TWO CONSTRAINED FORMULATIONS FOR DEBLURRING POISSON NOISY IMAGES

Mikael Carlavan and Laure Blanc-Féraud

ARIANA joint research group INRIA/I3S/UNSA
2004 route des Lucioles, 06902 Sophia-Antipolis, France
{Mikael.Carlavan, Laure.Blanc_Feraud}@inria.fr

ABSTRACT

Deblurring noisy Poisson images has recently been subject of an increasingly amount of works in many areas such as astronomy or biological imaging. Several methods have promoted explicit prior on the solution to regularize the ill-posed inverse problem and to improve the quality of the image. In each of these methods, a regularizing parameter is introduced to control the weight of the prior. Unfortunately, this regularizing parameter has to be manually set such that it gives the best qualitative results. To tackle this issue, we present in this paper two constrained formulations for the Poisson deconvolution problem, derived from recent advances in regularizing parameter estimation for Poisson noise. We first show how to improve the accuracy of these estimators and how to link these estimators to constrained formulations. We then propose an algorithm to solve the resulting optimization problems and detail how to perform the projections on the constraints. Results on real and synthetic data are presented.

Index Terms— Poisson deconvolution, discrepancy principle, constrained convex optimization.

1. INTRODUCTION

Deblurring images corrupted by Poisson noise is a challenging process which has devoted much research in many applications such as astronomical or biological imaging. If we consider a discrete version of a scene $\mathbf{x} \in \mathbb{R}^n$ (n being the number of pixels of the image) observed as an image $\mathbf{y} \in \mathbb{R}^n$ through an optical system with a Point Spread Function (PSF) h and corrupted by a Poisson noise process $\mathcal{P}$, then the image formation model can be written as:

$$\mathbf{y} = \mathcal{P}(\mathbf{Hx} + \mathbf{b}), \tag{1}$$

where $\mathbf{H} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ stands for the matrix notation of the convolution of the PSF h (we assume moreover $\mathbf{Hx} \geq 0 \, \forall \mathbf{x} \geq 0$) and $\mathbf{b} \in \mathbb{R}^n, \mathbf{b} \geq 0$ is a known constant background.
Using a bayesian approach, we want to retrieve the image $\mathbf{x}$ which maximizes the likelihood probability of (1). This probability can be expressed as:

$$p(\mathbf{y}|\mathbf{x}) = \prod_{i=0}^{n-1} \left(\frac{[(\mathbf{Hx}+\mathbf{b})^{\mathbf{y}}]_i \exp\left[-\left(\mathbf{Hx}+\mathbf{b}\right)\right]_i}{\mathbf{y}_i!} \right). \tag{2}$$

Maximizing (2) with respect to $\mathbf{x}$ is equivalent to minimize $-\log p(\mathbf{y}|\mathbf{x})$ that is to minimize:

$$J_L(\mathbf{x}, \mathbf{y}) = \mathbf{1}^T \left(\mathbf{Hx} + \mathbf{b}\right) - \mathbf{y}^T \log(\mathbf{Hx} + \mathbf{b}), \tag{3}$$

where $\mathbf{1}$ stands for a n-size vector whose components are all equal to 1.
As discussed previously, many works promote the introduction of explicit prior on the solution to regularize the ill-posed inverse problem. Maximizing the a posteriori probability $p(\mathbf{x}|\mathbf{y}) = p(\mathbf{y}|\mathbf{x}) \frac{p(\mathbf{x})}{p(\mathbf{y})}$, where $p(\mathbf{x})$ is the prior model on the object given by $p(\mathbf{x}) = \alpha \exp[-\tau J_R(\mathbf{x})]$ (α is a normalization constant and J_R is the regularizing term), is equivalent to solve:

$$\mathbf{x}^* = \begin{array}{cc} \arg\min & J(\mathbf{x},\mathbf{y}) := J_L(\mathbf{x},\mathbf{y}) + \tau J_R(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathbb{R}^n \end{array}, \tag{4}$$

τ being the regularizing parameter. The regularizing term can often be written as $J_R(\mathbf{x}) = \|\mathbf{Wx}\|_1$, where $\mathbf{W} : \mathbb{R}^n \rightarrow \mathbb{R}^p$ ($p \geq n$) is a linear transform which promotes the regularity of the image $\mathbf{x}$ in some domain. Common transforms are gradient operator giving Total Variation (TV) [1] and wavelet frame transforms ([2, 3] and references therein). Therefore the problem of deblurring Poisson noisy images can be written as:

$$\mathbf{x}^* = \begin{array}{cc} \arg\min & \mathbf{1}^T \left(\mathbf{Hx}+\mathbf{b}\right) - \mathbf{y}^T \log(\mathbf{Hx}+\mathbf{b}) + \tau\|\mathbf{Wx}\|_1 \\ \text{subject to} & \mathbf{x} \in \mathbb{R}^n, \mathbf{x} \geq 0 \end{array}. \tag{5}$$

