Change Detection in Sequences of Images by Multifractal Analysis

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ABSTRACT

In this work, we propose a multifractal approach to the problem of change detection in image sequences, such as registrated remotely sensed images of the same scene or sequences of medical images. We show that the multifractal analysis of images – based on a modelisation of the two-dimensional signal as measure – can be of great help if we want to detect changes without any *a priori* knowledge of the objects to be extracted. We first present a simple change detection method based on the classical multifractal analysis of images w.r.t. the Lebesgue measure. We then describe an improved method based on the analysis of images w.r.t. a reference measure, which in this case is the first image of the sequence. We finally show some results on real data.

1. INTRODUCTION

An important application of image analysis is to provide means to monitor a scene over a period of time and to detect changes in the content of the scene. In photo-interpretation, change detection consist of finding significant differences – most of the time man-made changes in opposition to natural and/or seasonal changes – between the new image and site models derived from the older images. In biomedical imagery, the aim is to control diseases evolution and its cure. In both contexts, the existing methods require a priori knowledge of the objects to be extracted in the new image, see [1, 2]. Here, we propose a multifractal approach to the change detection problem, which does not require such a assumption. In section 2, we recall some fundamental definitions of the multifractal theory. We describe in section 3 the way we apply it to image segmentation and to change detection. Some results of image sequence analysis w.r.t. the Lebesgue measure are also presented. We construct in section 4 a new way to analyse sequences of images w.r.t. a analysing measure and show some results, before concluding and proposing some desirable extensions.

2. FUNDAMENTALS OF MULTIFRACTAL THEORY

We define here and briefly recall some fundamental facts about the multifractal theory. More rigorous and complete definitions can be found in [3, 4, 5, 6]. Let a μ Borel probability measure laid upon a compact set $\mathcal{P}$. For each point x in $\mathcal{P}$, define the *Local Singularity* coefficient as:

$$\alpha(x) = \lim_{\delta \to 0} \frac{\log \mu(\mathcal{B}_\delta(x))}{\log \delta}, \tag{1}$$

where $B_\delta(x)$ is an open-ball of diameter δ centered on the point x and when the limit exist. $\alpha(x)$ is often called the *Hölder* coefficient. It reflects the local behavior of the measure μ around x. Points bearing the same coefficient can be gathered into sets, named *Iso-Local Singularity* sets, defined as follows:

$$E(\alpha) = \{x : \alpha(x) = \alpha\}. \tag{2}$$

As regards the preceeding definition, we may need a refinement to take into account some degenerate cases, which is Iso-Local Singularity sets *at the ϵ precision*:

$$E_\epsilon(\alpha) = \{x : \alpha - \epsilon \leq \alpha(x) < \alpha + \epsilon\}. \tag{3}$$

To characterize those sets, it is relevant to use a notion of set dimension, known as the *Hausdorff* dimension:

$$\begin{aligned} \dim_{\mathcal{H}} E &= \inf\left\{ s : \liminf_{\delta \to 0} \sum_{i=0}^{\infty} |E_i|^s = 0 \right\} \qquad (4) \\ &= \sup\left\{ s : \liminf_{\delta \to 0} \sum_{i=0}^{\infty} |E_i|^s = \infty \right\}, \end{aligned}$$

where $\{E_i\}_{1 \leq i \leq \infty}$ is a δ-cover of E: $E \subset \bigcup_{i=0}^{\infty} E_i$, $|E_i| < \delta$, $E_i \subset \mathcal{P}$, $\forall i$. Finally, define the following quantity:

$$f(\alpha) = \dim_{\mathcal{H}} E(\alpha). \tag{5}$$

The description $(\alpha, f_h(\alpha))$ is called the *Local Singularity* spectrum (sometimes known as the *Hölder* or *Hausdorff* spectrum) of the multifractal measure μ.

There are other possible multifractal descriptions of a measure, namely the *Large Deviation* spectrum $(\alpha, f_g(\alpha))$ and the *Legendre Transform* spectrum $(\alpha, f_l(\alpha))$, but none of these notions will be used in this paper (for further details on f_g or f_l see [3, 4]).

3. APPLICATIONS: IMAGE SEGMENTATION AND CHANGE DETECTION

3.1. Application to image segmentation

It is quite straightforward to apply multifractal tools to image analysis. Following equation (1), points are naturally

type	*name*	$\coprod$	$f(i,j)$	*parameter*
mix	sum	$\sum$	$g(i,j)$	none
altimetric	max	max	$g(i,j)$	none
	min	$1/\min$	$g(i,j)$	none
planimetric	iso	$1_{[-\Delta,\Delta[}()$	$\delta(i,j)$	Δ
	selfsim	$\exp(\frac{-()^2}{\gamma^2})$	$\delta(i,j)$	γ

Table 1. definition and parameters of the capacities.

associated to pixels of the images, open-balls to windows centered on each pixel, measures to functions of grey level intensities.

