy matrix M associated to the periodic orbit



Invariant manifolds of Halo orbits (3)

$$\mathbf{x}_0$$

- initial condition to generate a periodic orbit (given by the Richardson's method)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad , \quad \mathbf{x}(0) = \mathbf{x}_0$$

- integrate the dynamical system (of order 42) for $0 \leq t \leq T$ to obtain $\mathbf{x} = \mathbf{x}(t)$ and $M = M(t)$

$$\dot{M} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right] M \quad , \quad M(0) = \mathbf{I}$$

$$\mathbf{x}^i = \mathbf{x}(t_i), \quad M^i = M(t_i)$$

- generic point of the orbit

$$\mathbf{v}_s^i$$

- stable eigenvector of M

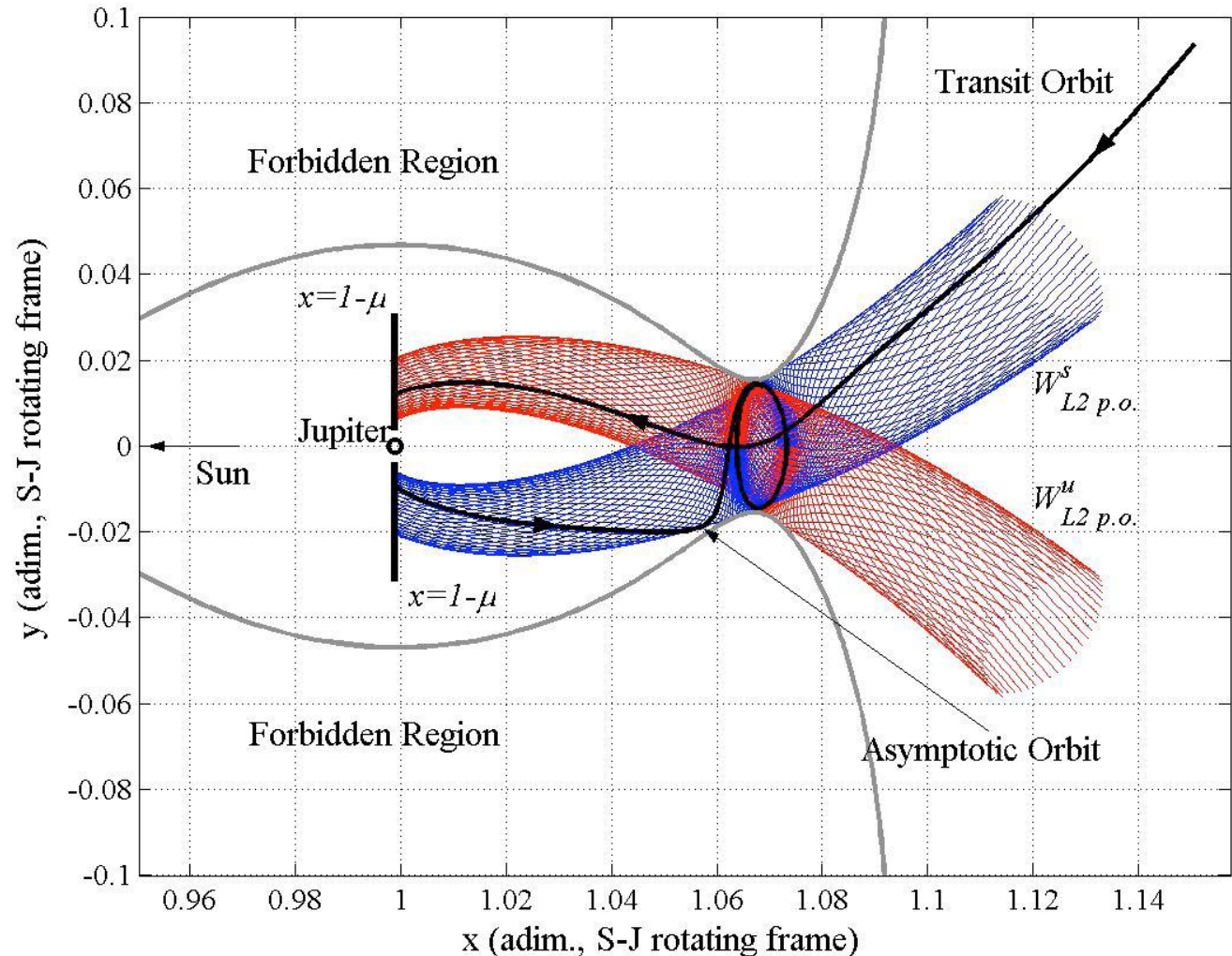
$$\mathbf{x}_{0,s}^i = \mathbf{x}^i \pm \varepsilon \mathbf{v}_s^i$$

- initial condition to integrate backward to get the stable manifold of $\mathbf{x}_i$



Invariant manifolds of Halo orbits (4)

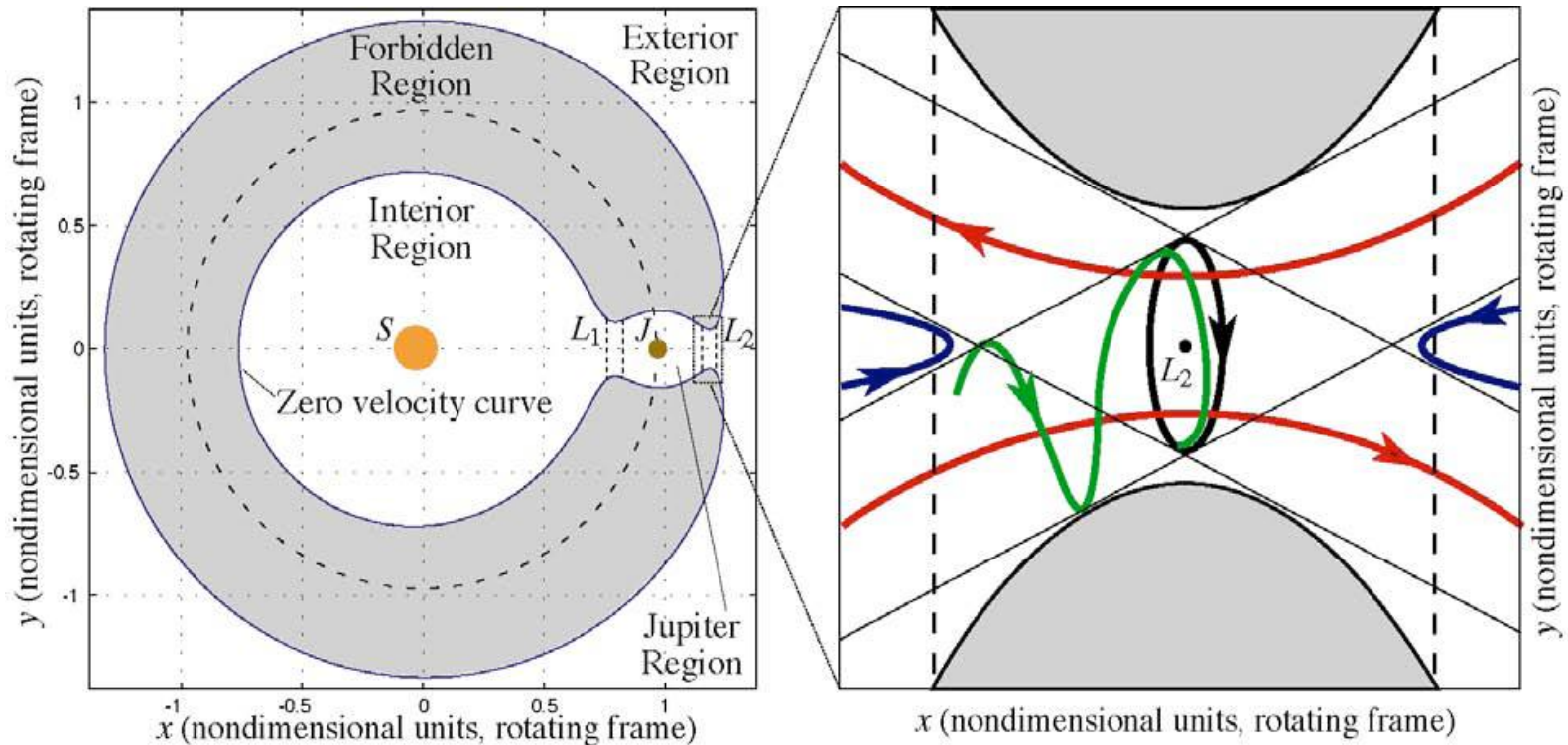
- Example of invariant manifold associated to a periodic orbit around L2 in the Sun-Jupiter system



