Multilevel Simulation under Uncertainty and Heterogeneity

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Simulation has become a central tool in almost all areas of science and engineering, and the ever increasing computational power of modern supercomputers allows us to tackle problems that were previously out of reach. However, heterogeneous and/or anisotropic diffusion processes (present in many applications) typically lead to extremely badly conditioned linear equation systems that can only be solved efficiently via multilevel methods, especially in realistic, highly resolved three dimensional computations. The robustness analysis of traditional multilevel preconditioners (such as multigrid or domain decomposition) in this context will form one part of this presentation.

Model parameters and coefficients are usually inferred from a small number of measurements and therefore largely uncertain. In a highly heterogeneous context, such as the flow of water underground, even small uncertainties in the model parameters can lead to large uncertainties in the predicted flow. Clearly the quantification of these uncertainties plays a central role in applications, such as the safety assessment of radioactive waste disposal or CO2 capture and storage underground. Stochastic modelling of data uncertainties in the rock permeabilities leads to elliptic PDEs with random coefficients. A typical computational goal is the estimation of the expected value or a higher order moment of some relevant quantity of interest, such as the breakthrough time of a plume of radionuclides. In the second part of the talk we will show how multilevel simulation can bring this even tougher computational goal within our reach, and how novel multilevel Monte Carlo methods can essentially reduce the computational cost of solving a stochastic elliptic PDE to that of a deterministic one.

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