The function to minimize is a closed convex function and strictly convex if $\mathbf{y}_i > 0$ and if the intersection of the null spaces of J_L and J_R is zero [4].
In most of the deconvolution methods proposed in the literature, the regularizing parameter τ has to be chosen such that it gives the best visual results. However, the interpretation of an image may be difficult in biology for example, specially when one use priors which could introduce artifacts. To overcome this problem, some authors proposed estimators of the distance of the restored image to the real unknown image $\mathbf{x}$ [2, 5, 6, 7]. For example, several methods based on Stein's principle have been proposed to minimize (in one instance) the *Mean Square Error* (MSE) in the case of Poisson noise [5]. However, these methods require the solution of the restoration algorithm to be expressed in closed-form and consist, most of the time, in a shrinkage of some wavelet coefficients. Consequently, the deconvolution process is rarely included in these techniques. Some authors have proposed non-linear algorithms as (5) to solve the restoration problem and to select the regularizing parameter using *discrepancy principles* which state that the distance of the restored image to the observation should be equal to the amount of noise [6, 7]. This implies however to run the restoration algorithm several times to find a "good" value of the regularizing parameter τ (that is, which verifies this principle). For this reason, we propose in the next section to solve the problem of deblurring and denoising Poisson noisy images using two new constrained formulations which avoid the iterative computation of the solution for different values of the regularizing parameter.

This research work has been funded by the ANR DetecFine. The authors gratefully acknowledge Pierre Weiss for several interesting discussions.

A part of this work has been submitted to IEEE Transactions on Image Processing.

2. CONSTRAINED FORMULATIONS

2.1. Gaussian approximation revisited

To the best of our knowledge, constrained formulations for Poisson noisy deblurring have been proposed only using a Gaussian approximation. More precisely, the Poisson noise in (1) is often approximated as an additive Gaussian noise with mean 0 and multidimensional variance $\mathbf{y}$ [8]. If we set [6]:

$$\mathbf{r} = (\mathbf{Hx} - (\mathbf{y} - \mathbf{b})) / \sqrt{\mathbf{y}}, \tag{6}$$

then $\mathbf{r}$ is a Gaussian random variable with mean 0 and variance $\mathbf{I}$ (the n-size identity matrix). In this case, a standard result gives $\|\mathbf{r}\|_2^2 \sim \chi^2(n)$, where $\chi^2(n)$ is the chi-square distribution with n degree of freedom which has a mean equal to n. Therefore:

$$E\left(\|\mathbf{r}\|_2^2\right) = n. \tag{7}$$

Then a restoration algorithm in its constrained formulation can be written as:

$$\mathbf{x}^* = \begin{array}{cl} \arg\min & \|\mathbf{Wx}\|_1 \\ \text{subject to} & \|\left(\mathbf{Hx} - (\mathbf{y} - \mathbf{b})\right) / \sqrt{\mathbf{y}}\|_2^2 \leq n \\ & \mathbf{x} \in \mathbb{R}^n, \mathbf{x} \geq 0 \end{array} . \tag{8}$$

However, a closer calculation of (7) can be made. As we deal with Poisson data, we have that, with no background, the image formation model writes $\mathbf{y} = \mathcal{P}(\mathbf{Hx})$. Thus for every pixel $\mathbf{x}_i$ inside a centered window (of the size of the kernel of the PSF) containing only 0 valued pixels, we can write that $[\mathcal{P}(\mathbf{Hx})]_i = 0$. Consequently, from (6) $(\mathbf{r})_i$ is a Gaussian random variable with mean 0 but variance 0. It seems thus more accurate to write that $\mathbf{r}$ is a Gaussian random variable with mean 0 but variance Σ with:

$$\Sigma = \begin{cases} 1 \text{ if } \mathbf{y}_i > 0, \\ 0 \text{ otherwise} \end{cases} . \tag{9}$$