blue: $W^s_{L2 p.o.}$
red: $W^u_{L2 p.o.}$



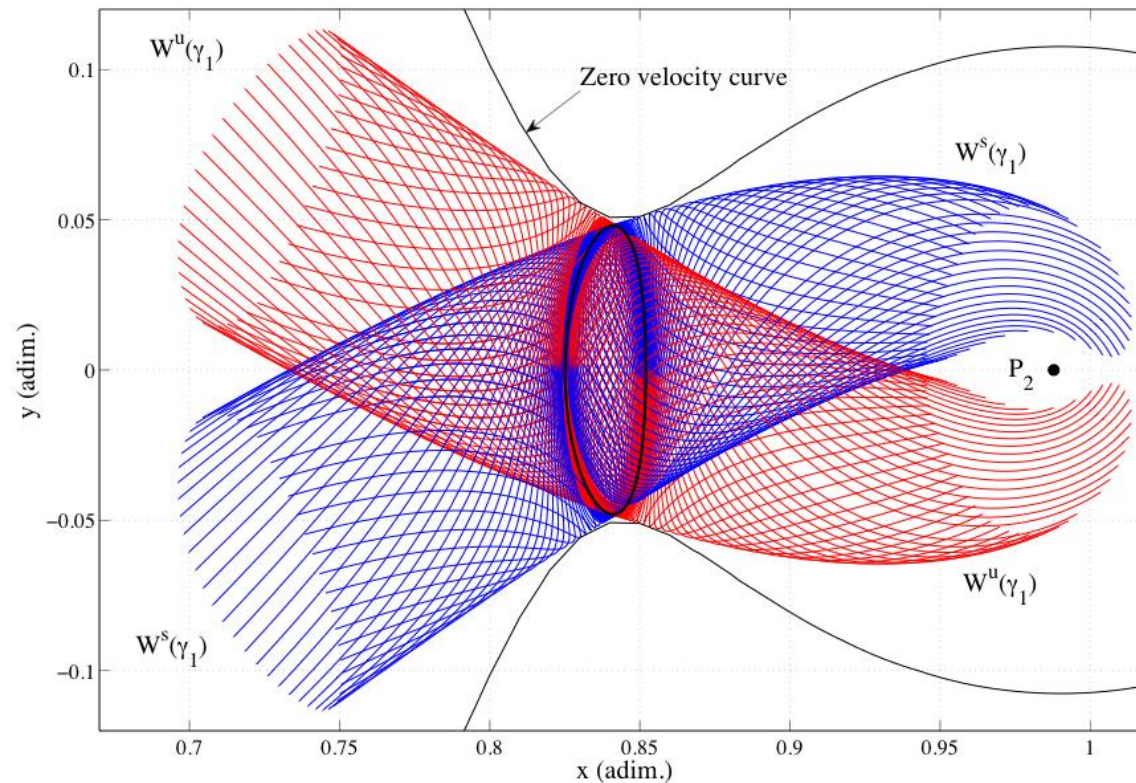
- Complete phase portrait
 - asymptotic orbits
 - periodic orbits
 - *transit orbits* (inside the tubes)
 - non-transit orbits



➤ Asymptotic orbits are useful for the trajectory design of libration point missions



- The phase space structure is exploited in a “natural” way.
- Equilibrium points, periodic orbits, invariant manifolds, etc. are used in place of orbital elements.



