Two Dimensional Linear Phase Multiband Chebyshev FIR Filters

Vinay KUMAR and Sunil BHOOSHAN

Abstract: *In this paper we discuss a method to design 2 dimensional (2D) FIR filters with the help of Chebyshev polynomials. The polynomial is first converted into a 2D function and later a mapping function is used to design the filters. By the proposed technique one can design filters with various types of passband.*

Keywords: *Chebyshev Polynomials, FIR Filters*

1. INTRODUCTION

For image processing operations filtering is one of the fundamental tasks. There are various methods available in the literature to design such type of filters. In the present paper we are going to present a technique using which one can design equiripple filters with user defined stop band levels. The issue Chebyshev polynomial based filter design is addressed by various authors Kamp and Thiran (1975); Harris and Mersereau (1977); Hu and Rabiner (1972); Lim (1990). Hu and Rabiner (1972) introduced a linear programming based technique. Fiasconaro's Fiasconaro (1975) reduced the amount computations associated with the method developed by Hu and Rabiner (1972). An equiripple linear phase FIR filter design technique was discussed by McClellan (1973). Lu (2002) and Lu and Hinamoto (May, 2005) introduced methods based on semidefinite programming (SDP) and sequential quadratic programming (SQP). These methods work quite well except for the fact that the design complexity becomes rather high even for filters of moderate order.

The design technique we are discussing in the present paper produces linear phase 2D FIR filters. The present method can be used to design filter having a variety of pass bands. The parameters involved in the design of these filters are few, therefore, computation time is also less. Another advantage of the present methods is the ease of understanding and application of present method. The transformation involved in the present discussion is much easier to understand and apply than discussed in Mecklenbrauker and Mersereau (1976); R.M. Mersereau and T.F. Quatieri (1976). Through some design examples we provide the effectiveness of the procedure.

Manuscript received October 08, 2010.

2. DESIGN

Design procedure for the FIR filters based on Chebyshev polynomials is discussed in the present section.

One dimensional Chebyshev polynomials are represented by

$$T_m(x) = \begin{cases} \cos\left(m\cos^{-1}x\right) & -1<|x|<1 \\ \cosh\left(m\cosh^{-1}x\right) & |x|>1 \end{cases} \tag{1}$$

where, m represents the order of the filter.

To design a 2D FIR filter we first have to represent the Chebyshev polynomial in 2D domain. We replace x with a new variable ρ and Chebyshev polynomial of Equation (1) becomes

$$T_m(\rho) = \begin{cases} \cos\left(m\cos^{-1}\rho\right) & -1<|\rho|<1 \\ \cosh\left(m\cosh^{-1}\rho\right) & |\rho|>1 \end{cases} \tag{2}$$

where ρ represents the mapping function; or in other words, ρ is mapped onto two variables by $\rho^2 = x^2 + y^2$, where x–y is the plane of reference.

Some of these polynomials, Equation (2), are represented in Figure 2.

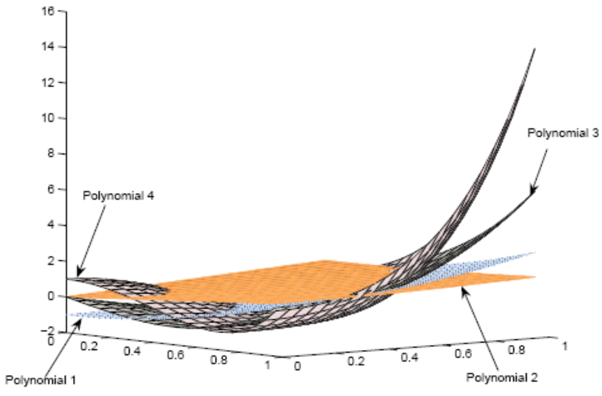

Fig. 1. Chebyshev Polynomials in 2 Dimension.

These polynomials will be used to design the required FIR filters. To convert these polynomials into frequency domain (ω-domain); or to get the corresponding filter, we use a special transformation function given by Bhooshan and Kumar (21-24 April, 2008, 2006, 2007)

$$\rho = \rho_0 \cos\left(\frac{\omega}{2}\right) \quad -\pi \le \omega \le \pi \tag{3}$$

where ρ_0 is the maximum value of ρ. Following paragraph discusses the transformation for better understanding of the procedure.