Then:

$$\|\mathbf{r}\|_2^2 \sim \chi^2(m) \quad \text{with} \quad m = \#\{\mathbf{y}_i, \mathbf{y}_i > 0\}, \tag{10}$$

and (8) becomes:

$$\mathbf{x}^* = \begin{array}{cl} \arg\min & \|\mathbf{Wx}\|_1 \\ \text{subject to} & \|\left(\mathbf{Hx} - (\mathbf{y} - \mathbf{b})\right) / \sqrt{\mathbf{y}}\|_2^2 \leq m \\ & \mathbf{x} \in \mathbb{R}^n, \mathbf{x} \geq 0 \end{array} . \tag{11}$$

2.2. Poisson constrained formulation

Even if the previous formulation avoids to manually select a regularizing parameter, it does not handle the Poisson noise statistics properly. For this reason, from (5) we propose the following constrained formulation:

$$\begin{array}{cl} \arg\min & \|\mathbf{Wx}\|_1 \\ \text{subject to} & \Upsilon_{\mathbf{y}}(\mathbf{Hx} + \mathbf{b}) \leq \alpha \\ & \mathbf{x} \in \mathbb{R}^n, \, \mathbf{x} \geq 0 \end{array} , \tag{12}$$

with:

$$\Upsilon_{\mathbf{y}}(\mathbf{x}) = \mathbf{1}^T(\mathbf{x}) - \mathbf{y}^T \log(\mathbf{x}) + \mathbf{y}^T \log(\mathbf{y}) - \mathbf{1}^T \mathbf{y}. \tag{13}$$

In that case, the estimation of α comes from the recent advances of [7] which introduced a discrepancy principle for Poisson data. The work of [7] is based from [9] in which the authors proposed, for their numerical simulations, to select the regularizing parameter by *means of a discrepancy principle*, that is it should verify (using our notations):

$$\Upsilon_{\mathbf{y}}(\mathbf{Hx}^* + \mathbf{b}) = \Upsilon_{\mathbf{y}}(\mathbf{Hx} + \mathbf{b}), \tag{14}$$

where $\mathbf{x}^*$ denotes a solution of (5) for a given τ and $\mathbf{x}$ is the true object which verifies (1). However, in the case of real data $\Upsilon_{\mathbf{y}}(\mathbf{Hx}+\mathbf{b})$ remains unknown. [7] showed that $\Upsilon_{\mathbf{y}}(\mathbf{Hx}+\mathbf{b})$ can be estimated using the following technique. First, they considered the function:

$$f(\mathbf{Y}_\lambda) = 2\left(\lambda - \mathbf{Y}_\lambda \log(\lambda) + \mathbf{Y}_\lambda \log(\mathbf{y}) - \mathbf{Y}_\lambda\right), \tag{15}$$

where $\mathbf{Y}_\lambda$ is a Poisson random variable with mean λ. They showed that, for large λ:

$$E(f(\mathbf{Y}_\lambda)) = 1 + O\left(\frac{1}{\lambda}\right). \tag{16}$$

In the case of deconvolution, we have $\mathbf{y} = \mathcal{P}(\mathbf{Hx} + \mathbf{b})$ and thus $\mathbf{y}$ is a Poisson random variable with mean $\mathbf{Hx} + \mathbf{b}$. Therefore:

$$E(\Upsilon_{\mathbf{y}}(\mathbf{Hx} + \mathbf{b})) \simeq \frac{n}{2}. \tag{17}$$

So, from this statement [7] proposed to select the regularizing parameter τ such that the solution $\mathbf{x}^*$ of the optimization problem (5) verifies:

$$\Upsilon_{\mathbf{y}}(\mathbf{Hx}^* + \mathbf{b}) \simeq \frac{n}{2}. \tag{18}$$

Thus, we propose to write our restoration algorithm as:

$$\mathbf{x}^* = \begin{array}{cl} \arg\min & \|\mathbf{Wx}\|_1 \\ \text{subject to} & \Upsilon_{\mathbf{y}}(\mathbf{Hx} + \mathbf{b}) \leq \frac{n}{2} \\ & \mathbf{x} \in \mathbb{R}^n, \, \mathbf{x} \geq 0 \end{array} . \tag{19}$$

Using the same remark as previously, we also propose a slight modification of this constrained formulation. Back to (15), if we consider that $0 \log(0) = 0$, then $f(\mathbf{y}_\lambda) = 0$ for $\lambda = 0$. In consequence, the proposed constrained formulation of (5) writes:

$$\mathbf{x}^* = \begin{array}{cl} \arg\min & \|\mathbf{Wx}\|_1 \\ \text{subject to} & \Upsilon_{\mathbf{y}}(\mathbf{Hx} + \mathbf{b}) \leq \frac{m}{2} \\ & \mathbf{x} \in \mathbb{R}^n, \, \mathbf{x} \geq 0 \end{array} , \tag{20}$$

where m is given by (10). Solving problems (11) and (20) is not trivial and we propose an algorithm in the next section to solve them.