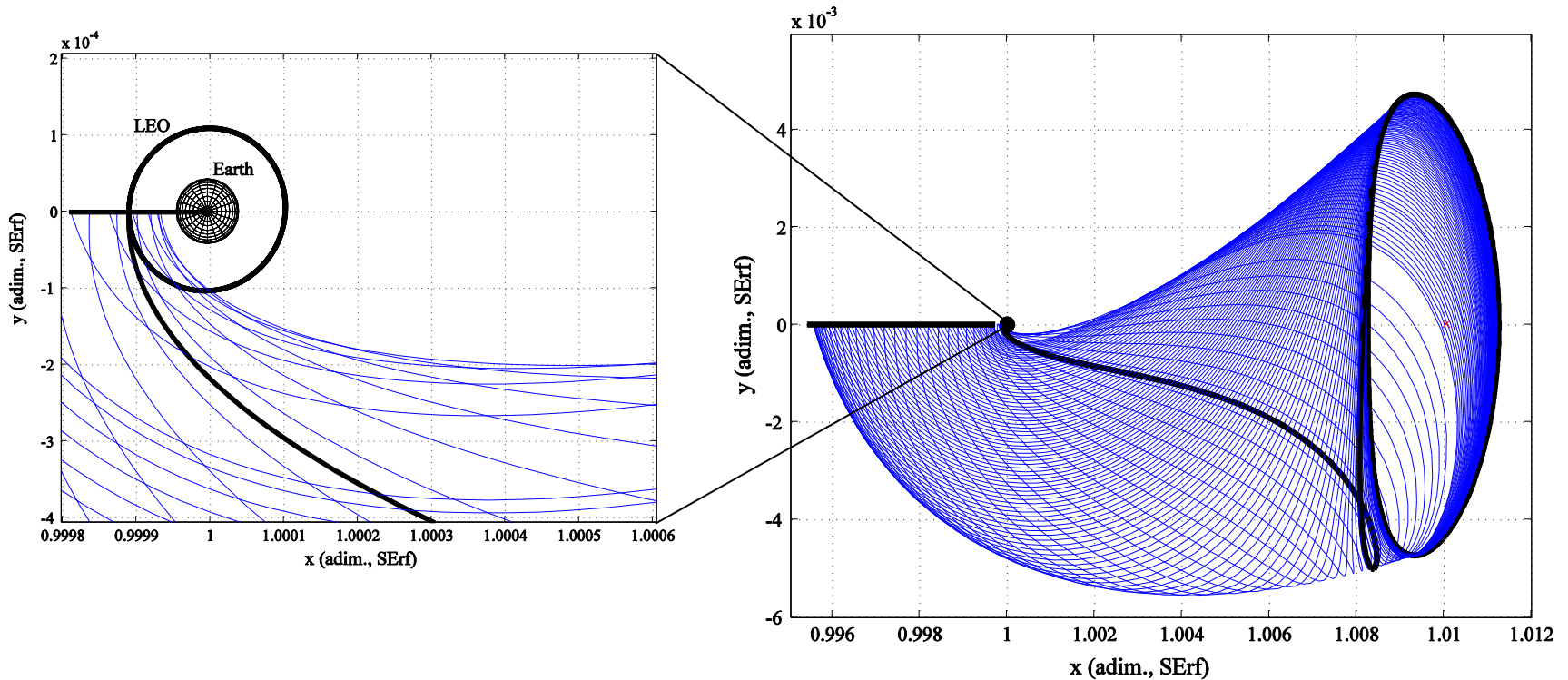
Typical case studies

- Transfers to **halo orbits**
- **Low energy** transfers to the **Moon**



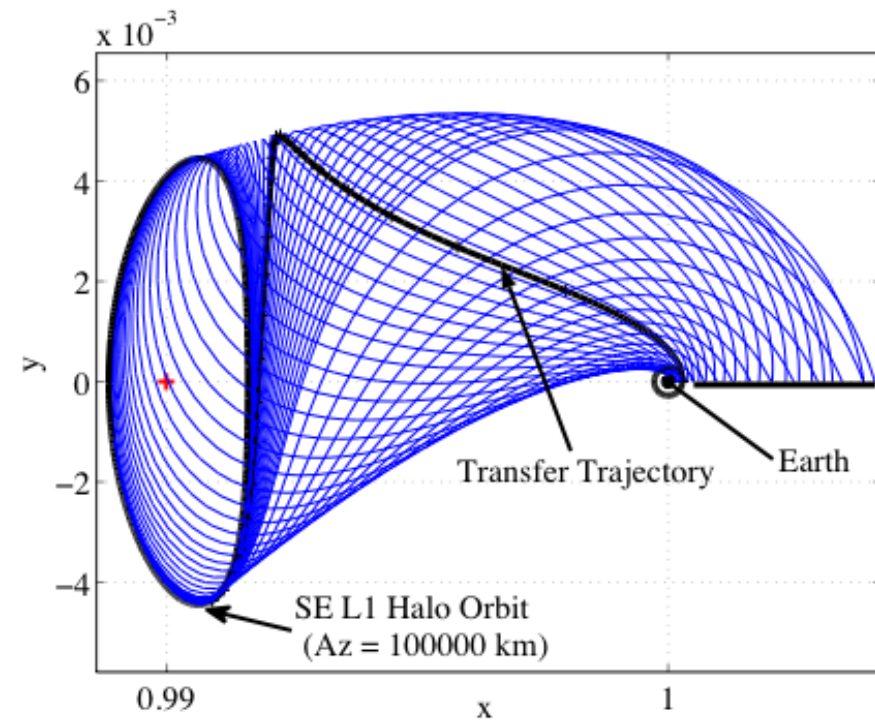
Transfer to Halo orbits (1)

- The concept of **stable manifold** is exploited for a transfer to Halo orbits
 - place the S/C on the stable manifold associated to that orbit
 - *Wait* (the S/C wraps on the final orbit)
 - perform an orbit injection maneuver to stabilize in a halo



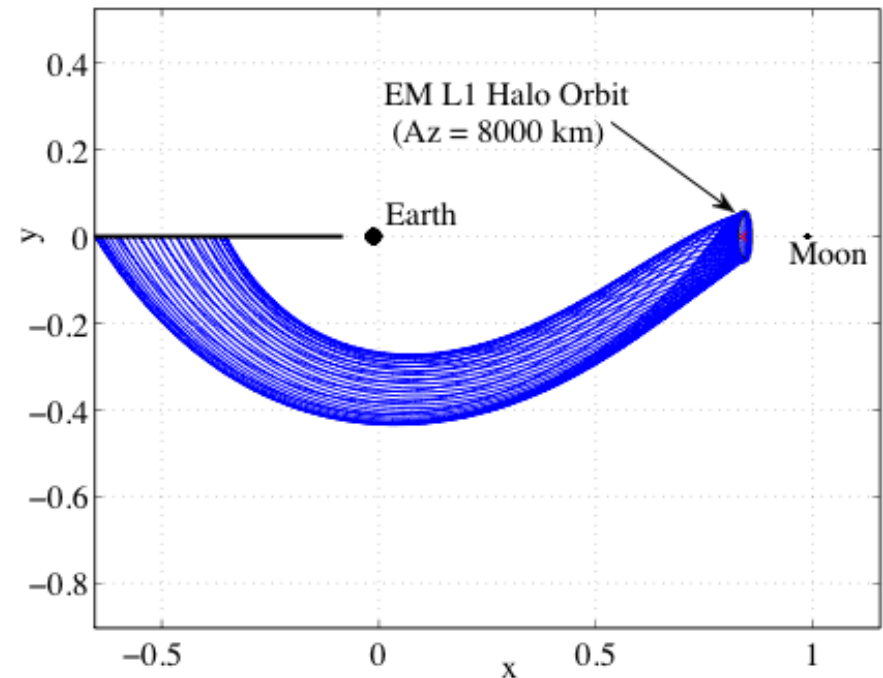


Transfer to Halo orbits (2)