When we consider the value of $\omega = 0$, it gives $\rho = \rho_0$; that is, maximum value of ρ. As we increase the value of ω from 0 to $\pi/2$ the value of ρ comes out to be 0 and when the value of ω is increased further to π, the value of ρ becomes ρ_0. In other words, as the value of ω increases from 0 to π the value of ρ decreases from ρ_0 to $-\rho_0$. Similarly, when ω varies from $-\pi$ to 0, ρ will vary from $-\rho_0$ to ρ_0. Therefore, we conclude that this transform converts the polynomial to low pass filter; that is, lower values of the polynomial, variable ρ, will be converted to higher values in filter characteristics, variable ω, and vice versa.

After transformation the Chebyshev polynomial becomes

$$T_m(\omega) = \begin{cases} \cos\left[m\cos^{-1}\left\{\rho_0\cos\left(\omega/2\right)\right\}\right] & -1 < |\rho| < 1 \\ \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\omega/2\right)\right\}\right] & |\rho| > 1 \end{cases} \tag{4}$$

ω is a mapping function which represents 2D frequency domain, or $\omega^2 = u^2 + v^2$, where u and v represent axis in frequency domain. Therefore, Equation (4) becomes

$$T_m(u,v) = \begin{cases} \cos\left[m\cos^{-1}\left\{\rho_0\cos\left(\frac{\sqrt{u^2+v^2}}{2}\right)\right\}\right] & -1 < |\rho| < 1 \\ \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\frac{\sqrt{u^2+v^2}}{2}\right)\right\}\right] & |\rho| > 1 \end{cases} \tag{5}$$

Equation (5) represents transfer function of 2D linear phase FIR filter.

The value of ρ depends on the order of the filter and attenuation required by the user for side bands. If b represents the attenuation of the side band with respect to pass band.

Following the procedure given in E.C. Jordan and K.G. Balman (1968) we can state that the value of ρ_0 is given by

$$\rho_0 = \cosh\left(\cosh^{-1} b/m\right) \tag{6}$$

where, m is the order of the filter, and b is the absolute value of the attenuation and is given by

$$b = 10^{(\text{attenuation in dB}/20)} \tag{7}$$

3. CALCULATION OF ω_s AND ω_p

Suppose the attenuation of side bands is b, absolute value of side band attenuation. Suppose the value of stop band is represented by ω_s in Equation (4) then we may rewrite the equation as follows

$$T_m(\omega) = \begin{cases} \cos\left[m\cos^{-1}\left\{\rho_0\cos\left(\omega_s/2\right)\right\}\right] & -1 < |\rho| < 1 \\ \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\omega_s/2\right)\right\}\right] & |\rho| > 1 \end{cases} \tag{8}$$

at the end of the stop band the value of T_m will be b thus Equation (7) will become E.C. Jordan and K.G. Balman (1968)

$$b = \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\omega_s/2\right)\right\}\right] \quad |\rho| > 1 \tag{9}$$

by rearranging the terms we get

$$\omega_s = 2\cos^{-1}\left[1\Big/\left\{\cosh\left(1/m\cosh^{-1} b\right)\right\}\right] \tag{10}$$

Similarly at the start of the stop band value of T_m will be b/sqrt2. Therefore, Equation (16) will be

$$b/sqrt2 = \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\omega_p/2\right)\right\}\right] \quad |\rho| > 1 \tag{11}$$

after rearranging the terms we get

$$\omega_p = 2\cos^{-1}\left[\frac{\cosh\left\{(1/m)\cosh^{-1}\left(b/\sqrt{2}\right)\right\}}{\cosh\left(1/m\cosh^{-1} b\right)}\right] \tag{12}$$

The design procedure discussed above produces a filter with pass band centered at (0, 0). To make sure that the user has control over the placement of the pass band we change the Chebyshev polynomial of Equation (5) to

$$T_m(u,v) = \begin{cases} \cos\left[m\cos^{-1}\left\{\rho_0\cos\left(\frac{\sqrt{(u-u_0)(v-v_0)}}{2}\right)\right\}\right] & -1 < |\rho| < 1 \\ \cosh\left[m\cosh^{-1}\left\{\rho_0\cos\left(\frac{\sqrt{(u-u_0)(v-v_0)}}{2}\right)\right\}\right] & |\rho| > 1 \end{cases} \tag{13}$$

where, u_0 and v_0 represents location of center of the pass band. By recursively applying the above formula we can design a multi band filter also.