3. ALTERNATING DIRECTION METHOD

The algorithm we propose to use to solve (11) and (20) is based on the Alternating Direction Method (ADM). The idea of the ADM method is to split the original variable x into several variables and then to minimize the augmented Lagrangian following each splitted variable. The ADM method has been recently proposed to solve the unconstrained formulation (5) in [4]. Its presentation is however beyond the scope of this paper, and we refer the interested reader to [4] (and references therein). But this technique can also be used to solve our constrained Poissonian deconvolution problem, and we will directly give the resulting algorithm.

The functions (11) and (20) to minimize in $\mathbb{R}^n$ are coercive, lower semi-continuous and the constraint sets are non-empty closed convex sets. Then, for each problem, a solution $\mathbf{x}^*$ exists and an estimate of this solution can be computed by the algorithm 1. This algorithm introduces a relaxation parameter γ, which has to belong to $]0, \frac{\sqrt{5}+1}{2}[$

to ensure the convergence of the algorithm [10], and β which is the parameter which controls the constraint. Theoritically, the algorithm converges for any $\beta > 0$, but the speed of convergence strongly depends on this parameter. For our expriments, we will set $\beta = 10$ and $\gamma = 1$. Finally, Π_K is the orthogonal projection on the convex set K defined by:

$$K = \left\{ \mathbf{w} \in \mathbb{R}^n, \quad \mathbf{w}_i > 0, \quad \|(\mathbf{w} - \mathbf{y})/\sqrt{\mathbf{y}}\|_2^2 \leq m \right\}. \tag{21}$$

for the problem (11) and :

$$K = \left\{ \mathbf{w} \in \mathbb{R}^n, \quad \mathbf{w}_i > 0, \quad \Upsilon_{\mathbf{y}}(\mathbf{w}) \leq \frac{m}{2} \right\}. \tag{22}$$

for the problem (20). The orthogonal projection (21) can be seen as a projection on a weighted l^2-ball and can be found in [11] for example. So we do not detail any further the computation of this projection. We focus instead on the orthogonal projection (22) which is not obvious. Even if we can not give a closed-form solution of this projection, we propose an iterative scheme to solve it. We recall that the orthogonal projection problem is to find:

$$\mathbf{w}^* = \Pi_K(\mathbf{w}_0) = \begin{array}{cc} \arg\min & \frac{1}{2}\|\mathbf{w} - \mathbf{w}_0\|_2^2 \\ \text{subject to} & \Upsilon_{\mathbf{y}}(\mathbf{w}) \leq \frac{m}{2} \\ & \mathbf{w} \in \mathbb{R}^n \end{array}. \tag{23}$$

First notice that if $\Upsilon_{\mathbf{y}}(\mathbf{w}_0) \leq \frac{m}{2}$ then $\mathbf{w}^* = \mathbf{w}_0$. Otherwise, there exists $\delta \in]0, +\infty[$ such that:

$$\mathbf{w}^* = \begin{array}{cc} \arg\min & \frac{1}{2}\|\mathbf{w} - \mathbf{w}_0\|_2^2 + \delta\Upsilon_{\mathbf{y}}(\mathbf{w}) \\ \text{subject to} & \mathbf{w} \in \mathbb{R}^n \end{array}. \tag{24}$$

From [12], we get that:

$$\mathbf{w}^* = \frac{1}{2}\left[\mathbf{w}_0 - \delta + \sqrt{(\mathbf{w}_0 - \delta)^2 + 4\delta\mathbf{y}}\right] = \Phi(\delta). \tag{25}$$

