Cosparse Analysis Modeling

Sangnam Nam, Mike E. Davies, Michael Elad, Rémi Gribonval

Proposition 1. *Let Ω be an analysis operator in general position. Then, for* almost all $\mathbf{M}$ *(with respect to the Lebesgue measure), a necessary and sufficient condition for* (2) *with $\tau = 0$ to have a unique minimum is*

$$m \geq 2(d - \ell). \tag{3}$$

The proposition characterizes the collection of signals that can be recovered from (2) for the generic case. We also observe that by comparing with the uniqueness result for the sparse synthesis model, again in generic cases, $d - \ell$ is playing the role of the sparsity $k = \|\mathbf{z}_0\|_0$.

In the sparse synthesis model, armed with the fact that $\mathbf{x}_0$ has a sparse representation, one may try to recover/approximate $\mathbf{x}_0$ by greedily selecting atoms from $\mathbf{MD}$ and matching the observation $\mathbf{y}$. The Orthogonal Matching Pursuit (OMP) is an example of such approachs. Similarly, we can consider a greedy algorithm as an alternative to (2) in the cosparse analysis model. As such an algorithm, we propose the Greedy Analysis Pursuit (GAP). In this algorithm, contrary to the greedy algorithms for the synthesis model, we try to identify the rows of Ω that correspond to the *zeros* of $\Omega\mathbf{x}_0$. Moreover, again contrary to the typical greedy approach, GAP greedily—perhaps, generously—removes a row that is likely not to correspond the zeros from a collection of candidate rows that may do. This way, instead of building the support of $\mathbf{z}_0$ by growing the size of the support as is done in, for example, OMP, we are carving out unwanted rows in order to obtain the correct cosupport of $\mathbf{x}_0$, i.e., the set of the indices that correspond to the zeros of $\Omega\mathbf{x}_0$.

Finally, we run a synthetic experiment to demonstrate the effectiveness of the proposed algorith, GAP. Interestingly, we observe from the phase transition diagrams obtained from the experiment that GAP performs better than the analysis ℓ_1-minimization.

Keywords— Sparse Representation, Inverse Problems, Algorithms, Cosparse Analysis Model

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