Sun-Earth system

- $\Delta v \simeq 3200$ m/s (from LEO)
- Single-impulse transfer



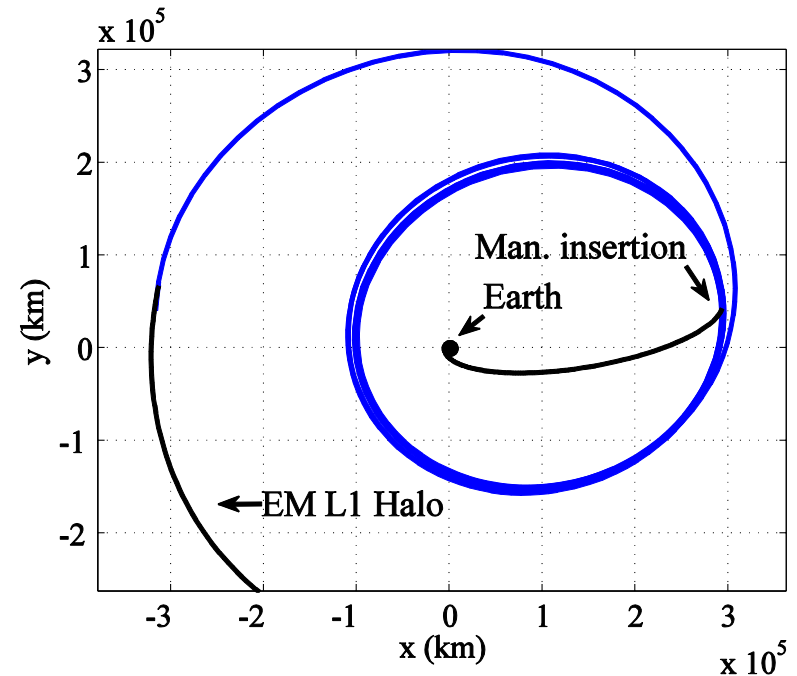
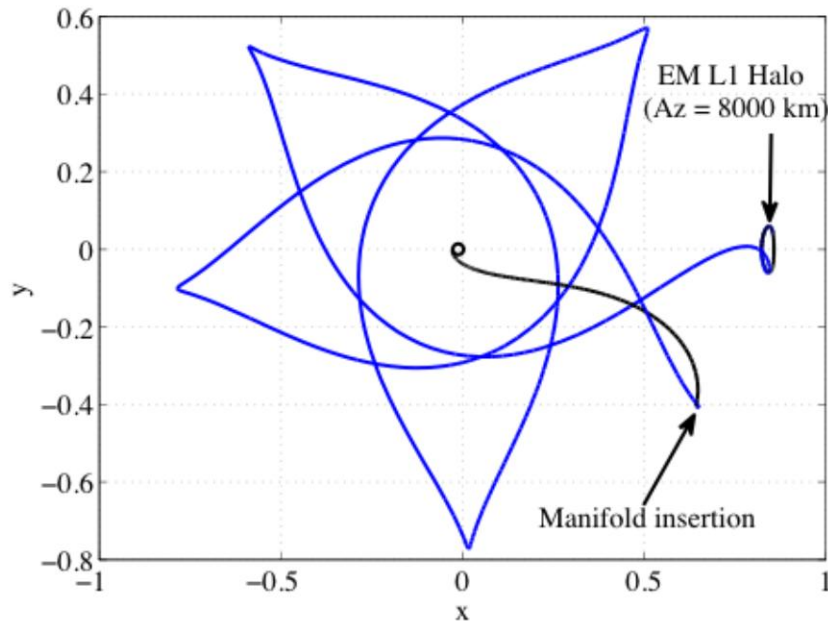
Earth-Moon system

- $\Delta v = ?$
- No single-impulse transfer



Transfer to Halo orbits (3)

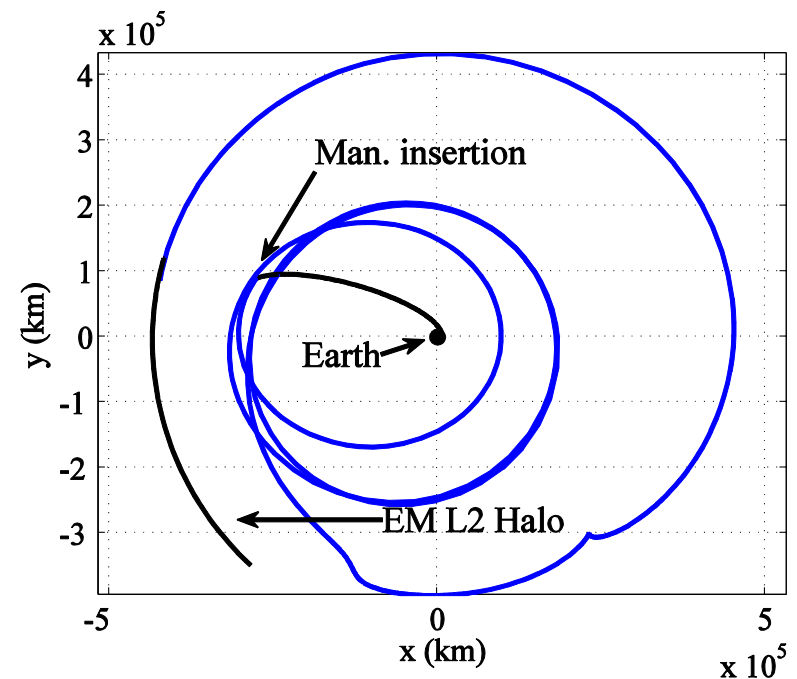
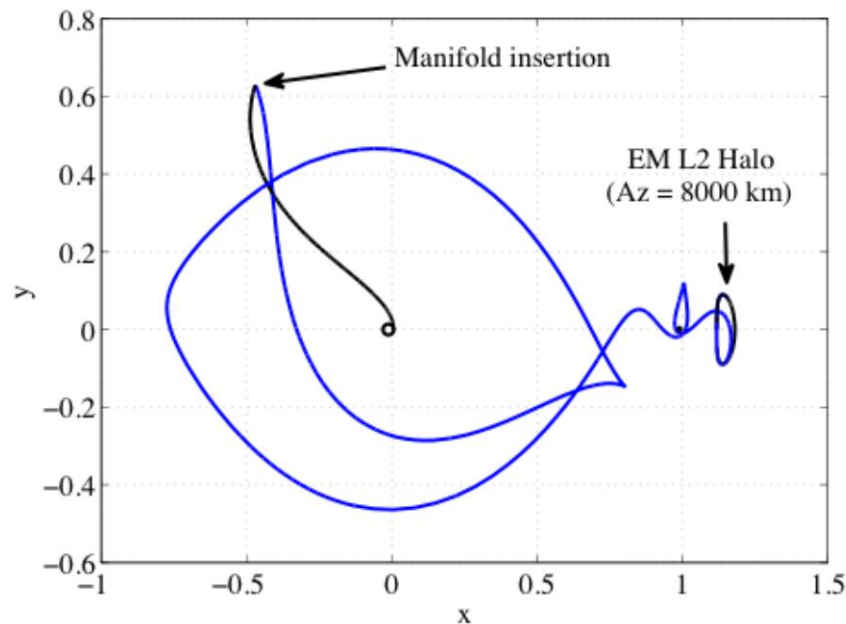
- A point on the **stable manifold** has to be targeted (two-impulse strategy)
- Transfer to L_1





Transfer to Halo orbits (4)

- A point on the **stable manifold** has to be targeted (two-impulse strategy)
- Transfer to L_2





Transfer to Halo orbits (5)



- In summary

Departing from circular, 200 km LEO

Type	A_z (km)	Δt (days)	Δv (m/s)	I_{sp} (s)	m_p/m_0
L_1	8000	67.3	3659	300	0.711
L_2	8000	65.0	3676	300	0.713



Introduction of Low-Thrust Propulsion (1)

Dynamical system theory allows us to define solutions with

- Unique **features** (e.g., libration point orbits)
- Less **propellant mass** (e.g., low energy transfers)

Rocket equation:

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{I_{sp}g_0}}$$

Invariant manifolds technique
is used to reduce Δv

Can we further reduce the propellant mass
using **low-thrust**, **high- I_{sp}** systems?