Another constraint related with the filter design discussed above is the size of pass band. The size of pass band is not user dependent to make it so we introduce another variable α. This variable is multiplied with ρ. Therefore, the updated Chebyshev polynomial will be

$$T_m(\alpha\rho) = \begin{cases} \cos\left(m\cos^{-1}\alpha\rho\right) & -1 < |\rho| < 1 \\ \cosh\left(m\cosh^{-1}\alpha\rho\right) & |\rho| > 1 \end{cases} \tag{14}$$

New values of ω_s and ω_p can easily be calculated as before and they come out to be

$$\omega_s = 2\cos^{-1}\left[1\Big/\alpha\left\{\cosh\left(1/m\cosh^{-1} b\right)\right\}\right] \tag{15}$$

value of ω_p remains the same.

Few examples discussed in the next section will clarify the procedure discussed in the present section.

4. EXAMPLES

To design a filter user have to provide order of the filter, *m*, value of α, value of attenuation of side bands and the coordinates of the center of the pass band.

4.1. Example I

Let us suppose that user needs to design a filter with its order 10, side band 40dB down, α=1 (no change in the size of passband) and center of the passband to be at (0,0). The values of ω_s and ω_p comes out to be 1.0133 and 0.3607, respectively.

The low pass and high pass filters for these values are shown in Figures 2 and 3, respectively.

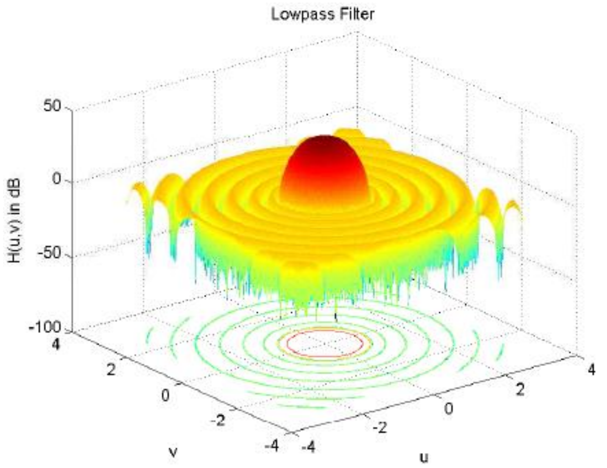

Fig. 2. 10th order lowpass filter with value of α = 1, sidebands 40dB down and centered at (0,0).

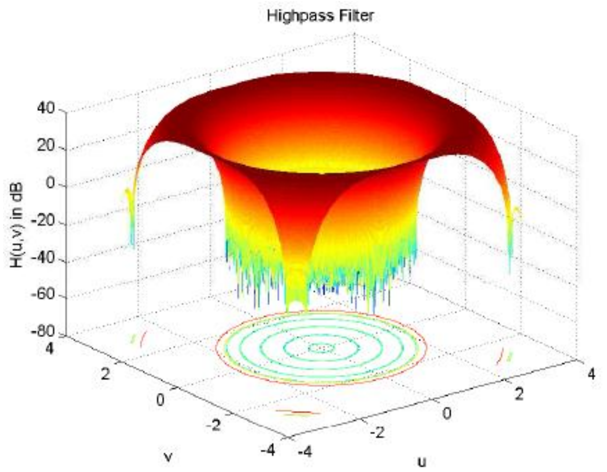

Fig. 3. 10th order high pass filter with value of α = 1, sidebands 40dB down and centered at (0,0).

If we shift the center of the passband to (π/2, π/2) the lowpass and high pass filters with all other values unchanged become as shown in Figures 4 and 5, respectively.