A first natural choice is to define the measure μ as the sum of the grey level intensities of pixels (i,j) contained in a window centered on pixel (x,y). This measure is of theoretical great interest, but is not sufficient for a fine and complete description of the image. Other functions of grey level intensities of the image can be defined. These functions are no longer measures but only capacities (for a proper founding of multifractal analysis of capacities, see [4]).

We introduce those capacities and summarize their definitions and parameters in table 1. The general form of these capacities is $\mu_{name}(x,y) = \coprod_{(i,j)\in\mathcal{B}_\delta(x,y)} f(i,j)$, where $\coprod$ is an given operator and $f(i,j)$ is either $g(x,y)$, which denotes the grey level intensity of pixel (x,y), or $\delta(i,j) = g(x,y) - g(i,j)$. Δ is a coefficient of under-quantization of the image, and γ can be seen as the image noise standard-deviation when the noise is of finite variance.

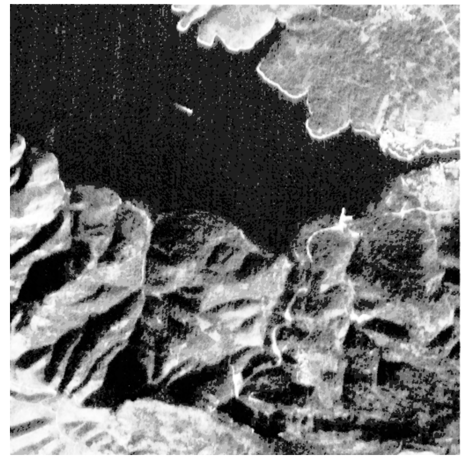
We distinguish three types of functions of grey level intensities according to their respective properties, the first one being the already defined "sum" measure. The two following ones are the "max" and "min" capacities leading to Local Singularity coefficients reflecting the sharpness of the image in the neighborhood of the pixel (x,y). We call them *altimetric* capacities. The two last capacities, which require the extra parameters Δ and γ are said to be *planimetric* since they are sensitive to the spatial distribution of the measure. "sum" is a *mix* measure since it responds to both sharpness and spatial distribution of the measure.

All pixels having the same Local Singularity coefficient can be grouped together to form a binary image of Iso-Local Singularities, as defined in (2) and (3) . These binary images can be characterized by a "fractal" dimension, defined in (4).

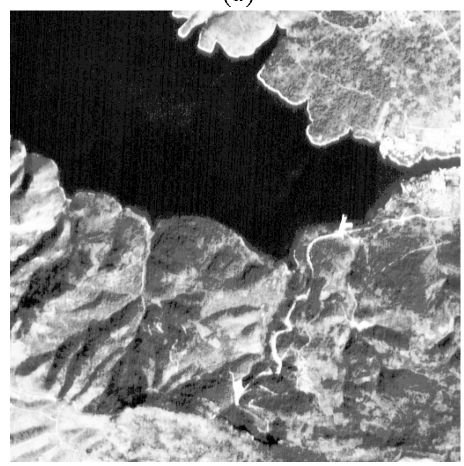
The multifractal description of (5) has the advantage of being at the same time *local* (with coefficients $\alpha(x,y)$) and *global* (with graph $(\alpha, f_h(\alpha))$), see graph (i) of fig. 2., corresponding to image (a) of fig. 1. analyzed w.r.t. Lebesgue measure. Thus, it appears to be a good way to solve the problem of image segmentation, as indicated in [7].

3.2. Application to change detection

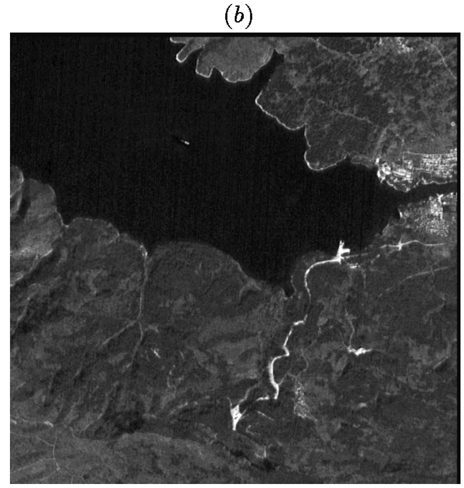
Such a description provides us with a powerful tool to detect changes in sequences of images and to extract and analyze those changes. If a change occurs in an incoming image, it is reflected in the global description provided by the graph of the multifractal spectrum. The abscissa α of the part of the spectrum $(\alpha, f_h(\alpha))$ that has changed allows the extraction



(a)



(b)



(c)

Figure 1. From top to bottom: (a) **analyzed image,** (b) **analyzing image et** (c) **absolute difference (pixel to pixel) between the two registrated images.**

of the binary image corresponding to this detected change.

An important topic in this approach is the definition and the computation of an *optimal* measure, which does not react too much to variation of the noise but only to effective changes. The "selfsim" measure represent a first step towards taking this requirement into account.