Can we further **improve** low-energy solutions?



Introduction of Low-Thrust Propulsion (2)

The effect of low-thrust systems is **no longer** modeled as instantaneous velocity changes !



Need to **re-formulate** existing “invariant manifolds technique” to incorporate **low-thrust propulsion** ...



Controlled Restricted Three-Body Problem

Equations of motion including propulsion

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_3}{\partial x} + \frac{T_x}{m}, \quad \ddot{y} + 2\dot{x} = \frac{\partial \Omega_3}{\partial y} + \frac{T_y}{m}, \quad \ddot{z} = \frac{\partial \Omega_3}{\partial z} + \frac{T_z}{m}, \quad \dot{m} = -\frac{T}{I_{sp} g_0}$$

Present thrust magnitude: $T = \sqrt{T_x^2 + T_y^2 + T_z^2}$

Variations of spacecraft mass, m , are considered through last eq.

The system is singular when $m \rightarrow 0$

 $\mathbf{T}(t) = (T_x(t), T_y(t), T_z(t)), t \in [t_0, t_f] \quad ?$

The **guidance law** is not given, but rather it is an **unknown** of the optimal control problem ...



$$x_0 = (x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0, m_0) \quad \text{initial condition}$$

$$\phi_{T(\tau)}(x_0, t_0; t) \quad \text{with} \quad T(\tau), \tau \in [t_0, t] \quad \text{controlled flow}$$

$$x(t) = \phi_{\bar{T}}(x_0, t_0; t) \quad \text{with} \quad T = \bar{T} \hat{v} \quad \text{tangential thrust flow}$$

$$\gamma_{\bar{T}}(x_0, t) = \{\phi_{\bar{T}}(x_0, t_0; \tau) | \tau < t\} \quad \text{tangential thrust orbit}$$

$$\boxed{\mathcal{A}_{\bar{T}}(t) = \bigcup_{x_0 \in \mathcal{X}} \gamma_{\bar{T}}(x_0, t)} \quad \text{Attainable set}$$

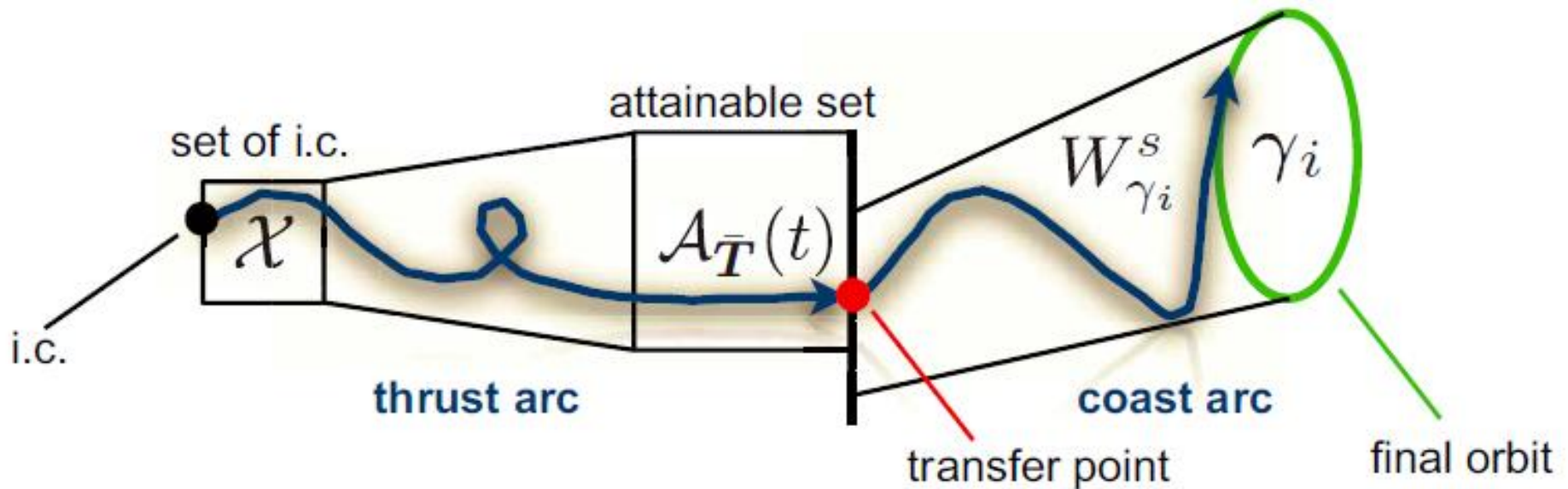


$\mathbf{A}_T$ is associated to an admissible set of initial conditions $\mathbf{X}$