Let us extend the example with a value of α other than 1. Suppose we take the value of α equal to 1.3 and we shift the center of the passband to (π/2, π/2) the above lowpass and high pass filters with all other

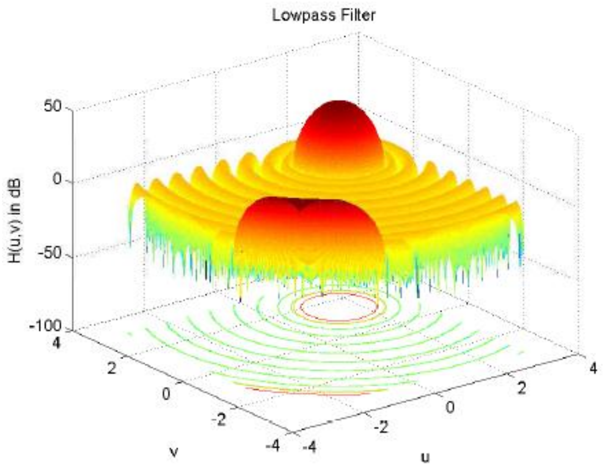

Fig. 4. 10th order lowpass filter with value of α = 1, sidebands 40dB down and centered at (π/2, π/2).

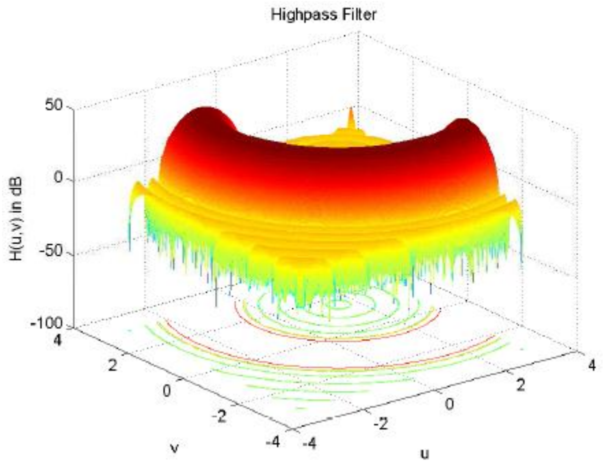

Fig. 5. 10th order high pass filter with value of α = 1, sidebands 40dB down and centered at (π/2, π/2).

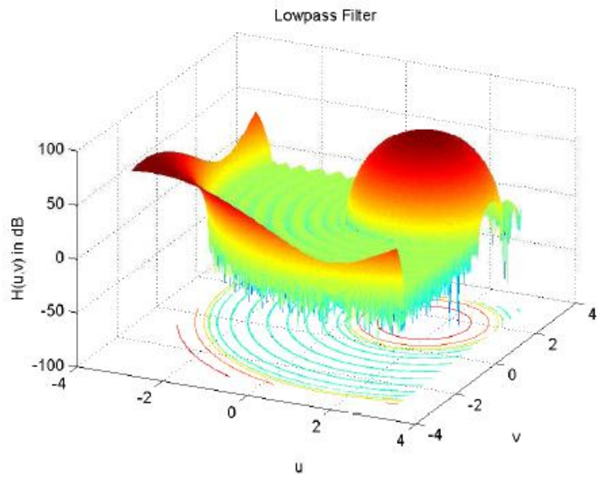

Fig. 6. 10th order lowpass filter with value of α = 1.3, sidebands 40dB down and centered at (π/2, π/2).

values unchanged become as shown in Figures 6 and 7, respectively.

Various figures in the present example show how various parameters are interdependent and they change

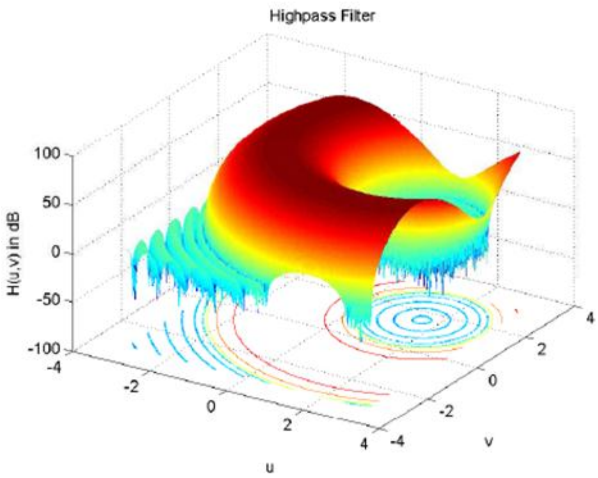

Fig. 7. 10th order highpass filter with value of α = 1.3, sidebands 40dB down and centered at (π/2, π/2).

the characteristics of the filter. In the next example we design a filter with non-circular passband.