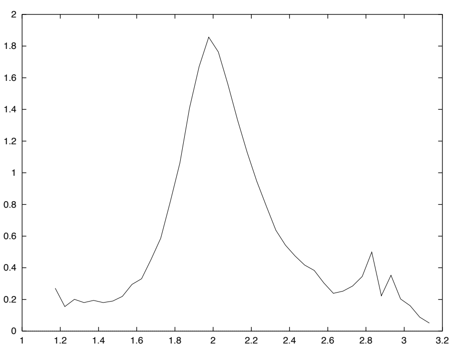
4. ANALYSING MEASURE METHOD

Another way to improve the results is to replace the analyzing measure (that means the measure in the denominator of expression (1), which in that case is the Lebesgue measure: $\mu(\mathcal{B}_\delta(x)) \propto \delta$) by another reference measure. We found it useful to be a reference image such as the first image of the sequence to be analyzed.

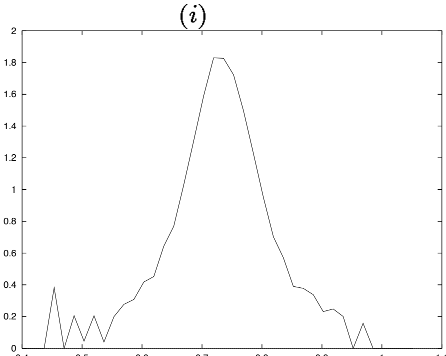
Thus, the corresponding Local Singularity coefficient *w.r.t. a reference measure* can be written as follows :

$$\alpha(x) = \lim_{\delta \to 0} \frac{\log c(\mathcal{B}_\delta(x))}{\log \mu(\mathcal{B}_\delta(x))}, \tag{6}$$

where μ is the analyzing measure and c the analyzed capacity and when the limit exist. Interested readers can refer to [4] for a complete description of *Analyzing Measure Method*. Analyzing the new incoming image w.r.t. the reference image emphasizes the changes: the multifractal spectrum will reflect the importance of the changes between the two compared images.



(i)

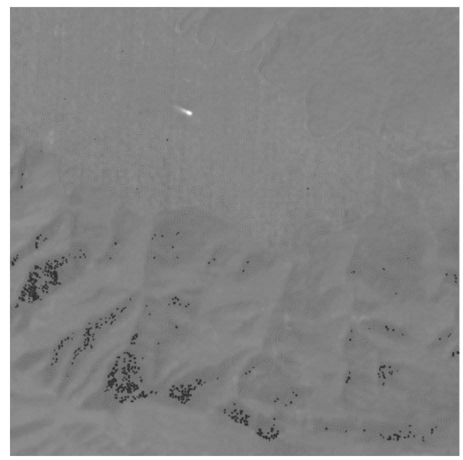


(ii)

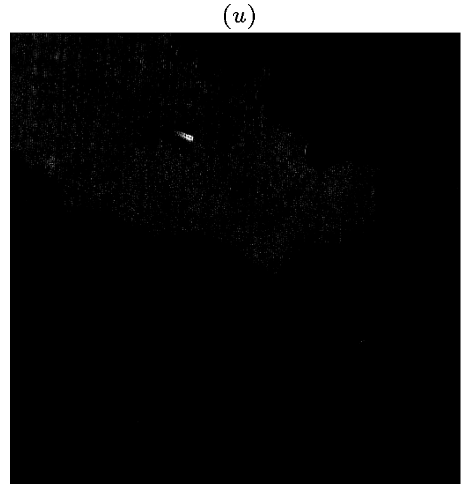
Figure 2. Local Singularity spectrums of: (i) analysed image with "sum" measure w.r.t. the Lebesgue measure, (ii) analysed image with "sum" measure w.r.t analysing image.

In our experiments, its main mode usually corresponds to the noise difference between images, whereas secondary modes correspond to effective changes. On graph (ii) of figure 2. can be seen the spectrum of image (a) analysed w.r.t. image (b). See image (u) of fig. 3. for Local Singularity coefficients image and image (v) for resulting extracted image of detected changes, corresponding to an Iso-Local Singularity image. As can be seen on image (c) of figure 1. and image (v) of figure 3., the extracted change using the multifractal analysis is much more relevant than the simple absolute difference the two images, without any geometric corrections.

Multifractal tools shows promises in the field of change detection. Some extensions of this work could be the use of Large Deviation and Legendre Transform spectrum extended to the Analyzing Measure Method context. It could be also interesting to define *optimality constraints* not only on the measures, but also on the corresponding spectrum.



(u)



(v)

Figure 3. (u) Local Singularity coefficients image and (v) extracted image of detected change between analysed and analysing image.

The authors wish to thank J. Degos, research engineer at Alcatel ISR, for his helpful advices and his great knowledge of the photo-interpretation context.

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