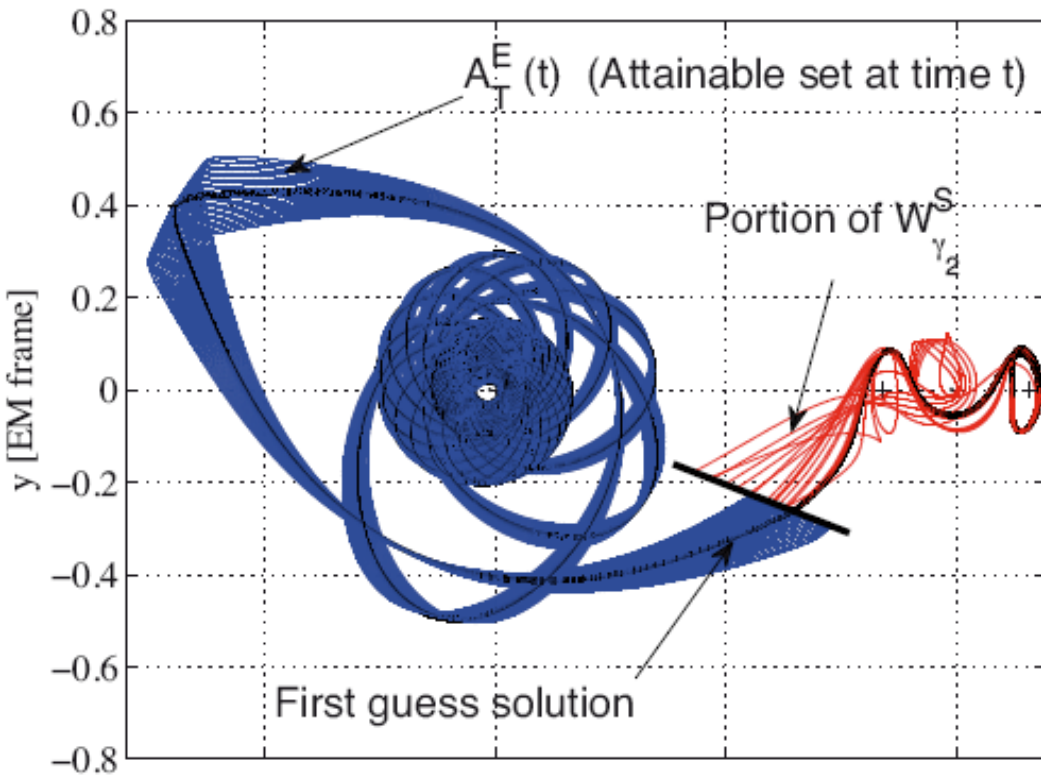
Attainable sets

- Contain a set of **tangential low-thrust** orbits associated to admissible initial conditions
- **Tangential thrust** is considered in this stage (preliminary solution later used as first guess of optimal control problem)
- **Attainable sets** are manipulated in the same fashion as **invariant manifolds**



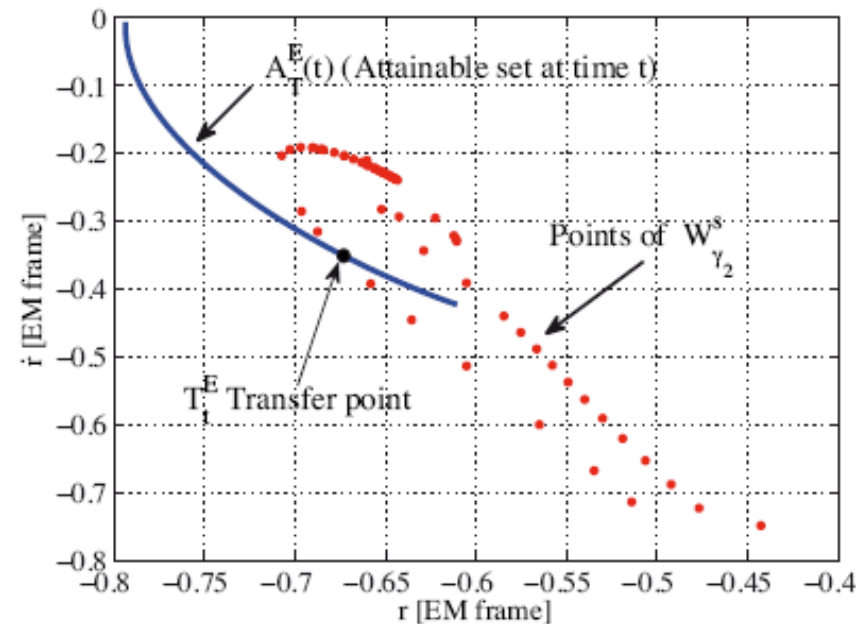


Attainable Sets for Transfers to Halo Orbits



$$\mathcal{X}^E = \{x_0(\omega^E) | \omega^E \in [0, 2\pi]\}$$

- Preliminary solution found simply by: $\mathcal{T}_t^E = \mathcal{A}_T^E(t) \cap W_{\gamma_i}^s$
- Small discontinuities are admitted, i.e., $\|x^A - x^W\| \leq \varepsilon$



$T = 0.5 \text{ N}; A_z = 8000 \text{ km}; t = 14.8$

- Departure: GTO
- Arrival: L₂ halo

same as
before !



Trajectory Optimization (1)

- Improve preliminary solutions found with attainable sets
- Eliminate possible discontinuities
- More refined dynamical model (controlled, bicircular RFBP):

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_4}{\partial x} + \frac{T_x}{m}, \quad \ddot{y} + 2\dot{x} = \frac{\partial \Omega_4}{\partial y} + \frac{T_y}{m}, \quad \ddot{z} = \frac{\partial \Omega_4}{\partial z} + \frac{T_z}{m}, \quad \dot{\theta} = \omega_S, \quad \dot{m} = -\frac{T}{I_{sp} g_0}$$

- Includes perturbation of the Sun in the EM model

$$\Omega_4(x, y) = \Omega_3(x, y, \mu_{EM}) + \frac{m_s}{r_s} - \frac{m_s}{\rho^2}(x \cos \theta + y \sin \theta).$$

- Sun-spacecraft distance

$$r_s^2 = (x - \rho \cos \theta)^2 + (y - \rho \sin \theta)^2$$

- Normalized Sun parameters:

- | | |
|--|----------------------------|
| • Sun-EM distance | $\rho = 3.8881 \cdot 10^2$ |
| • mass of the Sun | $m_s = 3.2890 \cdot 10^5$ |
| • angular velocity relative to EM system | $\omega_s = -0.9251$ |



Trajectory Optimization (2)

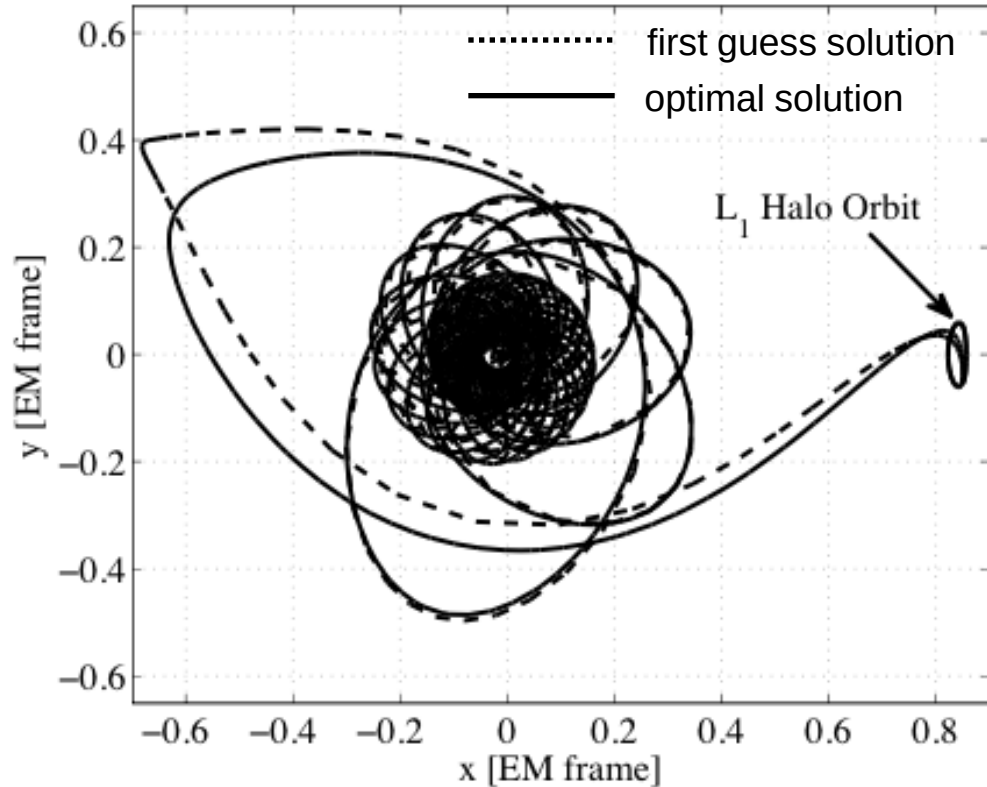
- Objective function (propellant mass minimization): $J = \int_{t_0}^{t_f} \frac{T(\tau)}{I_{sp} g_0} d\tau$
- Path constraints (saturation of thrusters): $T(t) \leq T_{\max}$
- Boundary conditions:
 - depart at periapsis of parking orbit
 - perigee and point on final halo orbit $x_f = x^W, \quad x^W \in W_{\gamma_i}^s$
- Solution by Direct Transcription and Multiple Shooting:
 - time domain split into N-1 intervals
 - each interval is split into M-1 segments
 - control discretized over the M nodes, using spline interpolation
 - nonuniform time grid