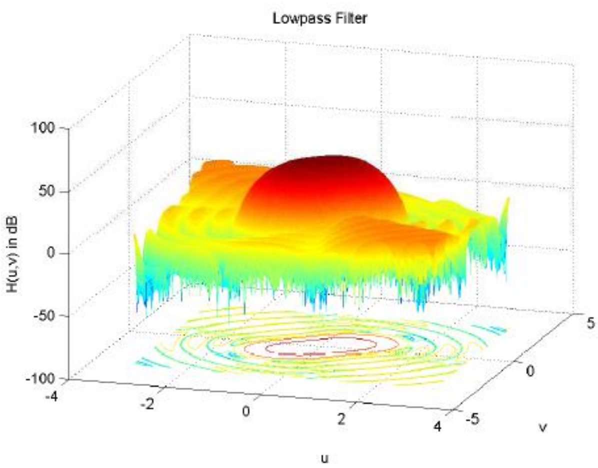

Fig. 8. 10th order lowpass filter with value of α=1, sidebands 40dB down.

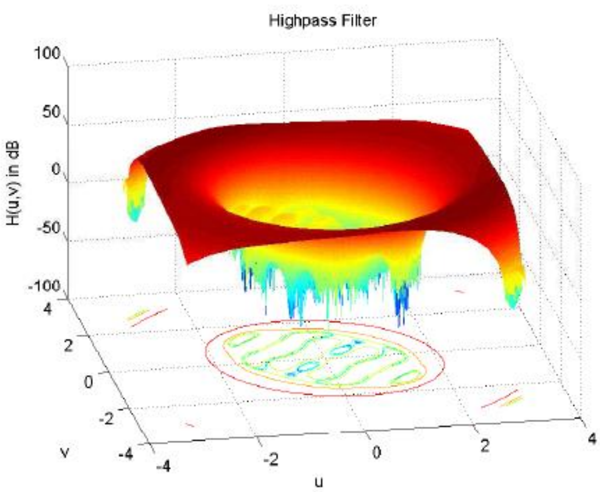

Fig. 9. 10th order highpass filter with value of α = 1, sidebands 40dB down.

4.2. Example II

In this example we consider that a bank of 7 filter is constituted with locations of center of the passband at (0,0) (0.1,0.1) (0.2,0.2) (0.3,0.3) (-0.1,-0.1) (-0.2,-0.2) (-0.3,-0.3), respectively, other values remains same as in the previous case. The filter characteristics of lowpass and highpass filters are shown in Figures 8 and 9, respectively.

To get the detailed view of the characteristics we show them on a frequency scale varying between -2π to 2π and are shown in Figures 10 and 11, respectively.

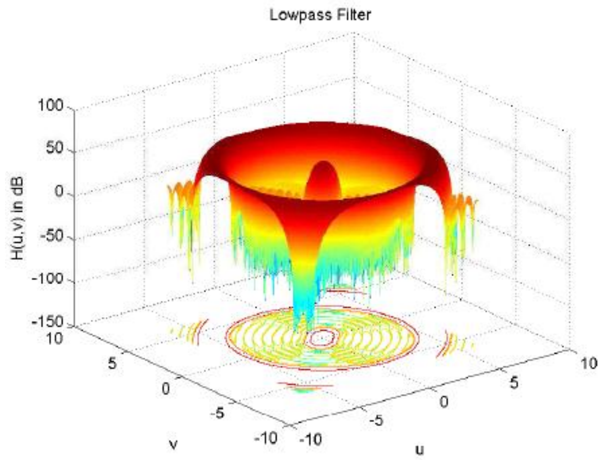

Fig. 11. 10th order lowpass filter with value of α = 1, sidebands 40dB down.