The problem is thus to find δ such that $\Upsilon_{\mathbf{y}}(\Phi(\delta)) \leq \frac{m}{2}$. Let us define:

$$f(\delta) := \Upsilon_{\mathbf{y}}(\Phi(\delta)) - \frac{m}{2}. \tag{26}$$

It can be shown that f is a convex and decreasing function with respect to δ. In order to find the root of the function f, we propose to use a Newton method and we only need to find $f^{'}(\delta)$. Simply remark that from the composition of functions, we have:

$$f^{'}(\delta) = \frac{\mathbf{1}^T}{2}\left\{ \left[\frac{\delta - \mathbf{x}_0 + 2\mathbf{y}}{\sqrt{(\mathbf{x}_0 - \delta)^2 + 4\delta\mathbf{y}}} - 1\right] \left[1 - \frac{2\mathbf{y}}{\mathbf{x}_0 - \delta + \sqrt{(\mathbf{x}_0 - \delta)^2 + 4\delta\mathbf{y}}}\right]\right\}. \tag{27}$$

The resulting algorithm is then given in the algorithm 2. In all our simulations, we checked that 20 iterations of this scheme are more than enough to get a machine precision.

Algorithm 1: ADM to solve (11) and (20)

Data: Number of iterations N;
Starting points $\mathbf{x}^0 = \mathbf{y},\ \lambda_1^0 = 0,\ \lambda_2^0 = 0,\ \lambda_3^0 = 0$;
Value of the parameters $\gamma > 0$ and $\beta > 0$;
Result: $\mathbf{x}^N$ an estimated of the solution of (11) and (20).
begin
- **1.** $\mathbf{s}^0 = \mathbf{H}\mathbf{x}^0 + \mathbf{b}$
- **2.** $\mathbf{t}^0 = \mathbf{W}\mathbf{x}^0$
- **for** k **from** 0 **to** $N-1$ **do**
  - **3.** $\mathbf{x}^{k+1} = \max\left(\mathbf{x}^k + \frac{\lambda_1^k}{\beta}, 0\right)$
  - **4.** $\mathbf{u}^{k+1} = \Pi_K\left(\mathbf{s}^k + \frac{\lambda_2^k}{\beta}\right)$
  - **5.** $\mathbf{v}^{k+1} = \text{sign}\left(\mathbf{t}^k + \frac{\lambda_3^k}{\beta}\right)\max\left(\left|\mathbf{t}^k + \frac{\lambda_3^k}{\beta}\right| - \frac{1}{\beta}, 0\right)$
  - **6.** $\mathbf{z}^{k+1} = \mathbf{x}^{k+1} - \frac{\lambda_1^k}{\beta} + \mathbf{H}^*\left(\mathbf{u}^{k+1} - \mathbf{b} - \frac{\lambda_2^k}{\beta}\right) + \mathbf{W}^*\left(\mathbf{v}^{k+1} - \frac{\lambda_3^k}{\beta}\right)$
  - **7.** $\mathbf{x}^{k+1} = (\mathbf{H}^*\mathbf{H} + \mathbf{W}^*\mathbf{W} + \mathbf{I})^{-1}\mathbf{z}^{k+1}$
  - **8.** $\mathbf{s}^{k+1} = \mathbf{H}\mathbf{x}^{k+1} + \mathbf{b}$
  - **9.** $\mathbf{t}^{k+1} = \mathbf{W}\mathbf{x}^{k+1}$
  - **10.** $\lambda_1^{k+1} = \lambda_1^k + \beta\gamma\mathbf{x}^{k+1}$
  - **11.** $\lambda_2^{k+1} = \lambda_2^k + \beta\gamma\mathbf{s}^{k+1}$
  - **12.** $\lambda_3^{k+1} = \lambda_3^k + \beta\gamma\mathbf{t}^{k+1}$
- **end**

end

Algorithm 2: Newton method to solve (23)

Data: Number of iterations N;
A starting point $\delta^0 = 0$;
Result: $\mathbf{w}^*$ an estimated of the solution of (23).
begin
- **for** k **from** 0 **to** $N-1$ **do**
  - **Step 1.** $\delta^{k+1} = \delta^k - \frac{f(\delta^k)}{f'(\delta^k)}$
- **end**
- $\mathbf{w}^* = \frac{1}{2}\left[\mathbf{w}_0 - \delta^N + \sqrt{(\mathbf{w}_0 - \delta^N)^2 + 4\delta^N\mathbf{y}}\right]$

end

4. RESULTS

We compared the results obtained using the Poisson constrained formulation (20) and the ones obtained using the weighted Gaussian constrained formulation (11) with the TV regularization. Results on synthetic image are given on figure 1. On this image, the blur $\mathbf{H}$ is a 7×7 Gaussian kernel. For low intensity images like the image (a) on the figure 1, the weighted Gaussan approximation is not efficient, and the results given by the Poisson formulation (20) are clearly better (the image (d) on the figure 1 shows an improvement of 2.5 dB). The Poisson formulation (20) may however be slightly outperformed by the the weighted Gaussian constrained formulation (11) on high intensity images for which the Poisson distribution is well approximated by a weighted Gaussian distribution (not shown here). Results on a real image are given on the figure 2. On this image, $\mathbf{H}$ is a confocal microscope PSF whose model is described in [1]. The image retrieved with the proposed formulation (image (d) on the figure 2) is less smoothed than the one retrieved with the weighted Gaussian approximation. We can distinguish more easily the details of the cells of the object.