First Guess vs Optimal Solution



Transfer to halo orbit

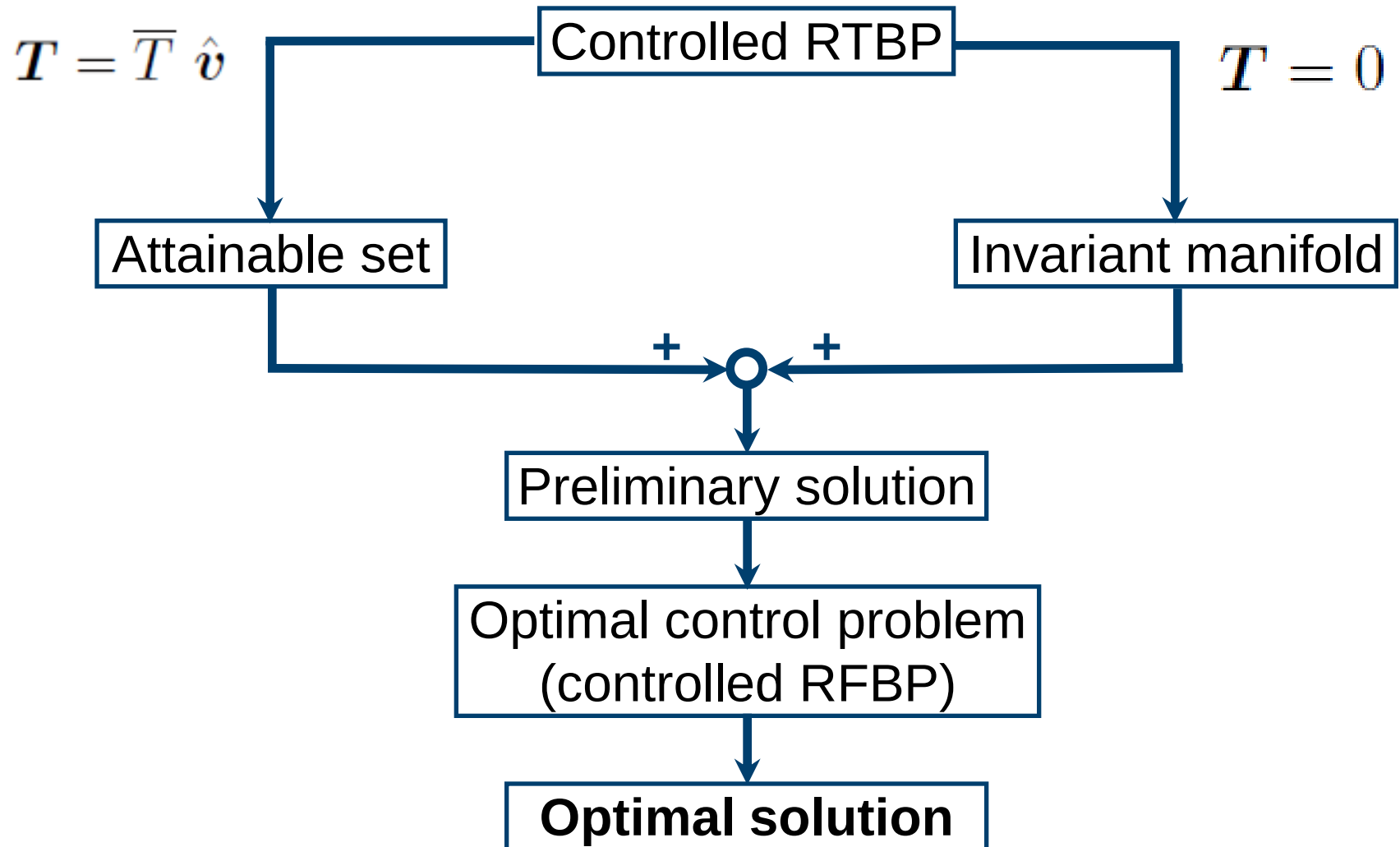


Direct transcription and multiple shooting (RKF78 + cubic spline)