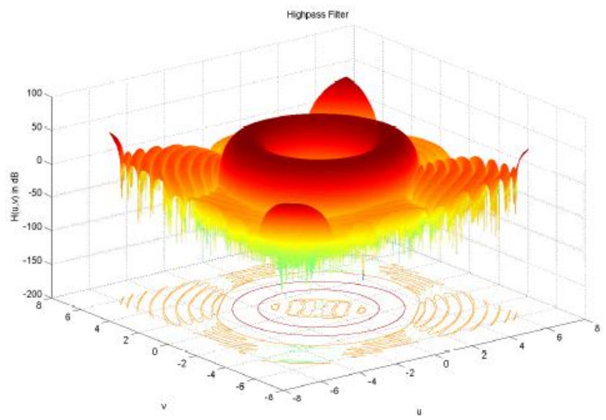

Fig. 10. 10th order highpass filter with value of α = 1, sidebands 40dB down.

From Figures 10 and 11 it is clear that the shape of the pass band is non-circular.

Next we create a multi band filter with pass bands centered at (0,0), (π/2, π/2), (-π/2, π/2), (π/2, -π/2), (-π/2, $-\pi$/2) while other values the same. The results of lowpass and highpass filters are shown in Figures 12 and 13, respectively.

5. APPLICATION

If we pass image shown in Figure 14 through a high pass filter of order 10 with values of α=1, Figure 15 and Figure 16, and passband centered at the origin. It is clear from the figures that when the value of alpha

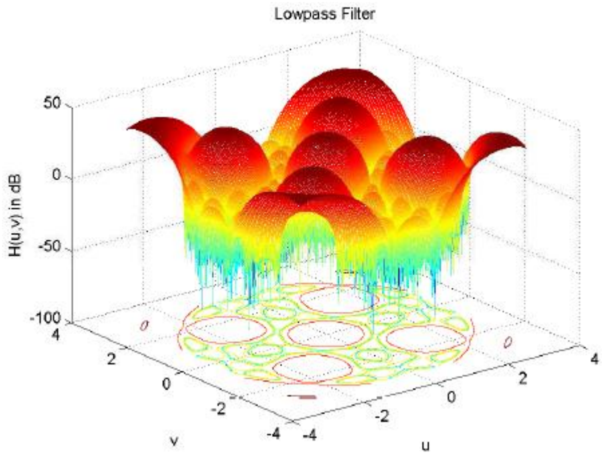

Fig. 12. 10th order lowpass filter with value of $\alpha = 1$, sidebands 40dB down.

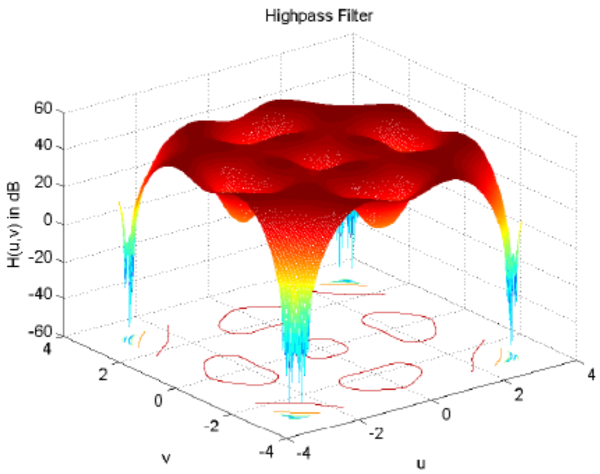

Fig. 13. 10th order highpass filter with value of $\alpha = 1$, sidebands 40dB down.

is increased the passband band becomes wider. From the fundamentals of image processing we know that