5. CONCLUSION

We have studied the problem of deconvolution of images corrupted by blur and Poisson noise and have proposed two new constrained formulations derived from recent regularizing estimation techniques. We have shown that the accuracy of these estimators can be im-

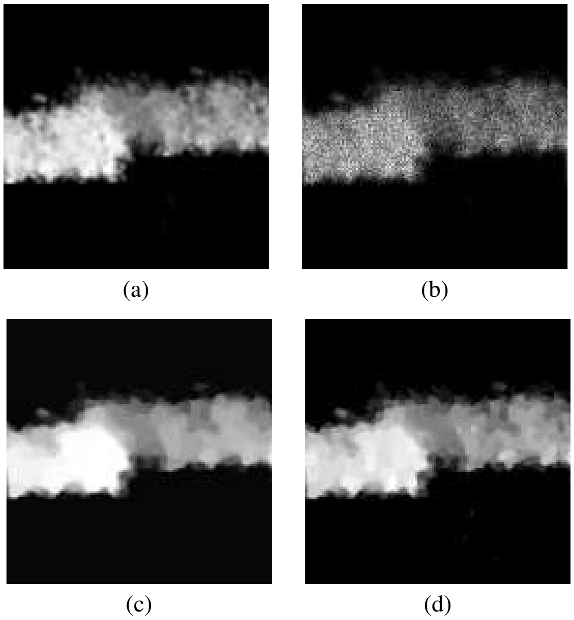

Fig. 1. Results of constrained formulations (11) and (20) on a low synthetic intensity image. (a) is the original image, (b) is the blur and noisy observation ($PSNR = 23.9\ dB$), (c) is the result with the weighted Gaussian constrained formulation (11) ($PSNR = 28.0\ dB$) and (d) is the result with the Poisson constrained formulation (20) ($PSNR = 30.5\ dB$). The original Gaussian constrained formulation (8) and the original Poisson constrained formulation (19) respectively give $PSNR = 22.8\ dB$ and $PSNR = 23.4\ dB$ (images not included here)

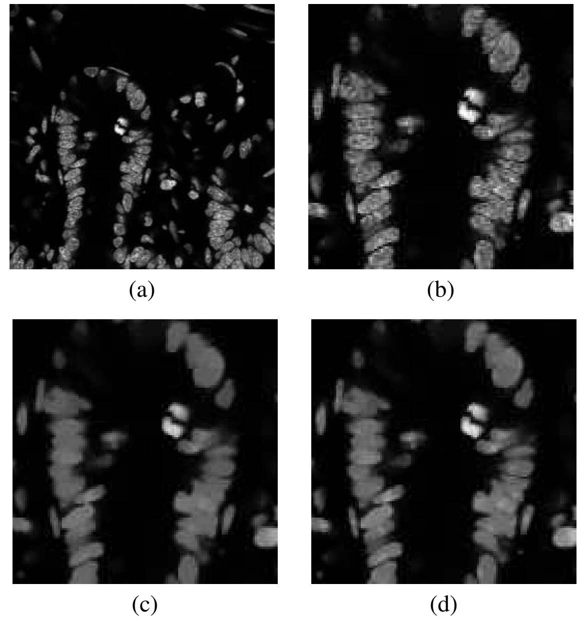

Fig. 2. Results of constrained formulations (11) and (20) on a $3D$ real biological image. (a) is the observed image (sample of mouse intestine), (b) is a zoom on the observation, (c) is the result with the weighted Gaussian constrained formulation (11) and (d) is the result with the Poisson constrained formulation (20)

proved by taking into account the properties of the Poisson statistics of the noise, and that these estimators can also be used to write constraint formulations which avoid the computational burden required by the regularizing parameter estimation in the unconstrained form. Finally, we have proposed an algorithm to solve the resulting optimization problems and their respective projections. A comparison of both formulations has been presented on synthetic and real data showing that the Poisson formulation is actually very promising for images with low intensity.

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