- end-to-end trajectory transcription
- non-uniform grid implementation



Approach Summary





Results

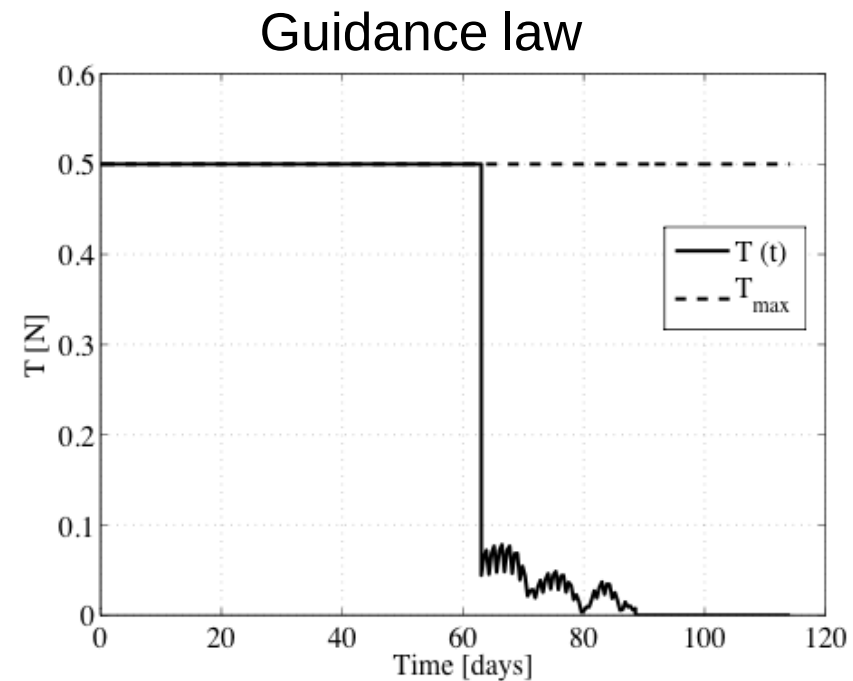
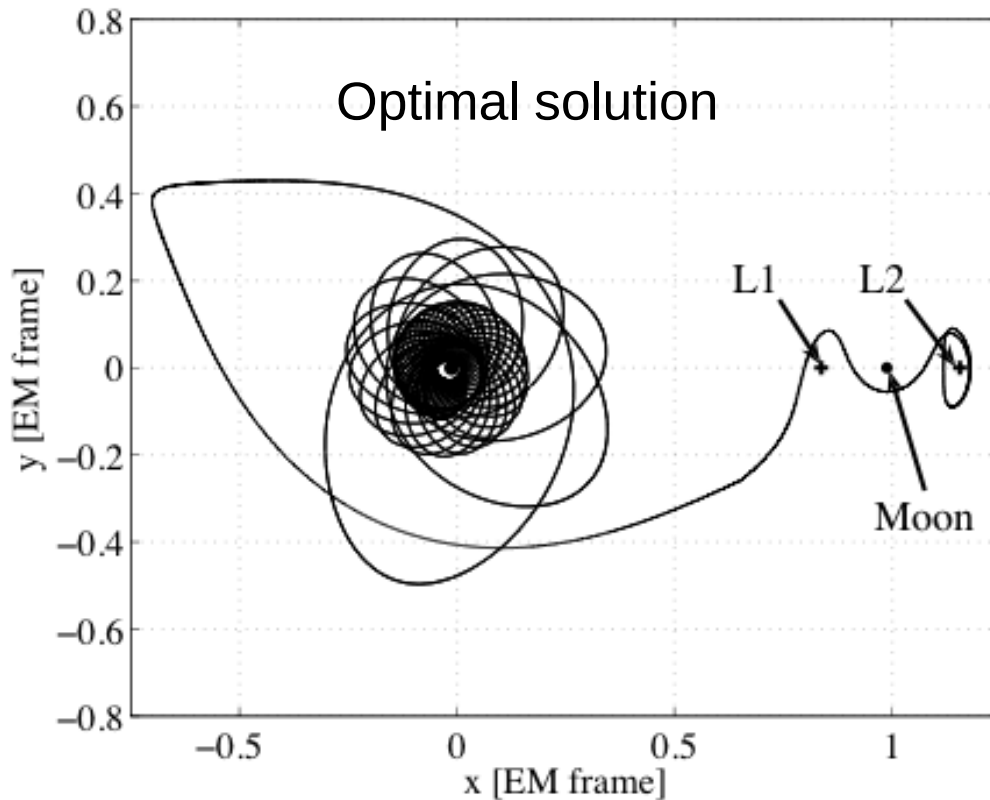
- Departure orbit
- Arrival orbit
- Propellant mass fraction
- Transfer time

Comparisons

- Optimal low-thrust, invariant-manifold solutions vs
 - Solutions achieved with impulsive maneuvers
 - Existing solutions in literature



Low-Thrust, Stable-Manifold Transfers to Halos



- ~ 300 multiple shooting segments (i.e., ~5100 var., 3300 constr.)
- The **signature** of the preliminary solution is clearly visible



Low-Thrust, Stable-Manifold Transfers to Halos

LTSM: Low-thrust, stable-manifold

HTSM: High-thrust, stable-manifold (impulsive maneuvers)

Type	Initial Orbit		Final Orbit		I_{sp} [s]	m_p/m_0	Δt [days]
	h_p^E [km]	e^E	A_z [km]	L_i			
LTSM#1	200	0	8000	L_1	3000	0.171	178
LTSM#2	400	0.72	8000	L_1	3000	0.090	91
LTSM#3	200	0	8000	L_2	3000	0.183	195
LTSM#4	400	0.72	8000	L_2	3000	0.091	114
HTSM#1	200	0	8000	L_1	300	0.711	67
HTSM#2	200	0	8000	L_2	300	0.713	65

- LEO: $h = 200$ km, $e = 0$
- GTO: $h_p = 400$, $e = 0.72$
- $m_0 = 1000$ kg



- A method to **include low-thrust** orbit pieces in the “standard” invariant manifolds frame has been derived
- Low-thrust **attainable sets** are manipulated in the same fashion as invariant manifolds
- Preliminary solutions are **optimized** in a four-body frame (end-to-end optimization)
- Solutions **reduce** the propellant mass (due to higher I_{sp})
- Presented solutions are **not achievable** neither with patched-conics method nor with standard invariant manifolds technique



Suggested readings

- Szebehely, V., Theory of Orbits: The Restricted Problem of Three Bodies, Academic Press Inc., New York, 1967, pp. 16–22
- Farquhar, R., “Lunar Communications with Libration-Point Satellites,” Journal of Spacecraft and Rockets, Vol. 4, 1967, pp. 1383–1384
- Belbruno, E. and Miller, J., “Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture,” Journal of Guidance, Control, and Dynamics, Vol. 16, 1993, pp. 770–775
- Gomez, G., Simo, C., Masdemont, J., and Jorba, A., Dynamics and Mission Design near Libration Points: Vol. III Advanced Methods for Collinear Points, World Scientific, London, 2001, pp. 69–92
- Gomez, G., Koon, W., Lo, M., Marsden, J., Masdemont, J., and Ross, S., “Invariant Manifolds, the Spatial Three-Body Problem and Space Mission Design,” Advances in the Astronautical Sciences, Vol. 109, 2001, pp. 3–22
- Howell, K. and Ozimek, M., “Low-Thrust Transfers in the Earth-Moon System Including Applications to Libration Point Orbits,” Paper AAS 07-43, Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Mackinac Island, MI, 2007
- Topputo, F., Low-Thrust Non-Keplerian Orbits: Analysis, Design, and Control, Ph.D. thesis, Politecnico di Milano, Italy, 2007.
- Mingotti, G., Trajectory Design and Optimization in Highly Nonlinear Astrodynamics, Ph.D. thesis, Politecnico di Milano, Italy, 2010



POLITECNICO
DI MILANO



Incorporating low-thrust propulsion into the invariant manifolds technique for space trajectories design

F. Bernelli-Zazzera, F. Topputo

Politecnico di Milano, Dipartimento di Ingegneria Aerospaziale

G. Mingotti

Universität Paderborn, Institut für Mathematik