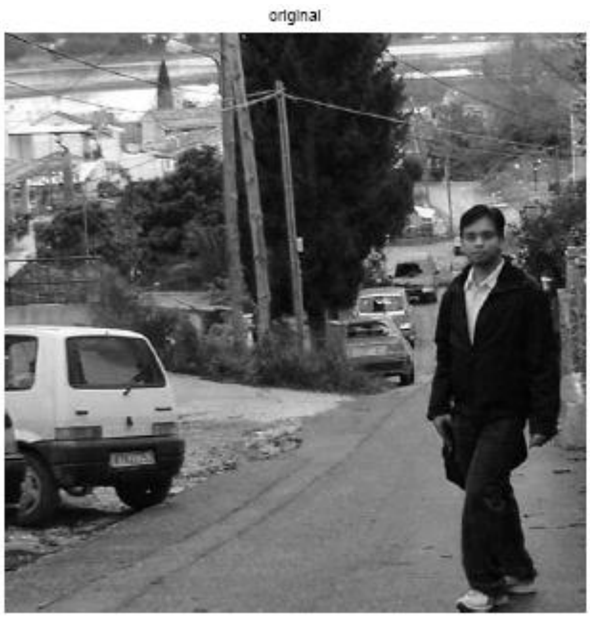

Fig. 14. Original image.

with an increase in size of passband of a highpass filter it allows to pass lower frequencies through. Thus if we compare results shown in Figures 15 and 16, respectively, we conclude that in Figure 15 together with edges (high frequency components in an image) some overview of the picture (low frequency components) is also visible.

6. CONCLUSION

A design procedure for 2D FIR filters is presented. The procedure uses 2D Chebyshev polynomials for creating image filters. The advantage of this type of filter is its ease of design and application. Although only few designs are presented in the paper, one can

Fig. 15. Image passed through a 10th order highpass filter with value of $\alpha = 1$, sidebands 40dB down.

Fig. 16. Image passed through a 10th order highpass filter with value of $\alpha = 1.3$, sidebands 40dB down.

design a range of different types of filters. As we increase the order of the filter the passband of the resulting filter becomes narrower.

REFERENCES

1. Bhooshan, S. and Kumar, V. (2006). A polynomial approach towards the design of linear phase fir filters.
2. Bhooshan, S. and Kumar, V. (2007). A novel approach towards the design of chebyshev fir filter with linear phase. *WSEAS TRANSACTIONS ON SIGNAL PROCESSING*, 3, Issue 2, 179–185.
3. Bhooshan, S. and Kumar, V. (21-24 April, 2008). A new method to design linear phase fir filter.
4. E.C. Jordan and K.G. Balman (1968). *Electromagnetic Waves and Radiating Systems*. Prentice Hall of India Pvt. Ltd., 2 edition.
5. Fiasconaro, J.G. (1975). Two-dimensional nonrecursive digital filters. *Picture Processing and Digital Filters, T. S. Huang*, Springer-Verlag.
6. Harris, D.B. and Mersereau, R.M. (1977). A comparison of algorithms for minimax design of two-dimensional linear phase fir digital filters. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, ASSP-25, 492–500.
7. Hu, J.V. and Rabiner, L.R. (1972). Design techniques for two-dimensional digital filters. *IEEE Transactions on Audio Electroacoustics*, AU-20, 249–257.
8. Kamp, Y. and Thiran, J.P. (1975). Chebyshev approximation for two dimensional nonrecursive digital filters. *IEEE Transactions on Circuits and Systems*, CAS-22, 208–218.
9. Lim, J.S. (1990). *Two-Dimensional Signal and Image Processing*. Englewood Cliffs, NJ: Prentice-Hall.
10. Lu, W.S. (2002). A unified approach for the design of 2-d digital filters via semidefinite programming. *IEEE Transactions on Circuits System*, 49, 814–826.
11. Lu, W. and Hinamoto, T. (May, 2005). A new minimax design for 2-d fir filters with low group delay.
12. McClellan, J.H. (1973). The design of twodimensional digital filters by transformations.
13. Mecklenbrauker, W.F.G. and Mersereau, R.M. (1976). Mcclellan transformations for twodimensional digital filtering: Ii. implementation. *IEEE Transactions on Circuits System*, CAS-23, 414–422.
14. R.M. Mersereau, W.F.G.M. and T.F. Quatieri, J. (1976). Mcclellan transformations for twodimensional digital filtering: I.design. *IEEE Transactions on Circuits System*, CAS-23, 405–414.

Vinay KUMAR
Sunil BHOOSHAN

Signal Processing Group
ECE Department
Jaypee University of Information Technology
Waknaghat, Solan173215, INDIA
E-mail: vinay.kumar@juit.ac.in